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NUMERICAL INVESTIGATION OF DEVELOPING LAMINAR FLUID FLOW THROUGH RECTANGULAR ANNULUS DUCT

Takwah Talib Hasan[1], Dr. Raed G. Saihood[2]

[1]BSC: Mechanical Engineering, Engineering College, Wassit University

[2]Ph.D. Mechanical Engineering, Engineering College, University of Baghdad

Email: [1]t.hasan1209@coeng.uobaghdad.edu.iq, [2]raed.hassan@coeng.uobaghdad.edu.iq

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Abstract

The laminar fluid flow of water through the annulus duct was investigated numerically by ANSYS fluent version 15.0 with height (2.5, 5, 7.5) cm and constant length (L=60cm). With constant heat flux applied to the outer duct. The heat flux at the range (500,1000,1500,2000) w/m^2 and Reynolds number values were ≤ 2300. The problem was 2-D investigated. Results revealed that the Nusselt number decreased and the wall temperature increased with the increase of heat flux. Also, the average Nusselt number increase as Re increases. And as the height of the annulus increase, the values of the temperature and the local and average Nusselt number increase.

Keywords: Forced convection, Laminar flow, annulus ducts, developing flow, entrance region.

NOMENCLATURE:

A	Cross-sectional area	m^2
a_o	The duct's height	m
b_o	The duct's width	m
L	The duct's length	m
b	Height of the annulus	m
P	Perameter	m
K	Thermal conductivity	w/m.k
V	velocity	m/s
T_w	Wall temperature	k
T_b	Bulk Temperature	k
q_w	Heat flux	w/m^2
D_h	Hydraulic diameter	m
X	Axial coordinate	m
h	Coefficient of heat transfer	w/m^2.k
U_m	Mean velocity	m/s

GREAK SYMBOLS:

μ	Dynamic viscosity	kg/m.s
ρ	water density	kg/m^3

DIMENSIONLESS GROUP:

Re	Renolds number
Nu	Local nusselt number
Nu_{avg}	Average nusselt number

I. Introduction

The investigation of the entrance region in laminar flow heat transfer in a non-circular annulus is of urgent need in engineering applications such as (polymer processing industries, compact heat exchangers design and axial-flow turbo machinery). Many studies were done on laminar flow heat transfer. Some of these studies showed below: **Javeri (1977), [**VIII**]** applied the Galerkin-Kantorowich method of variational calculus to solve the energy equation for laminar forced convection heat transfer to investigate the impression of the thermal boundary condition of the third class at the thermal entrance region of the rectangular channel. It was the inference that the local Nusselt number could be affected by the Biot number. The developing steady flow through a square duct was studied by **Emery et al. (1980), [**VI**]** to determine the velocity and temperature profiles**.** The thermal boundary condition was constant wall temperature, constant heat flux, or asymmetric heating. The results revealed that both temperature and velocity field prediction showed the transport effects of local wall shear stress, secondary flow, and heat flux distribution. These effects illustrated the peaking behavior between the duct's midplane and the corner region. Average Nusselt number predictions were approximately ten percent less than the Dittus–Boleter experimental correlation. **Cotta and Ozisik (1986), [**IV**]** considered specified wall heat fluxes. Alternating direction implicit (ADI) finite difference method was used by **Soh(1988),** [XII] to solve numerically the governing equation for developing laminar flow in a curved square duct. **Maia and Gasparetto (2003),** [X] found a difference in the entrance region of annulus geometries by using the finite difference method for the power-law fluid. The flow and heat transfer in horizontal and an inclined rectangular duct was studied experimentally by **Daotong et al., (2007)** [V]. Uniform heat flux was projected on a plate fixed at the middle of the cross-section. They found that by increasing the Re, the average Nusselt number increase. The effect of uniform heat flux on the laminar mixed convection heat transfer was studied by **Ahmed et al. (2013),** [II] for a slanted rectangular cylinder**.** the upper surface was heated and the other surfaces were insulated. They inferred that the Richardson number had a major role in the heat transfer process. **Rajamohan et al. (2015),** [XI] analyzed Combined convection heat transfer in a thermally developing flow inside a heated square duct with different inclination angles. Results revealed that the variations of inclination angle, the velocity of the flow, Reynolds number and the surface radiation greatly affected the enhancement of heat transfer and convective Nusselt number. **Kandasamy (2015), [**IX**]** studied the flow and heat transfer in the entrance region for simultaneously developing boundary layer in the concentric annulus for a Bingham fluid by presenting the velocity, temperature and pressure distributions. **Akeel & Mustafa (2018),** [III] investigated experimentally the flow and heat transfer characteristics of the entrance region for a uniformly heated horizontal duct with a flat plate at the bottom. The results illustrated the effect of heat flux and Reynolds number on the Nusselt Number.

