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CHARACTERIZATION OF THE NONLINEAR BEHAVIOR OF FLEXIBLE ROAD PAVEMENTS

Irfan Ullah[1], Dr. Rawid Khan[2], Manzoor Elahi[3], Ajab Khurshid[4]

[1,2,3,4] Department of Civil Engineering, University of Engineering and Technology Peshawar, Pakistan.

[*1]irfanullah@iiu.edu.pk, [2]dr.rawid@uetpeshawar.edu.pk, [3]manzoor.elahi@uetpeshawar.edu.pk,[4]13pwciv@ uetpeshawar.edu.pk

Corresponding Author: **Irfan Ullah**

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Abstract

In this paper, the Asphalt Concrete is analyzed by finite element modeling in Abaqus. The nonlinear viscoelastic behavior of Asphalt Concrete is simulated in Finite Element Analysis (FEA). The X-Ray Computed Tomography scans of the laboratory specimen are converted to a 3D virtual model in image processing software (Simpleware Scan IP). The 3D model is used in FEA by applying boundary conditions and giving mechanical properties, considering the Asphalt Concrete as a viscoelastic material. The mechanical properties of the Asphalt Concrete were determined from the laboratory test performed on the same sample. Three different types of model were analyzed. The representative 3D meshed model and Abaqus meshed model were analyzed for recovery of stress under constant strain and compared. It was concluded that the analysis on a model without considering the actual geometry of the Asphalt Concrete, gives a similar pattern of results but differ by 18% from a laboratory test. The actual 3D geometry of the Asphalt Concrete specimen can be obtained by converting 2D X-ray CT scans. It was also found that the nonlinear viscoelastic analysis on a 3D virtual model of Asphalt Concrete gives 96% similar results to the laboratory tests.

Keywords: Micromechanical Modeling, asphalt concrete, finite element, discrete element, X-ray CT.

I. Introduction

A composite material consists of aggregate, asphalt binder (bitumen), and air voids termed Asphalt Concrete. Aggregate is a collective term that includes crushed stone, gravel, sand and mineral materials in their processed or natural state. Asphalt binder is composed of bitumen which is used as cementations material. Asphalt is available naturally and can be produced by the refining of petroleum. The flexible pavements constructed of asphalt concrete are considered the largest component of infrastructures, all over the world. This material (Asphalt Concrete) has other uses in structural engineering, Asphalt concrete is generally used for paving airports and roads etc. Asphalt concrete revetments have been also researched including coastal embankments and cores of asphalt concrete needed for the construction of embankment dams.

Asphalt Concrete is a complex heterogeneous material. The load-carrying behavior and analysis of asphalt concrete are very complex. Asphalt Concrete has been treated as pure elastic solids for flexible road pavements in mechanistic analyses, however, they more act like viscoelastic materials in response. Up till now the theory of linear viscoelasticity to analysis and design of flexible pavement has not been clearly implemented, though the concept of viscoelasticity is old.

The pavement is a dynamic structure containing many layers of various materials, varying configurations of unpredictable flows of traffic and various environmental factors. A practical evaluation of the actions and efficiency of the asphalt concrete surface is thus of main significance to a Transportation engineer. The result or performance of asphalt concrete flexible pavements is directly linked to the performance of asphalt concrete. Asphalt concrete performance models include links between different processes involved in construction and rehabilitation, pavement design, and asphalt mix design.

The asphalt concrete behaves such that it is more dependent on loading frequency and temperature, contrasting to elastic solids. This is also observed at high loading frequency and relatively low-temperature asphalt concrete behave like elastic solids. While at low loading frequency and a high temperature it behaves like viscous fluids. At average loading frequencies and temperatures, AC behaves like viscoelastic materials that usually exhibit a substantial level of elastic solid stiffness.

The key objective of the study presented here is to integrate viscoelastic material properties calculated by the laboratory test into a 3D finite element (FE) model. This model effectively simulates the nonlinear behavior of asphalt concrete and hence flexible pavement to strain under traffic loading at different conditions. Importantly, the findings of this study greatly enhance the efficiency of existing methods of design and analysis that are frequently used in predicting pavement responses and providing realistic guidance for the design and analysis of flexible pavement in Pakistan, India and related countries in the region.

