JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-16, No.-1, January (2021) pp 19-26 ISSN (Print) 0973-8975

COMPOUND PROPOSITIONAL LAW FOR LOGICAL EQUIVALENCE, TAUTOLOGY AND CONTRADICTION

Umair Khalid Qureshi[1], Parivish Sami Lander[2], Shahzad Ali Khaskheli[3], Manzar Bashir Arain[4], Zubair Ahmed Kalhoro[5], Syed Hasnain Ali Shah[6], Amir Khan Mari[7], Saifullah Bhatti[8]

[1]Department of Business Administration, Shaheed Benazir Bhutto University, Sanghar, Sindh, Pakistan

[2,3,4,6,7,8]Department of Information Technology, Shaheed Benazir Bhutto University, Sanghar, Sindh, Pakistan

[5]Institute of Mathematics and Computer Science, University of Sindh, Jamshoro, Sindh, Pakistan

Corresponding Author: Umair Khalid Qureshi

umair.khalid_sng@sbbusba.edu.pk

https://doi.org/10.26782/jmcms.2021.01.00002

(Received: December 5, 2020; Accepted: January 13, 2021)

Abstract

This paper presents a Compound Propositional Law for Logical Equivalence, Tautology and Contradiction. The proposed Law is developed with the help of negation, disjunction, conjunction, exclusive or, conditional statement and bi-conditional statement. The idea of research is taken from de-Morgan law. This proposed law is important and useful for Logical Equivalence, Tautology and Contradiction for the research purpose because these are the rare cases in the field of research. This article aims to help readers understand the compound proposition and proposition equivalence in conducting research. This article discusses propositions that are relevant for proposition equivalence. Six main compound propositions are distinguished and an overview is given in the article. Hence, it is observed from the result and discussion that the compound proposition law is a good achievement in discrete structure for the logical Equivalence, Tautology and Contradiction purpose.

Keywords: Proposition Equivalence, Compound Proposition, Truth Table, Result Analysis, Logical Symbols.

I. Introduction

Discrete mathematics is a branch of applied mathematics that deals with arrangements of discrete objects which are separated from each other, such as sets, functions, relations, matrix algebra, combinatory and finite probability, graph theory, finite differences and recurrence relations, logic, mathematical induction, and algorithmic thinking [II], [X]. It has many applications in computer science and software engineering, such as how to describe a static structure and dynamic behavior of the software system, and how to verify a software specification by logic statements, etc. In computer science, it can help people understand and construct a computer system to solve a given problem. Discrete mathematics has applications to all fields of computer science, it is used extensively in telecommunications and information processing. We learn concepts associated with them, properties and relationships among them. Because of this variety of topics, it is perhaps preferable to study all these contents of discrete mathematics. They concluded that the Discrete mathematics material should be taught with examples and applications from computer science because the applications would enhance the understanding of Discrete mathematics [IV], [VI], [V], [III]. Similarly, in this research, we focused on discrete mathematics' main topic of Logic. Logic is a language for reasoning for some assertion. It is a collection of rules which can be used when doing logical reasoning. Human reasoning has been observed over centuries from at least the times of Greeks, and patterns appearing in reasoning have been extracted and abstracted. The foundation of the logic was laid down by a British mathematician Boole in the middle of the 19th century. Logic is interested in the true or false of statements, and how the truth/false of a statement can be determined from other statements. We use symbols to represent arbitrary statements so that the results can be used in many similar but different situations, so logic can promote clarity of thought and eliminate ambiguity and mistakes. There are various topics in logic such as a proposition, tautology, contradiction, consistency, logical equivalence predicate, quantifiers, etc. In this research, we are concern with Logical Equivalence, Tautology and Contradiction, and which we are discussed one by one. Firstly tautology, a tautology is a compound statement that is true for every value of the individual statements [IX], [I]. The word tautology is derived from the Greek word ‘tauto’ means ‘same’ and ‘logy’ means ‘logic’. A compound statement is made with two more simple statements by using some conditional words such as ‘and’, ‘or’, ‘not’, ‘if’, ‘then’, and ‘if and only if’. Now contradiction, contradiction is just the opposite of tautology or you can it contradict the tautology statement. When a compound statement formed by two simple given statements by performing some logical operations on them, gives the false value only is called a contradiction or in different terms, it is called a fallacy [VII]. Logical equivalence, compound propositions are said to be logical equivalence if they have the same truth value [VII]. Correspondingly, in this research, we have proposed a compound propositional law for Logical Equivalence, Tautology and Contradiction. The compound propositional law depends on negation, disjunction, conjunction, exclusive or, conditional statement and bi-conditional statement. The proposed compound propositional law validity shows from methodology and result.