II. Formulation of the Problem

The geometry of the problem is explained in Fig.1. The Cartesian coordinates system was considered and the origin point is considered at the inlet section on the central axis of the annulus. Water enters the horizontal rectangular annulus at a uniform velocity profile along the axial direction x. The flow is steady laminar forced convection without internal heat generation. Also, the flow is incompressible and viscous dissipation is negligible. All physical properties of water are assumed constant.

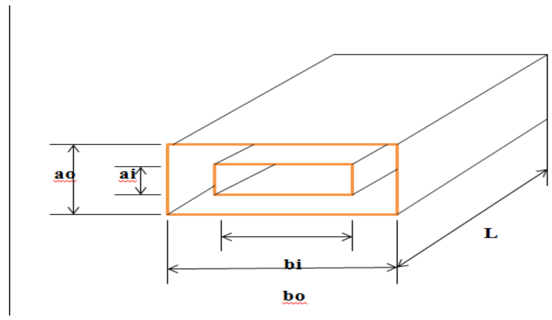


Fig 1. The geometry of the problem.

The governing equations:

Under the above hypotheses and for 2D-flow, the governing equations for the Cartesian coordinate system (x, y) at the entrance region are:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

X-momentum:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2a}$$

Y-momentum:

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)) \tag{2b}$$

Energy equation:

$$\rho Cb\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=-\frac{\partial p}{\partial x}+k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

Boundary conditions:

The boundary conditions is summarized in Figure (2):

Inlet section boundary conditions:

The inlet boundary conditions as follows:-

$$\left.\begin{aligned} u(0,y) &= u_o \\ v(0,y) &= 0 \\ T(0,y) &= T_o \end{aligned}\right\} \qquad (4)$$

Exit section boundary conditions:

The outlet boundary conditions as follows:-

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0 \ , \ \frac{\partial T}{\partial x} = \text{constant}, \quad \frac{\partial P}{\partial x} = \text{constant} \qquad (5)$$

Wall boundary conditions:

At the walls, all velocity components are zero according to no-slip boundary condition, hence:-

$$\left.\begin{aligned} u(x, a_i) &= v(x, a_i) = 0 \\ u(x, a_o) &= v(x, a_o) = 0 \end{aligned}\right\} \qquad (6)$$

and, for q_w =constant , the boundary conditions will be:-

$$\left.\begin{aligned} -k\frac{\partial T}{\partial y}\Big|(x, a_o) &= q_w \\ \frac{\partial T}{\partial y}\Big|(x, a_i) &= 0 \end{aligned}\right\} \qquad (7)$$