II. Methods of Micromechanical Modeling of Asphalt Concrete:

The micromechanical approach is grounded upon discretizing the asphalt concrete microstructure and micromechanical modeling or representation of the material properties of its ingredients. The benefit of this methodology is that it represents the material anisotropy and heterogeneity coming about because of the aggregate shape and placing within the asphalt concrete. Two mathematical methodologies have been used to mimic the micromechanical conduct, specifically the discrete element method (DEM) and the finite element method (FEM). Recent research in finite element modeling, techniques of X-ray computed tomography scans and photography are utilized to investigate the interior structure of asphalt concrete specimens. In DEM modeling contact between aggregates (elastic constituent) is represented using a viscoelastic model for contact. It is determined from research that the intricate behavior of the aggregate and Bitumen is the principal cause of the nonlinear inclinations in the mechanical response of asphalt concrete and, and hence susceptible to rutting. With the increase of angularity, the increase in strength of Asphalt Concrete was reported. In this study, the micromechanical behavior of Asphalt Concrete is studied by the finite element modeling method (FEM). The finite element method was preferred due to its accuracy and time-dependent simulation.

III. Finite Element Micromechanics Models of Asphalt Concrete

Most of the models that computational-micromechanical are based on the FE method, overcome the limitation of the models being described earlier in history.

The numerical analysis of the micromechanical behaviour of asphalt concrete was carried out by a numerical modeling scheme based on micromechanical simulation using the system of finite elements [VI]. A network of special frame elements built from an approximate elasticity solution was first introduced into the finite element scheme. Within this solution, a damage mechanics methodology was then implemented and this directed to the development of a model able to envisage AC (Asphalt Concrete) inelastic behaviour. In order to perform simulations of real laboratory specimens, this principle was then applied within the ABAQUS FEA code. Indirect tension tests (IDT) were performed in a sequence of model simulations. Simulation results of the overall sample behavior compared with experimental results. Simulation outcomes were correlated with the experimental data obtained. Compared with the results, the simulation of load-deformation of the overall sample compared favorably.

The damage model was capable of predicting the extensive softening activity observed in real asphalt materials correctly. Lastly, a short exploration of the evolution of internal micro damage within the models of indirect tension tests was performed. In [III] developed a finite element model to research the impact of local distribution of strain on Hot Mix Asphalt (HMA) by using the microstructure obtained by image analysis, which identified particles greater than 0.3 mm as aggregates and others as mastics. The [V] used the two-phase microstructure of asphalt concrete mixtures to forecast the dynamic shear modulus and phase angle by the finite element technique (FEM) and confirmed by comparing the results to those from the super-pave shear tester. (Abbas et al. 2005) used the discrete element method (DEM) to predict the dynamic modulus of asphalt mastics made from different binders and minerals passing through 0.075 mm sieve in different volumetric proportions and compared them to other micromechanics models and the dynamic shear Rheometer results. This model appeared to be more accurate than other models. In [II] studied the two distinct aggregate and mastic components of the HMA microstructure, which were obtained by cutting the HMA and scanning them. They subjected the microstructure to oscillatory loads to simulate the simple performance tests (SPT) in two dimensions using the DEM for analysis. They found that their results overestimated the dynamic modulus of mixtures with original binders but underestimated for modified binders.

The [VII] also presented micromechanical FE and DE models to predict the overall stiffness of asphalt concrete by using the creep stiffness properties of the two distinct aggregate and mastic components. To determine the creep stiffness, they performed uniaxial compressive tests of aggregates and creep compliance tests of mastic samples fabricated by mixing binder and fine aggregates passing through a 2.36 mm sieve. By comparing the results of the two models with the laboratory test results, they concluded that both methods were applicable for predicting the creep stiffness of the mixtures.

Recently, [IV] predicted the dynamic modulus of asphalt mixtures by using two semi-empirical, models, one analytical model and one computational micromechanics model. The researchers obtained the properties of the fine aggregate matrix with hydrated lime by sweeping the oscillatory frequency at several temperatures and then obtained the characteristic Prony Series coefficients from their master curves.

