JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-16, No.-1, January (2021) pp 74-86 ISSN (Print) 0973-8975

THE MAXIMUM RANGE COLUMN METHOD – GOING BEYOND THE TRADITIONAL INITIAL BASIC FEASIBLE SOLUTION METHODS FOR THE TRANSPORTATION PROBLEMS

Huzoor Bux Kalhoro [1], Hafeezullah Abdulrehman[2], Muhammad Mujtaba Shaikh[3], Abdul Sattar Soomro[4]

[1,4] Institute of Mathematics and Computer Science, University of Sindh, Jamshoro, Pakistan

[2] Education and Literacy Department, Government of Sindh, Pakistan

[3] Department of Basic Sciences and Related Studies, Mehran University of Engineering and Technology, Jamshoro, Pakistan

[1]huzoorbux.1971@gmail.com [2]hafeez1969@hotmail.com, [3]mujtaba.shaikh@faculty.muet.edu.pk, [4]dr_sattarsoomro@yahoo.co.in

Corresponding Author: **Muhammad Mujtaba Shaikh**

https://doi.org/00.00000/jmcms.2021.01.00006

(Received: December 11, 2020; Accepted: January 18, 2021)

Abstract

The transportation problems (TPs) are a fundamental case-study topic in operations research, particularly in the field of linear programming (LP). The TPs are solved in full resolution by using two types of methods: initial basic feasible solution (IBFS) and optimal methods. In this paper, we suggest a novel IBFS method for enhanced reduction in the transportation cost associated with the TPs. The new method searches for the range in columns of the transportation table only, and selects the maximum range to carry out allocations, and is therefore referred to as the maximum range column method (MRCM). The performance of the proposed MRCM has been compared against three traditional methods: North-West-Corner (NWCM), Least cost (LCM) and Vogel's approximation (VAM) on a comprehensive database of 140 transportation problems from the literature. The optimal solutions of the 140 problems obtained by using the TORA software with the modified distribution (MODI) method have been taken as reference from a previous benchmark study. The IBFSs obtained by the proposed method against NWCM, LCM and VAM are mostly optimal, and in some cases closer to the optimal solutions as compared to the other methods. Exhaustive performance has been discussed based on absolute and relative error distributions, and percentage optimality and nonoptimality for the benchmark problems. It is demonstrated that the proposed MRCM is a far better IBFS method for efficiently solving the TPs as compared to the other discussed methods, and can be promoted in place of the traditional methods based on its performance.

Keywords : Transportation problem, optimal solution, MODI method, TORA software, Minimum cost, Initial basic feasible solution, Maximum range.

I. Introduction

Many problems in mathematical, applied and engineering sciences demand the determination of an unknown quantity subject to certain restrictions, and the objective is to get the best answer under the specified restrictions. Such problems are often referred to as optimization problems, and the unknown quantity to be determined is known as the value of an objective function under stated constraints [II], [XIV], [XXIV], [XXVIII].

In transportation problems (TPs), which are a special section in linear programming, such constraints are usually the non-negativity restrictions on the decision variables along with the supply and demand capacities. The objective function is the transportation cost, to determine the minimum transportation cost under similar conditions so that the constraints are met [XXIV], [XXVII], [XXVIII], [XIX].

There are two stages in solving a TP when the transportation table listing the number of sources, number of destinations, the supply capacities of sources, the demand of the destinations, and the transportation costs per unit of product is given. The first stage attempts to find initial approximations to the required minimum cost by using IBFS methods. Traditionally, the most basic, ancient and widely used IBFS methods in the literature are the north-west-corner method (NWCM), least cost method (LCM) and Vogel's approximation method (VAM). Of these, mostly the VAM is considered better [XXIV]. Several attempts have also been made by researchers to efficiently solve the TPs with less effort and time, but with more accuracy. Besides, many methods that were proposed were tested for the optimality and the claimed performance as well. For example, [I], [IV], [V], [VI], [VII], [VIII], [IX], [X], [XI], [XII] [XV], [XVI], [XVII], [XX], [XXI], [XXII], [XXIII], [XXVI], [XVIII].