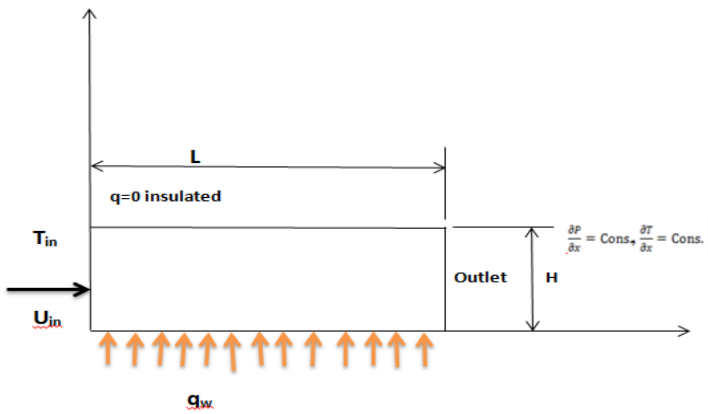


Fig 2. The boundary conditions on the symmetry lines

III. Further calculation

A numerical simulation for the heat transfer process has been developed successfully done by using ANSYS FLUENT software package (version 15.0). A two-dimensional model was used to simulate the cases of the present work. The simulation was done with four steps which are geometry formation, meshing, post-processing and finally results in analysis. The local Nusselt number was computed for velocity inlet values initially at Re=500 and gradually increased to 2000. The heat flux values were (500, 1000, 1500 and 2000) w/m^2.

Calculation of mean velocity:--

Since the velocity throughout the cross-section is varied and there is no clear definition for the free stream velocity in internal flows, it is important to use the mean velocity concept to express the velocity in such flows. The mean velocity (u_m) is calculated from Reynolds number law, where:

$$Re = \frac{\rho\, u_m D_h}{\mu} \tag{8}$$

Then, the Mean velocity will be:

$$u_m = \frac{Re\, \mu}{\rho\, D_h} \tag{9}$$

Where (D_h) is the hydraulic diameter which defined as:

$$D_h = \frac{4\,A}{perameter} \tag{10}$$

Where:

$$\text{A=The cross-section area} = (2a_o \times 2b_o) - (2a_i \times 2b_i) \tag{11}$$

$$perameter = 2 \times (2a_o + 2b_o) + 2 \times (2a_i + 2b_i) \tag{12}$$

Calculation of local and mean nusselt numbers:

The Nusselt number calculated as follow **Akeel and Mustafa, (2018)[11]**

$$Nu = \frac{h\, D_h}{k} \tag{13}$$

Where

$$q_w = h\,(T_w - T_b) \tag{14}$$

Where: Tw= wall temperature and Tb=balk temperature

$$T_b = \frac{\int \rho\, u\, T\, dy}{\int \rho u\, dy} \tag{15}$$

Then (h) the coefficient of heat transfer will be:

$$h = \frac{q_w}{(T_w - T_b)} \tag{16}$$

And the local Nusselt number will be then:

$$Nu = \frac{q_w D_h}{k\,(T_w - T_b)} \tag{17}$$

Average Nusselt number can be calculated by integrating the local Nusselt number throughout the channel, as follow

$$Nu_{avg} = \frac{1}{L}\int Nu\, dx \tag{18}$$

IV. Result and Discussion

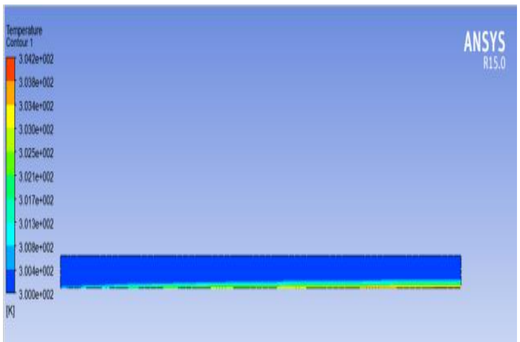


Fig 3. Sample of temperature field.

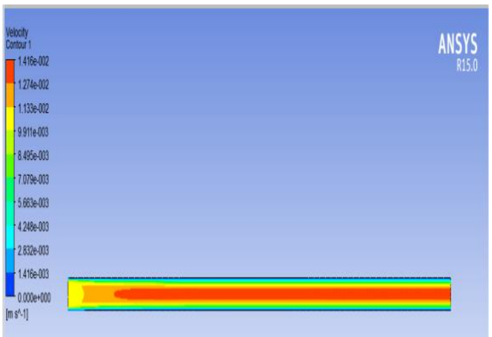
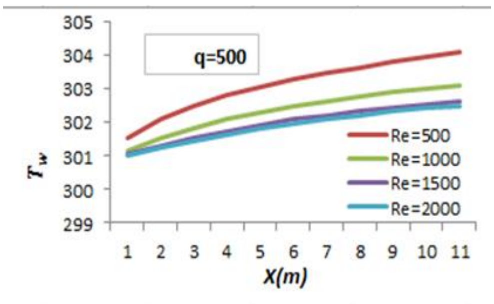