II. Proposed Compound Propositional Law

In this section, we have presented the proposed compound proposition law for logical equivalence, tautology and contradiction with the help of negation, disjunction, conjunction, exclusive or, conditional statement and bi-conditional statement. The proposed compound propositional laws can be defined as:

i. $p \vee (p \rightarrow q)$
ii. $p \vee (\neg p \vee q)$
iii. $p \vee (\neg q \vee q)$
iv. $\neg(p \wedge q) \rightarrow (\neg p \oplus q)$
v. $\neg(p \vee q) \wedge (p \oplus q)$
vi. $\neg(p \vee q) \rightarrow (\neg p \leftrightarrow q)$
vii. $\neg(p \oplus \neg q) \rightarrow [\neg(p \wedge q)]$
viii. $\neg(p \vee q) \rightarrow (\neg p \leftrightarrow \neg q)$
ix. $(\neg p \rightarrow q) \vee (q \oplus \neg q)$
x. $(p \oplus q) \leftrightarrow (\neg p \oplus \neg q)$
xi. $\neg p \leftrightarrow q$
xii. $p \wedge \neg p$
xiii. $p \wedge (\neg p \wedge q)$
xiv. $(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$
xv. $(\neg p \wedge \neg q) \leftrightarrow (p \vee q)$
xvi. $(p \leftrightarrow q) \vee (\neg p \leftrightarrow \neg q)$
xvii. $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow \neg q)$
xviii. $(\neg p \leftrightarrow \neg q) \vee (\neg p \rightarrow q)$
xix. $(\neg p \oplus \neg q) \vee (\neg p \rightarrow q)$
xx. $\neg(p \leftrightarrow q) \equiv \neg p \oplus \neg q$
xxi. $\neg(p \wedge q) \equiv \neg(p \wedge q) \vee (\neg p \oplus \neg q)$
xxii. $\neg p \leftrightarrow \neg q \equiv \neg(p \wedge q) \rightarrow (\neg p \leftrightarrow \neg q)$
xxiii. $\neg p \leftrightarrow \neg q \equiv \neg(p \vee q) \rightarrow (\neg p \leftrightarrow \neg q)$
xxiv. $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
xxv. $p \oplus q \equiv \neg p \oplus \neg q$
xxvi. $(p \oplus q) \equiv (p \oplus q) \vee (\neg p \oplus \neg q)$
xxvii. $p \leftrightarrow q \equiv (p \leftrightarrow q) \vee (\neg p \leftrightarrow \neg q)$
xxviii. $p \leftrightarrow q \equiv (p \leftrightarrow q) \wedge (\neg p \leftrightarrow \neg q)$
xxix. $\neg(p \oplus q) \equiv p \leftrightarrow q$
xxx. $\neg p \leftrightarrow \neg q \equiv \neg(p \wedge q) \rightarrow (\neg p \leftrightarrow \neg q)$

Hence these are the compound propositional law for logical equivalence, tautology and contradiction. Where i to ix are the tautology and contradiction and x to xviii are the logical equivalence. Proposed laws are used all the symbol the same as de-morgan law. From these laws, it can be observed that this research is a good achievement in discrete Mathematics, and it is also used to further research purposes. In the below segment, we have produced the results of the proposed compound propositional law.