To compare the performance of the ancient methods [XXIV] and the new methods, a set of standard transportation problems, i.e. a database of 140 transportation problems [XVIII] was used in [XI] to provide ready and worked-out optimal solutions.

In this research work, we propose a new IBFS method, referred to as the maximum range column method (MRCM) for obtaining the minimum transportation cost in TPS. The proposed method has been tested against the traditional NWCM, LCM and VAM on a benchmark test database of 140 balanced and unbalanced TPs [XVIII]. The optimal solutions [XI] are used to discuss the error analysis and accuracy of the results by proposed and the three traditional methods. The proposed MRCM method is found to be an efficient and more accurate alternative to the traditional IBFS methods.

II. Material and Methods

For a known number of sources and destinations, the cost table of transportation problems together with the supply and demand capacities of the sources and destinations is usually defined in form of the transportation array. For example, for a transportation problem with three sources: A, B, C and four destinations D, E, F, G with supply and demand capacities of S1-S3 and D1-D4, respectively is represented in Table 1, where the $C_{i,j}$, i=A,B,C and j=D,E,F,G are the costs required per unit transportation of goods from a source "i" to a destination "j".

The objective is to find the optimal values for the decision variables X's which represent total units of products to be transported from sources to destinations such that the total transportation cost is minimized subject to the supply, demand and non-negativity constraints. The total transportation cost is defined as the sum of the products of per-unit costs together with the optimal values of decision variables for all interior cells in the transportation table.

Table 1: Example transportation problem array with 3 sources and 4 destinations

Destinations / Sources	D		*E*		*F*		*G*		Supply
A	X_1	C_{AD}	X_4	C_{AE}	X_7	C_{AF}	X_{10}	C_{AG}	*S1*
B	X_2	C_{BD}	X_5	C_{BE}	X_8	C_{BF}	X_{11}	C_{BG}	*S2*
C	X_3	C_{CD}	X_6	C_{CE}	X_9	C_{CF}	X_{12}	C_{CG}	*S3*
Demand	*D1*		*D2*		*D3*		*D4*		Balanced if Total supply = Total demand

The Proposed Maximum Range Column Method (MRCM)

Given a transportation problem, and its transportation array is defined as per the example form described in Table 1, the proposed MRCM uses the following seven simple steps.

Step 1. Balance the model, if required.

Step 2. Determine the range cost for each column by taking the difference between the largest cost and lowest cost.

Step 3. Select the column with maximum range. If a tie occurs, choose any row/column with the lowest range.

Step 4. Allocate min(s,d) to the cell having minimum unit transportation cost in the selected column or row.

Step 5. Note that no further consideration is required for the row or column which is satisfied. If a row or column is satisfied at a time, delete only one and assign zero to the remaining row or column.

Step 6. Continue the process until all rows and columns are satisfied.

Step 7. Calculate total transportation cost for the feasible allocations using the original transportation array.

The proposed MRCM can be used to quickly and more accurately to determine the IBFS to transportation problems as compared to the traditional IBFS methods: North-West-Corner method (NWCM), Least cost method (LCM) and Vogel's approximation method (VAM) [XXIV], [XXVII], [XIX], [XI]. The algorithms of these can be found in [XI]. Moreover, [XI] also contains the algorithm of the optimal MODI method and its implementation details with TORA software. The NWCM uses a simple fixed direction search strategy, due to which it does not attain the smallest cost or closer to the optimum cost. The LCM proceeds in the variable direction of the lowest costs within the cells, and onwards, so mostly it gives better results than NWCM. Sometimes LCM also gives better results than the VAM, which is not the case always. The VAM considers the penalty of the two smallest numbers across each row and column and usually attains better results than NWCM and LCM, but we will show in the next section that the proposed MRCM is better than the VAM through several examples. Generally, the MRCM finds the range for each column only which is a quicker search strategy as compared to searching for the two smallest numbers in each row and each column to determine penalties row-wise and column-wise in VAM. Consequently, the MRCM implementation can be done quickly as compared to the VAM with more accuracy.