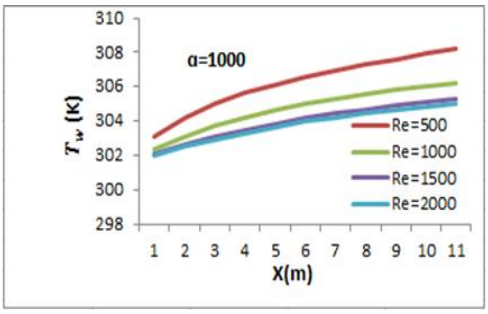
Fig 4. Sample of velocity field

The surface temperature and local Nusselt number distributions for all cases are shown below:

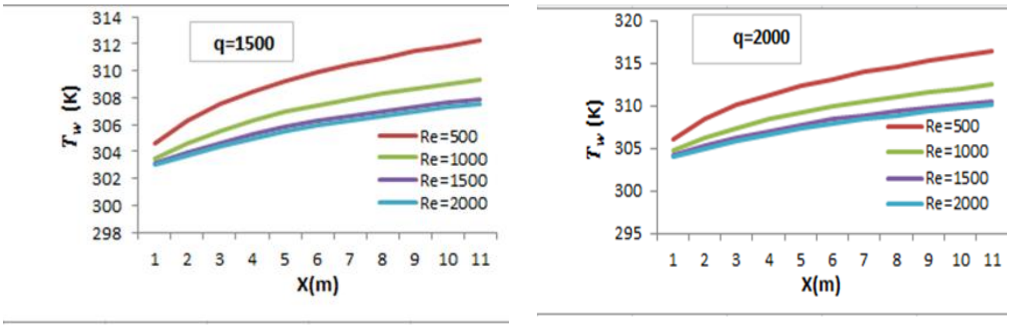
CASE (1): L=60cm and b=0.025m:



(a) at q=500 w/m^2

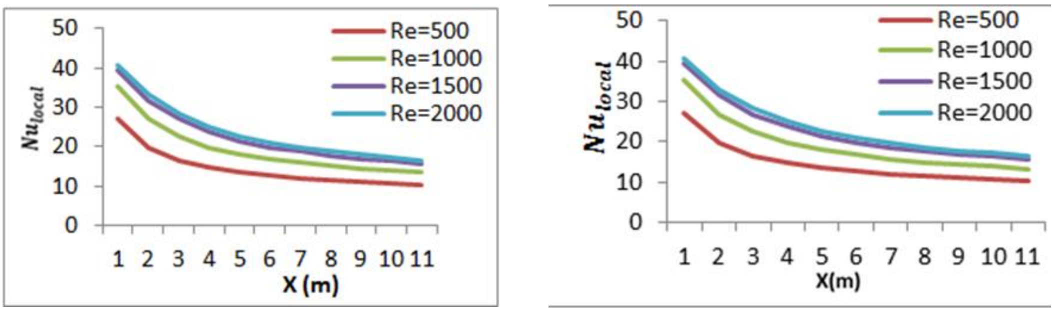


(b) at q=1000 w/m^2

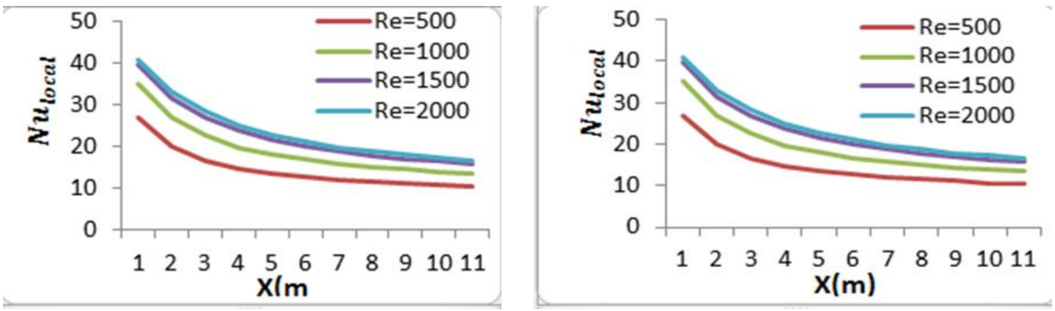


(c) at q=1500 w/m^2 (d) at q=2000 w/m^2

Fig 5. The effect of q_w on wall temperature distribution for various values of Re.