III. Results and Discussions

This segment is producing the results of the proposed compound propositional law. The results were created by truth table to better understand the proposed laws, but before that let us learn about the logical operations performed on given statements. Logical operations are used to connect the two simple statements in the form of compound statements and this process is called logical operations. There are 5 major logical operations performed based on respective symbols, such as AND, OR, NOT, if and then, if and only if. Let us perform one by one with their logical operation.

i. $p \lor (p \to q)$

p	q	p → q	p ∨ (p → q)
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

ii. $\neg(p \lor q) \land (p \oplus q)$

p	q	p ∨ q	p ⊕ q	¬(p ∨ q)	¬(p ∨ q) ∧ (p ⊕ q)
T	T	T	F	F	F
T	F	T	T	F	F
F	T	T	T	F	F
F	F	F	F	T	F

iii. $\neg p \leftrightarrow q$

p	q	¬p	¬p ↔ q
T	T	F	F
T	F	T	F
F	T	F	F
F	F	T	F

iv. $\neg(p \lor q) \leftrightarrow (\neg p \to q)$

p	q	p ∨ q	¬(p ∨ q)	¬p	¬p → q	¬(p ∨ q) ↔ (¬p → q)
T	T	T	F	F	T	F
T	F	T	F	F	T	F
F	T	T	F	T	T	F
F	F	F	T	T	F	F

v. $\neg(p \oplus \neg q) \lor [\neg(p \land q)]$

p	q	¬q	p ⊕ ¬q	¬(p ⊕ ¬q)	p ∧ q	¬(p ∧ q)	¬(p ⊕ ¬q) ∨ [¬(p ∧ q)]
T	T	F	F	T	T	F	T
T	F	T	T	F	F	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T

vi. $\neg(p \lor q) \to (\neg p \leftrightarrow \neg q)$

p	q	p ∨ q	¬(p ∨ q)	¬p	¬q	¬p ↔ ¬q	¬(p ∨ q) → (¬p ↔ ¬q)
T	T	T	F	F	F	T	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

vii. $(\neg p \rightarrow q) \vee (q \oplus \neg q)$

p	q	¬p	¬q	¬p → q	q ⊕ ¬q	(¬p → q) ∨ (q ⊕ ¬q)
T	T	F	F	T	T	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	T	T

viii. $(p \oplus q) \leftrightarrow (\neg p \oplus \neg q)$

p	q	p ⊕ q	¬p	¬q	¬p ⊕ ¬q	(p ⊕ q) ↔ (¬p ⊕ ¬q)
T	T	F	F	F	F	T
T	F	T	F	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	F	T

ix. $\neg(p \leftrightarrow q) \equiv \neg p \oplus \neg q$

p	q	¬p	¬q	p ↔ q	¬(p ↔ q)	¬p ⊕ ¬q
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

x. $\neg(p \wedge q) \equiv \neg(p \wedge q) \vee (\neg p \oplus \neg q)$

p	q	¬p	¬q	p ∧ q	¬(p ∧ q)	¬p ⊕ ¬q	¬(p ∧ q) ∨ (¬p ⊕ ¬q)
T	T	F	F	T	F	F	F
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	F	T

xi. $\neg p \leftrightarrow \neg q \equiv \neg(p \wedge q) \rightarrow (\neg p \leftrightarrow \neg q)$

p	q	¬p	¬q	p ∧ q	¬(p ∧ q)	¬p ↔ ¬q	¬(p ∧ q) → (¬p ↔ ¬q)
T	T	F	F	T	F	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	F	F
F	F	T	T	F	T	T	T

xii. $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

p	q	¬p	¬q	p↔q	¬p↔¬q
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

xiii. $p \oplus q \equiv \neg p \oplus \neg q$

p	q	¬p	¬q	p⊕q	¬p⊕¬q
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

xiv. $(p \oplus q) \equiv (p \oplus q) \vee (\neg p \oplus \neg q)$

p	q	¬p	¬q	p⊕q	¬p⊕¬q	p⊕q∨¬p⊕¬q
T	T	F	F	F	F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	F	F

xv. $p \leftrightarrow q \equiv (p \leftrightarrow q) \wedge (\neg p \leftrightarrow \neg q)$

p	q	¬p	¬q	p↔q	¬p↔¬q	p↔q∧¬p↔¬q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	F

xvi. $\neg(p \oplus q) \equiv p \leftrightarrow q$

p	q	p ⊕ q	¬(p ⊕ q)	p ↔ q
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

xvii. $\neg(p \wedge q) \equiv \neg p \oplus \neg q$

p	q	¬p	¬q	p ∧ q	¬(p ∧ q)	¬p ⊕ ¬q
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