III. Detailed Working of the Proposed MRCM

To understand the working of the proposed MRCM to attain an IBFS to transportation problems which either optimal or very close to the optimal solution, we discuss its detailed application on Examples 1-4 here. The application details step-by-step of the proposed MRCM on Example 1 are summarized in Table 2. For Examples 2-3, the given transportation and the final allocation tables by the proposed MRCM method are given in Tables 4-5, respectively. Besides, in Tables 2-5, the minimum cost obtained by NWCM, LCM, VAM and optimal MODI is also quoted for reference for Examples 1-3. It can be seen that the proposed MRCM in Examples 1-3 attains the same IBFS as attained by the VAM but with lesser cost, since the proposed MRCM uses only column ranges, whereas the VAM uses penalties for rows as well as columns. Moreover, the results by MRCM in Examples 1,3 are optimal, whereas in Example 2 closer to optimal. However, the proposed MRCM takes an edge over the VAM method in most of the problems, as to be highlighted in the exhaustive application in the next section. In Table 6, the original transportation array for Example 4 along with the final allocation tables on the same by NWCM, LCM, VAM and the proposed MRCM is presented, whereas the optimal MODI cost is also mentioned for reference. It is evident from Example-4 via Table 6, that the proposed MRCM proves to be best of all as it attains an IBFS same as the optimal solution with lesser time and computation cost as compared to other methods. In the next section, we give a more extended application of the proposed MRCM and its thorough comparison with the NWCM, LCM and VAM for 140 test transportation problems from [XVIII] keeping the corresponding MODI optimal solutions from [XI] as reference.

Table 2: MRCM working on Example-1

	X	Y	Z	Supply
1	16	20	12	200
2	14	8	18	160
3	26	24	16	90
Demand	180	120	150	450

(a). Given transportation array.

	X	Y	Z	Supply
1	16	20	12	200
2	14	8(120)	18	40
3	26	24	16	90
Demand	180	0	150	330
Range	***12***	***16***	***6***	

(b). Assigned 120 to (2,Y) due to maximum column range and lowest cost. Deleting Y.

	X	Z	Supply
1	16	12	200
2	14(40)	18	0
3	26	16	90
Demand	140	150	290
Range	***12***	***6***	

(c). Assigned 40 to (2,X) due to maximum column range and lowest cost. Deleting '2'.

	X	Z	Supply
1	16(140)	12	60
3	26	16	90
Demand	0	150	150
Range	***10***	***4***	

(d). Assigned 140 to (1,X) due to maximum column range and lowest cost. Deleting 'X'.

	Z	Supply
1	12(60)	0
3	16(90)	90
Demand	90	90
Range	***4***	

(e). Assigned 60 to (1,Z). Deleting '1'.

	Z	Supply
3	16(90)	0
Demand	0	0
Range	-	

(f). Assigned 90 to (3,Z). Deleting '3', 'Z'.

	X	Y	Z	Supply
1	16 (140)	20	12 (60)	200
2	14 (40)	8 (120)	18	160
3	26	24	16 (90)	90
Demand	180	120	150	450

(g). Final allocation table by MRCM

(h). Total minimum transportation cost by MRCM

Z_{MRCM}

$= 16\,(140) + 12(60)$

$+ 14(40)\ \ 8(120) + 16(\,90\,) = 5920$

Note. $Z_{NWCM} = 6600$, $Z_{LCM} = 6460$

$Z_{VAM} = 5920$, $Z_{MODI} = 5920$

Table 3: MRCM allocation in Example-2

	W	X	Y	Z	Supply
O1	6	1	9	3	70
O2	11	5	2	8	55
O3	10	12	4	7	90
Demand	85	35	50	45	215