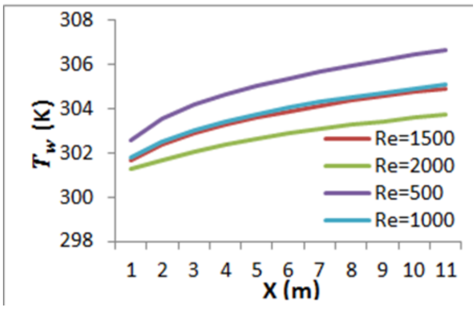
(a) at q= 500 w/m^2 (b) at q= 1000 w/m^2



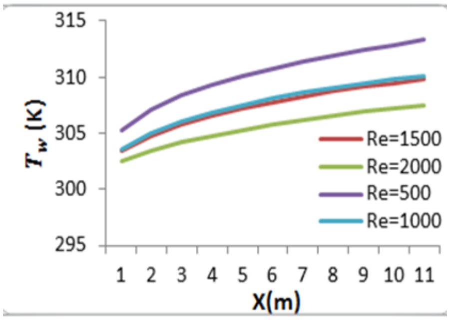
(c) at q= 1500 w/m^2 (d) at q= 2000 w/m^2

Fig 6. The effect of q_w on the local Nusselt number for various values of Re.

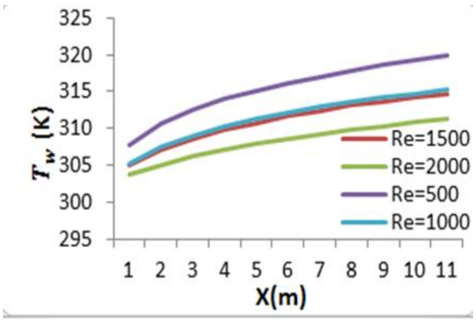
CASE (2) L=60cm, b=0.05m:



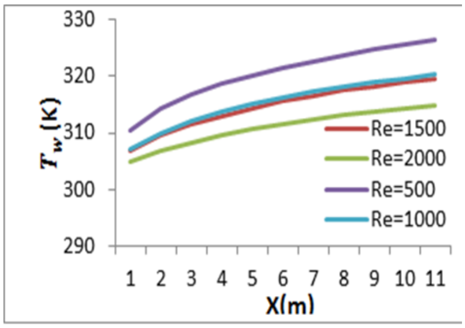
(a) at q=500 w/m^2



(b) at q= 1000 w/m^2

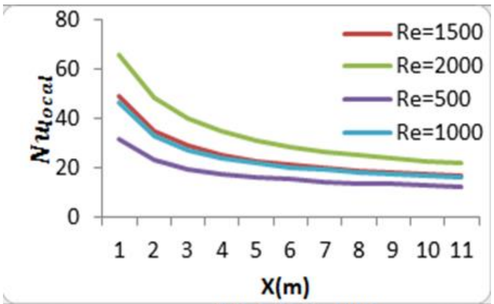


(c) at q= 1500 w/m^2

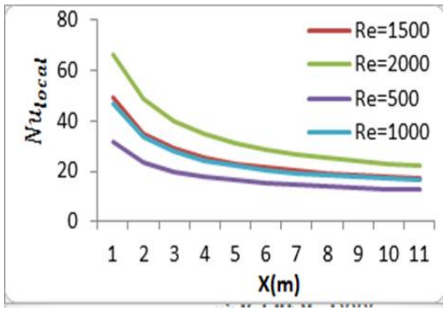


(d) at q=2000 w/m^2

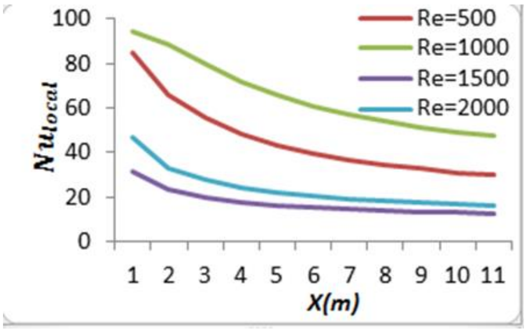
Fig 7. The effect of q_w on wall temperature distribution for various values of Re.