xviii. $\neg(p \leftrightarrow q) \equiv \neg p \oplus \neg q$

p	**q**	**¬p**	**¬q**	**p ↔ q**	**¬(p ↔ q)**	**¬p ⊕ ¬q**
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

xix. $\neg p \leftrightarrow \neg q \equiv \neg(p \wedge q) \rightarrow (\neg p \leftrightarrow \neg q)$

p	**q**	**¬p**	**¬q**	**¬p ↔ ¬q**	**p ∧ q**	**¬(p ∧ q)**	**¬(p ∧ q) → (¬p ↔ ¬q)**
T	T	F	F	T	T	F	T
T	F	F	T	F	F	T	F
F	T	T	F	F	F	T	F
F	F	T	T	T	F	T	T

From the results shows in the truth table it has been clarified that the proposed compound propositional law is useful and very important for logical equivalence, tautology and contradiction. Throughout the results and discussions, it can be detected that the compound proposition law is good achievement for logical equivalence, tautology and contradiction and it is also good for further research purposes in discrete structure.

IV. Conclusion

This research has developed a compound proposition law estimated for logical equivalence, tautology and contradiction. De-Morgan law is an insight that helped to guide this study. The results and discussion appear to show support for compound proposition law and directions for further research in this area. Furthermore, the study suggests judging the validity of logical equivalence, tautology and contradiction. This research has been discussed some useful topics —i.e. logical equivalence, tautology, contradiction, negation, disjunction, conjunction, exclusive or, conditional statement and bi-conditional statement. Throughout the article, it can be observed that the compound propositional law is more effectively and efficiently for logical equivalence, tautology and contradiction.

V. Acknowledgment

We would like to special thanks to class BS-19IT, Department of Information Technology, Shaheed Benazir Bhutto University, Sanghar, Sindh, Pakistan who have been done tremendous work with us. We would like to thank faculty members of Shaheed Benazir Bhutto University, Sanghar, Sindh, Pakistan for encouraging our research and inspiring us to grow as researchers. We would like to thank Dr. Amanat Ali Jalbani Vice-chancellor of Shaheed Benazir Bhutto University, Shaheed Benazirabad, Sindh, Pakistan who allotted us time for research work. Finally, special thanks to our family for supporting us for everything at every stage.

Conflict of Interest:

There is no conflict of interest regarding this article.

References

I. Agassi, J., Tautology and Testability in Economics, Philosophy of the Social Sciences, Phil. Soc. Sci. vol: 1, pp. 49-63.

II. Avan, B. I. and F. White, The Proposition: An insight into research, Journal of the Pakistan Medical Association February, 2013.

III. Crisler, N., P. Fisher. Discrete mathematics through Applications. W. H. Freeman and Company, 1994.

IV. Kiran, K., Computational Thinking and Its Role in Discrete Mathematics, Biz and Bytes, Vol: 7(1), 2016.

V. Kwon, I., A Tautology is a Tautology: Specificity and Categorization in Nominal Tautological Constructions, In the Proceedings of the 35th annual meeting of berkeley linguistics society, 2009.

VI. Liu, J. and L. Wang, Computational Thinking in Discrete Mathematics, Second International Workshop on Education Technology and Computer Science, 2010.

VII. Ljnda, C. M., R. I Horvotzt and A. R. Fhnstbn, A Collection of 56 Topics with Contradictory Results in Case Control Research, International Journal of Epidemiology, vol: 17(3), 2015.

VIII. Marcela, P., L. Osorio and Á. A. Caputi, Perceptual Judgments of Logical Propositions, Asian Journal of Research and Reports in Neurology, vol: 2(1), pp 1-14, 2019.

IX. Marion, B., Final Oral Report on the SIGCSE Committee on the Implementation of a Discrete Mathematics Course. In SIGCSE Technical Symposium on Computer Science Education, vol: 2006, pp-268-279, 2006.

X. Selva, J. A. N., J. L. U. Domenech and H. Gash, A Logic-Mathematical Point of View of the Truth: Reality, Perception, and Language, 2014.