(a). Given transportation array

	W	X	Y	Z	Supply
O1	6 (35)	1 (35)	9	3	70
O2	11	5	2 (50)	8 (5)	55
O3	10 (50)	12	4	7 (40)	90
Demand	85	35	50	45	215

(b). Final allocation by MRCM

(c). $Z_{MRCM} = 6(35) + 1(35) + 2(50) + 8(5) + 10(50) + 7(40) = 1165$
Note. $Z_{NWCM} = 1265$, $Z_{LCM} = 1165$, $Z_{VAM} = 1165$, $Z_{MODI} = 1160$

Table 4: MRCM allocation in Example-3

	W	X	Y	Z	Supply
O1	5	2	4	3	22
O2	4	8	1	6	15
O3	4	6	7	5	8
Demand	7	12	17	9	45

(a). Given transportation array

	W	X	Y	Z	Supply
O1	5	2(12)	4(2)	3(8)	22
O2	4	8	1(15)	6	15
O3	4(7)	6	7	5(1)	8
Demand	7	12	17	9	45

(b). Final allocation by MRCM

(c). $Z_{MRCM} = 2(12) + 4(2) + 3(8) + 1(15) + 4(7) + 5(1) = 104$
Note. $Z_{NWCM} = 131$, $Z_{LCM} = 105$, $Z_{VAM} = 104$, $Z_{MODI} = 104$

Table 5: Example-4 with allocations by NWCM, LCM, VAM and proposed MRCM

		Destination				
		D	E	F	G	Supply
Factory	A	20	22	17	4	120
	B	24	37	9	7	70
	C	32	37	20	15	50
Demand		60	40	30	110	240

(a). Original transportation array

	D	E	F	G	Supply
A	20 / 60	22 / 40	17 / 20	4	120
B	24	37	9 / 10	7 / 60	70
C	32	37	20	15 / 50	50
Demand	60	40	30	110	240

(b). IBFS allocation by NWCM

	D	E	F	G	Supply
A	20 / 10	22	17	4 / 110	120
B	24 / 40	37	9 / 30	7	70
C	32 / 10	37 / 40	20	15	50
Demand	60	40	30	110	240

(c). IBFS allocation by LCM

	D	E	F	G	Supply	Row Penalty			
A	20	22 / 40	17	4 / 80	120	(13)	(13)	--	--
B	24 / 10	37	9 / 30	7 / 30	70	(2)	(2)	(2)	17
C	32 / 50	37	20	15	50	(5)	(5)	(5)	17
Demand	60	40	50	110	240				
	(4)	(15)	(8)	(3)					
	(4)	--	(8)	(3)					
	(8)	--	(11)	(8)					
	(8)			(8)					

(d). Final allocation with details by VAM

	D	E	F	G	Supply
A	20 / 60	22 / 40	17	4 / 20	120
B	24	37	9 / 30	7 / 20	70
C	32	37	20	15 / 50	50
Demand	60	40	50	110	240
	(12)	(15)	(11)	(11)	
	(12)	--	(11)	(11)	
		--	(11)	(11)	

(e). Final allocation details by MRCM

NWCM	LCM	VAM	MODI	MRCM
3860	3670	3520	3460	3460

(f). Minimum costs obtained by IBFS methods: NWCM, LCM, VAM and MRCM and the optimal MODI with proposed MRCM being optimal here