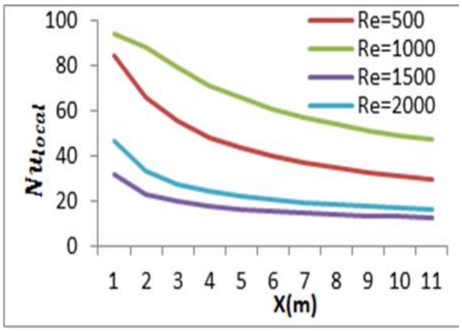
(a) at q=500 w/m^2



(b) at q=1000 w/m^2



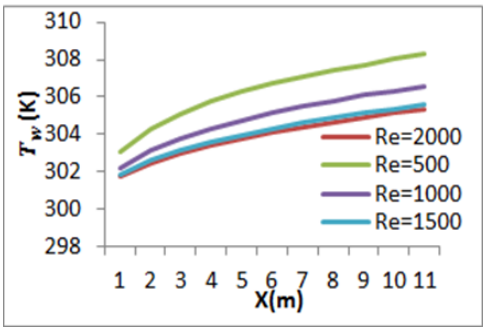
(c) at q=1500 w/m^2



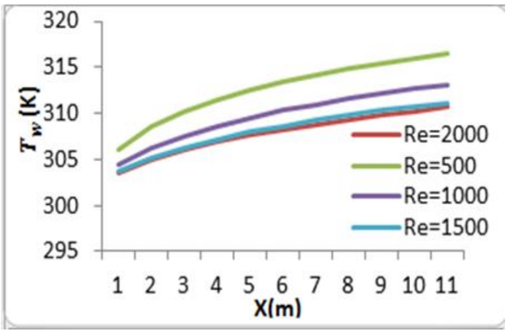
(d) at q=2000 w/m^2

Fig 8. The effect of q_w on the local Nusselt number for various values of Re.

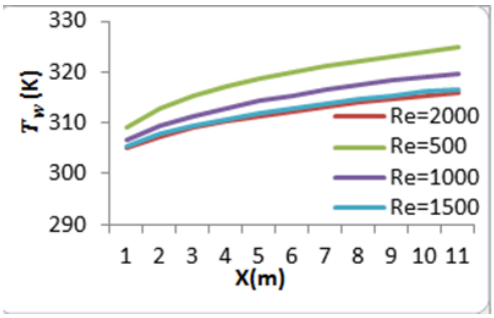
CASE (3) L=60cm, b= 0.075m:



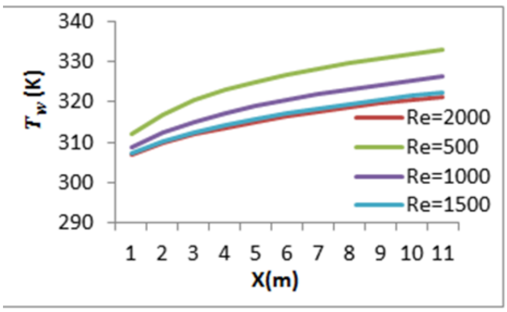
(a) at q= 500 w/m^2



(b) at q=1000 w/m^2

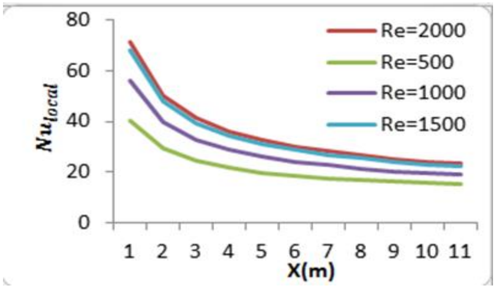


(c) at q=1500 w/m^2

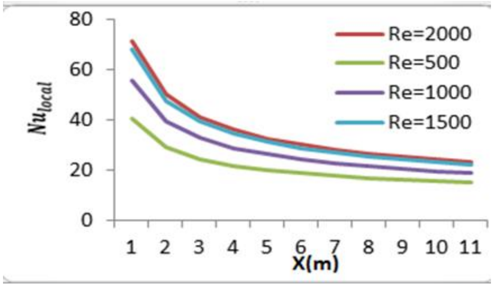


(d) at q=2000 w/m^2

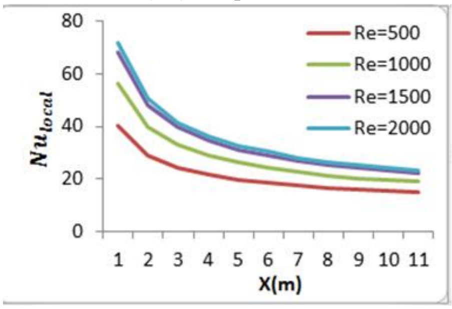
Fig 9. The effect of q_w on wall temperature distribution for various values of Re.



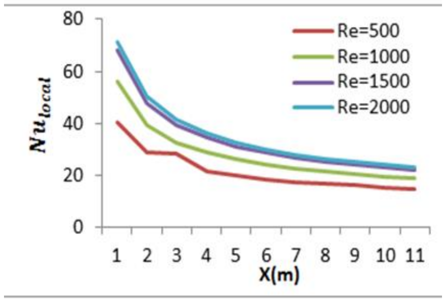
(a) at q=500 w/m^2



(b) at q= 1000 w/m^2



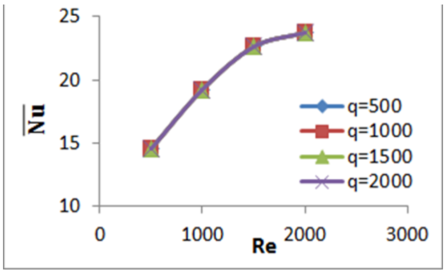
(c) at q=1500 w/m^2



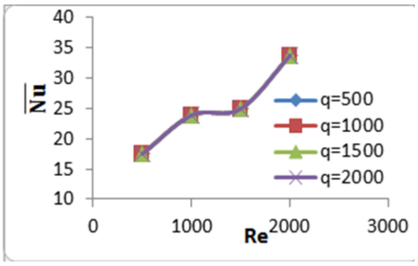
(b) at q=2000 w/m^2

Fig 10. The effect of q_w on the local Nusselt number for various values of Re..

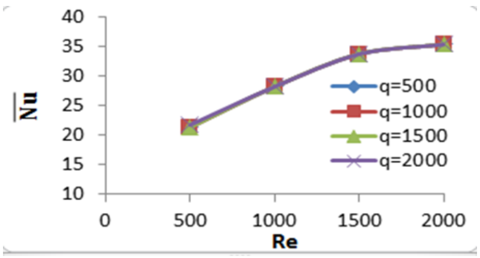
The average nusselt number distribution for the four cases are shown below:



Case1: b=0.025 m



Case2: b= 0.05 m



Case3: b=0.075 m

Fig 11. Influence of the wall heat flux on average Nusselt number.

Generally, for all cases the temperature increases along the flow direction due to the influence of the heat flux which makes the water fluid in the core cooler than the water fluid near the wall. For **Nu** distribution, it is clear that Nu increases by increasing the heat flux and the height of the annulus for all four cases. Results showed that $\mathbf{Nu_{avg}}$ linearly increased by increasing the Re at the same heat flux due to the dominant forced convection in the heat transfer process. Also, the increase of heat flux leads to an increase in the average local Nusselt number.

V. Conclusions

I. Temperature increases along the path of the flow.
II. As the heat flux and the annulus height increase, the temperature values also increase.
III. Nu decreases along the flow direction and increases with the increase of the annulus hight.
IV. The average Nusselt number increases directly with the increase of Re and the annulus size.

Conflict of Interest

There was no relevant conflict of interest regarding this paper.

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