IV. Results and Discussion

To demonstrate the efficiency of the proposed MRCM over the traditional IBFS methods: NWCM, LCM and VAM, we use a set of 140 transportation problems as given in [XVIII] as a benchmark set of tests TPs. The problem set [XVIII] contains 103 balanced TPs, 25 unbalanced TP and remaining 12 from some research papers mentioned therein [III], [V], [X], [XII], [XIII], [XVII], [XIX], [XX], [XXII], [XXV], [XXVII]. For a ready reference, the optimal solutions of the 140 test problems are taken from a recent study [XI], where the IBFSs attained through NWCM, LCM and VAM were used to get the optimal solutions using the MODI method through TORA software for all 140 problems in [XVIII]. Here, we use the optimal solutions from [XI] to compare the error distributions between the IBFSs obtained by the proposed MRCM and the three traditional methods: NWCM, LCM and VAM. The IBFSs attained by the proposed MRCM method for test transportation problems 1-140 [XVIII] are summarized in Table 6. In most of the problems, the proposed MRCM directly finds the optimal solutions. The percentage comparison of the used methods:

NWCM, LCM, VAM and proposed MRCM as to whether being able to obtain optimal solution directly or not are shown in Fig. 1. It can be seen that the proposed MRCM attain directly the optimal solution for 102 test problems out of 140, whereas the optimality count for the NWCM, LCM and VAM are 6, 26 and 62, respectively. The absolute errors between the IBFSs obtained by used methods and the optimal MODI solutions in cases where the methods have not been optimal are shown in Fig. 2 on a logarithmic scale. For the problems 1-72, 104-125, 129-130, 132-136 and 140 the proposed MRCM was optimal, whereas in the remaining problems absolute errors in the IBFSs by MRCM are almost smaller than all other methods including VAM. In the stated 102 problems where the MRCM was found to be optimal, there as many instances where the VAM could not be optimal. The relative errors for all test problems 1-140 are shown in the original scale in Fig. 3, and it is easily visible that the proposed MRCM method exhibits smaller relative errors as compared to the traditional methods NWCM, LCM and VAM. While the cost and time effectiveness of the proposed MRCM is also established in section III, the proposed MRCM is a way forward to get an efficient and more accurate IBFS to the transportation problems as compared to the traditional methods.

Table 6: Proposed MRCM IBFS of test Problems 1-140 from [XI], [XVIII]

T.P	MRCM IBFS solution	T.P	MRCM IBFS solution	T.P	MRCM IBFS solution	T.P	MRCM IBFS solution
1.	24	36.	920	71.	390	106	980
2.	35	37.	730	72.	830	107	4450
3.	20	38.	235	73.	47250	108	1140
4.	62	39.	44100	74.	2517	109	920
5.	4525	40.	102	75.	415	110	735
6.	505	41.	89	76.	60448	111	1620
7.	2350	42.	7350	77.	12200	112	68
8.	15650	43.	3400	78.	781	113	80
9.	380	44.	772	79.	535	114	740
10.	1200	45.	2595	80.	1005	115	17300
11.	1130	46.	2221	81.	2550	116	4840
12.	1350	47.	799	82.	113	117	630
13.	14150	48.	47	83.	20970	118	1160
14.	143	49.	173	84.	80	119	515
15.	143	50.	559	85.	150	120	13650
16.	173	51.	2365	86.	25	121	790
17.	743	52.	412	87.	26	122	43476
18.	610	53.	960	88.	134	123	4525
19.	3460	54.	674	89.	112	124	9200
20.	506	55.	76	90.	332	125	2750
21.	886	56.	172	91.	230	126	1650
22.	118	57.	1680	92.	1860	127	6445
23.	2100	58.	420	93.	56	128	13695
24.	10730	59.	5950	94.	800	129	412
25.	11500	60.	184	95.	80	130	743
26.	140	61.	267	96.	290	131	60448
27.	10680	62.	695	97.	750	132	80

28.	796	63.	490	98.	140	133	610
29.	635	64.	6400	99.	271	134	3460
30.	3100	65.	2850	100.	380	135	76
31.	56	66.	3320	101.	988	136	506
32.	12220	67.	1390	102.	391	137	208
33.	7350	68.	555	103.	68500	138	152
34.	5300	69.	625	104.	2424	139	327
35.	900	70.	590	105	3300	140	172

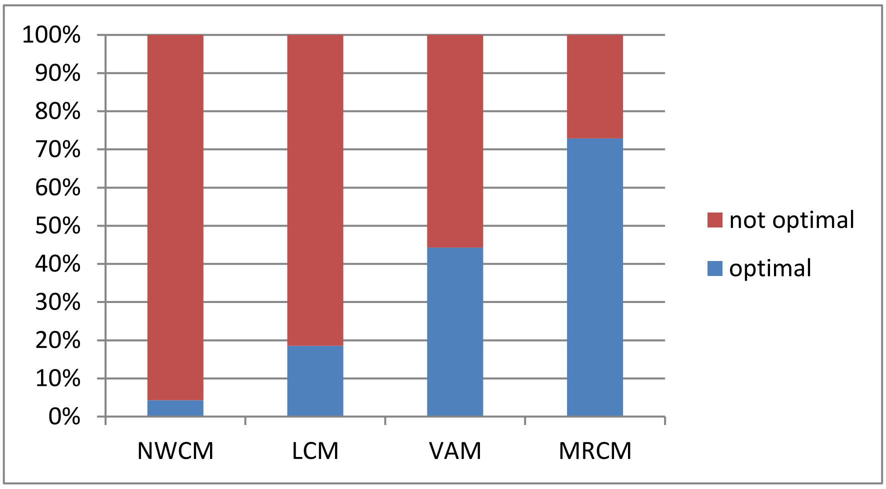

Figure 1. Optimal and non-optimal % comparison of IBF solution methods versus MODI optimal solutions

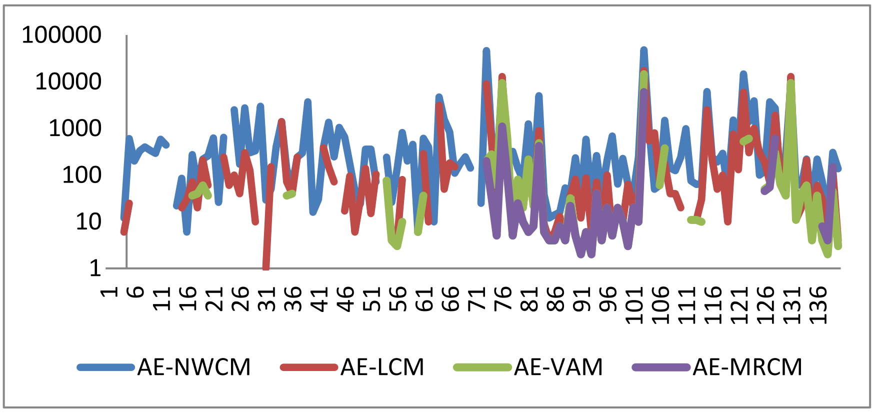

Figure 2. Absolute errors in logarithmic scale between IBFSs and MODI for non-optimal T.Ps

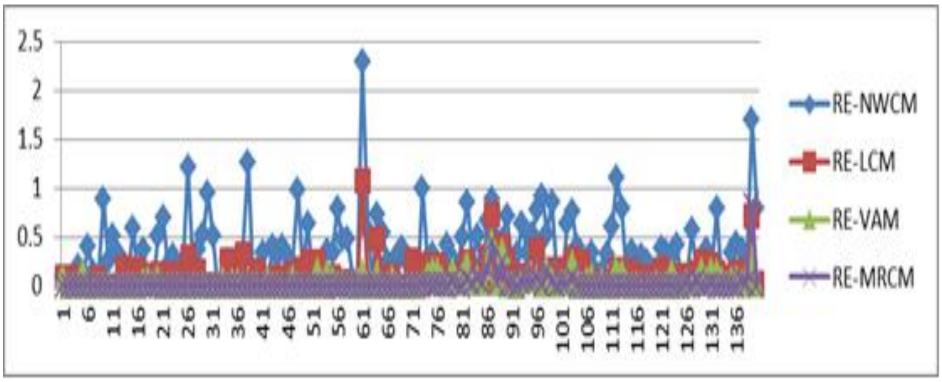

Figure 3. Relative errors between IBFSs and MODI for test problems 1-140

V. Conclusion

In this study, a new IBFS method has been presented for a more accurate and quick approximation of the optimal solution of the transportation problems as compared to the widely known traditional IBFS methods: NWCM, LCM and VAM. The proposed method has been referred as MRCM (maximum range column method) as it uses the maximum of the ranges in for columns only and allocates to the smallest cost corresponding to the maximum range column. The search strategy of the proposed MRCM is quicker than the VAM since in the VAM the penalties are computed for both rows and columns in the transportation array. The example working of the proposed MRCM has been explained on four example transportation problems. Besides, a comprehensive set of results have been established in this study for the proposed MRCM and traditional NWCM, LCM and VAM on 140 test transportation problems from the literature. The comprehensive results demonstrate the efficiency of the proposed MRCM over the discussed methods in terms of finding the optimal or nearly optimal solution more quickly and accurately. The results in terms of optimality and non-optimality percentage, absolute error distributions in case of non-optimal solutions in logarithm scale, and relative error distributions for all problems in original scales have been used to show that the proposed MRCM is better than NWCM, LCM and VAM, and may be used to get more accurate and reliable IBFS to transportation problems as compared to the traditional NWCM, LCM and VAM.

VI. Acknowledgements

Authors are thankful to their institutes for providing facilities for the conduct of this research.

Conflict of Interest:

There is no conflict of interest regarding this article

References

I. Adlakha, Veena, and Krzysztof Kowalski. "Alternate solutions analysis for transportation problems." *Journal of Business & Economics Research* 7.11 (2009): 41-49.

II. Bhan, Veer, Ashfaque Ahmed Hashmani, and Muhammad Mujtaba Shaikh. "A new computing perturb-and-observe-type algorithm for MPPT in solar photovoltaic systems and evaluation of its performance against other variants by experimental validation." *Scientia Iranica* 26, no. Special Issue on machine learning, data analytics, and advanced optimization techniques in modern power systems [Transactions on Computer Science & Engineering and Electrical Engineering (D)] (2019): 3656-3671.

III. Chungath Linesh, "Comparison of Transportation Problems Solved by Vogel's Approximation Method (VAM-1958), Revised Distribution Method (RDI -2013) & The New Method", available online, 2004 @ https://www.academia.edu/1137498

IV. Das, Utpal Kanti, et al. "Logical development of vogel's approximation method (LD-VAM): an approach to find basic feasible solution of transportation problem." *International Journal of Scientific & Technology Research* 3.2 (2014): 42-48.

V. Deshmukh, N. M. "An innovative method for solving transportation problem." *International Journal of Physics and Mathematical Sciences* 2.3 (2012): 86-91.

VI. Goyal, S. K. "Improving VAM for unbalanced transportation problems." *Journal of the Operational Research Society* 35.12 (1984): 1113-1114.

VII. Hakim, M. A. "An alternative method to find initial basic feasible solution of a transportation problem." *Annals of pure and applied mathematics* 1.2 (2012): 203-209.

VIII. Islam Md Amirul, Aminur Rehman Khan, Sharif Uddin M and Abdul Malek M Islam. "Determination of basic feasible solution of transportation problem: a new approach." *Jahangirnagar University Journal of Science* 35.1 (2012): 101-108.

IX. Jamali, S., Shaikh, M. M., & Soomro, A. S. (2019). Overview of Optimality of New Direct Optimal Methods for the Transportation Problems. Asian Research Journal of Mathematics, 15(4), 1-10.

X. Jamali S., Soomro, A. S., & Shaikh, M. M. (2020). The Minimum Demand Method – A New and Efficient Initial Basic Feasible Solution Method for Transportation Problems., : J. Mech. Cont.& Math. Sci., Vol. 15, No.10, pp. 94-105.

XI. Kalhoro H. B., Abdulrehman H., Shaikh, M. M., Soomro, A. S. (2020). A pioneering and comprehensive database of balanced and unbalanced transportation problems for ready performance evaluation of existing and new methods. J. Mech. Cont.& Math. Sci., Vol. 15, No.11, pp. 149-159.

XII. Korukoğlu, Serdar, and Serkan Ballı. "A Improved Vogel's Approximation Method for the Transportation Problem." *Mathematical and Computational Applications* 16.2 (2011): 370-381.

XIII. Mamidi, Pushpa Latha. "Ones method for finding an optimal solution for transportation problem." In Proceedings International Conference On Advances In Engineering And Technology, International Association of Engineering & Technology for Skill Development, 41-45, ISBN NO: 978 – 1503304048,

XIV. Massan, S.-u-R., Wagan, A. I., & Shaikh, M. M.. "A new metaheuristic optimization algorithm inspired by human dynasties with an application to the wind turbine micrositing problem." *Applied Soft Computing* 90 (2020): 106176.

XV. Pandian, P., and G. Natarajan. "A new method for finding an optimal solution for transportation problems." *International J. of Math. Sci. and Engg. Appls* 4 (2010): 59-65.

XVI. Pandian P. and Natarajan G. "A new approach for solving transportation problems with mixed constraints", *Journal of Physical Sciences* 14 (2010): 53-61.

XVII. Quddoos, Abdul, Shakeel Javaid, and Mohd Massod Khalid. "A new method for finding an optimal solution for transportation problems." *International Journal on Computer Science and Engineering* 4.7 (2012): 1271.

XVIII. Shaikh, Muhammad Mujtaba; Soomro, Abdul Sattar; Kalhoro, Huzoor Bux (2020), "Comprehensive database of test transportation problems (balanced and unbalanced) ", Mendeley Data, V1, doi: 10.17632/b73b5kmcwm.1

XIX. Sharma, S. D., Sharma Himanshu *Operations Research*, Kedar Nath Ram Nath, 2010

XX. Soomro, A.S., S. Jamali, and M. M. Shaikh. "On Non-Optimality of Direct Exponential Approach Method for Solution of Transportation Problems." *Sindh University Research Journal-SURJ (Science Series)* 49.1 (2017): 183-188

XXI. Soomro, Abdul Sattar, Gurudeo Anand Tularam, and Ghulam Murtaa Bhayo. "A comparative study of initial basic feasible solution methods for transportation problems." *Mathematical Theory and Modeling* 4.1 (2014): 11-18.

XXII. Soomro, Abdul Sattar, Muhammad Junaid, and Gurudev Anand Tularam. "Modified Vogel's Approximation Method for Solving Transportation Problems." *Mathematical Theory and Modeling* 5.4 (2015): 32-42.

XXIII. Sudhakar, V. J., N. Arunsankar, and T. Karpagam. "A new approach for finding an optimal solution for transportation problems." *European journal of scientific Research* 68.2 (2012): 254-257.

XXIV. Taha, Hamdy A. *Operations research: An introduction (for VTU)*. Pearson Education India, 2005.

XXV. Unit 1, Lesson 15: "Methods for finding initial solution for a transportation problem" @ https://www.coursehero.com/file/10473072/3TransportationProblem/

XXVI. Vannan, S. Ezhil, and S. Rekha. "A New Method for Obtaining an Optimal Solution for Transportation Problems." *International Journal of Engineering and Advanced Technology* 2 (2013).

XXVII. Winston, Wayne L. "Transportation, Assignment, and Transshipment Problems." *Operations Research Applications and Algorithms, Duxbury Press, California* (1994): 338.

XXVIII. Yousaf, M., Shaikh M. M., & Shaikh A. W. (2020). Some Efficient Mathematical Programming Techniques for Balancing Equations of Complex Chemical Reactions., : J. Mech. Cont. & Math. Sci.,Vol. 15, No.10, pp. 53-66.