IV. Experimental Models of Asphalt Concrete:

Asphalt concrete mixture is the common complex material that contained two dissimilar components with completely different behavior, i.e. aggregates which elastic and bitumen which is viscous or sometimes viscoelastic. The response and properties of constituent material and their interaction with each other define the overall behavior of the blend (Asphalt Concrete). The behavior of viscoelastic mixtures is represented by a characteristic master curve which is generated simply by shifting the frequency sweep at numerous temperatures to one reference temperature until the resultant curve is smooth.

There are several standards in use for determining the dynamic modulus, which conduct frequency sweeps from 25 Hz to 0.01 Hz at -10°C to 54.4°C in sequence. [VIII] Suggested characterizing dynamic modulus as the most important property of hot mix asphalt mixtures by performing the indirect tensile (IDT). They recommended increasing the testing frequencies by two and decreasing the number of testing temperatures to three to decrease the total testing time without compromising the quality of the tests.

The composite Asphalt Concrete is intricate in properties and its behavior is affected by the level of stress/strain, rate of stress/strain and temperature at the time. Also, viscoelastic (recoverable) and viscoplastic (irrecoverable) strain response can be observed are included in the overall reaction of asphalt materials subjected to applied stress. As the temperature drops and the loading rate increases, the viscoelastic portion of the reaction becomes more predominant.

Depending on the applied stress/strain limits and temperature, the relationship between stress and the recoverable strain variable can be nonlinear. As the viscoplastic reaction is also complex and as temperature rises and the rate of loading decreases, it becomes more dominant.

To establish the constitutive relationships for representing these components (elastic and viscous) and decide the parameters of the model associated with each part, it is important to distinguish the recoverable and irrecoverable pressure. Therefore, a mechanistic model that incorporates all the elastic, viscous and viscoelastic components of the material response needs to be developed. In order to forecast efficiency under practical boundary conditions describing the laboratory and the onsite situation, the mechanistic model needs to be implemented in finite elements.

The key objective of the study presented here is to integrate viscoelastic material properties calculated by the laboratory test into a 3D finite element (FE) model. This model effectively simulates the nonlinear behavior of asphalt concrete and hence flexible pavement to strain under traffic loading at different conditions. Importantly, the findings of this study greatly enhance the efficiency of existing methods of design and analysis that are frequently used in predicting pavement responses and providing realistic guidance for the design and analysis of flexible pavement in Pakistan, India and related countries in the region.

V. Laboratory Specimen Preparation:

The Asphalt mixture laboratory specimen were prepared with the crushed mineral stone aggregate and 60-70 grade bitumen as a binder. The selected crushed stone aggregate had a "Wear percentage" of 26% by the Los Angeles Abrasion test (AASHTO T-96). Aggregate crushing value (ACV) was 19% according to experiment BS-812 (1990). The weight loss was recorded as 7.5% when subjected to five cycles of sodium-sulfate soundness testing as calculated by AASHTO T-104. The asphalt binder (Bitumen) was homogeneous, free from water, of 60-70 grade with Flash Point of 232°C and 100 cm ductility at 25°C according to (AASHTO M 20-70). The aggregate gradation is shown in Figure 2.

The binder and the aggregates were mixed at a temperature of 165ºC. A cylindrical specimen was prepared to have a height of 100 mm and a diameter of 100 mm in the SuperPave^{TM} gyratory compactor. At a temperature of 150°C, the sample was compacted in the Superpave Gyratory Compactor (SGC). The binder (Bitumen) content was 5.5% and air content was targeted at about 6% according to the general specifications of the National Highway Authority (NHA).

VI. X-ray CT Imaging:

The non-destructive CT X-ray technology typically characterizes a heterogeneous, opaque, composite material's internal structure. In particular, the use of X-ray CT for Asphalt Concrete applications is constantly growing.

In this research, a special X-ray computed Tomography (CT) method was used to characterize the internal microstructure of the Asphalt Concrete specimen. The slices (sequence of horizontal planar images of the specimen) were obtained having two-dimensional information at a specific interval across the section of the specimen. The linear distance between two successive 2D scans across the height of the specimen is termed as slice interval. The slice interval was kept at 83 micrometers all over the height of the prepared sample.

All the data in the form of 2D images (scans) are saved in the memory of the X-ray Computed Tomography (CT) system, for further processing. The representation of 2D scans is shown in figure 1.

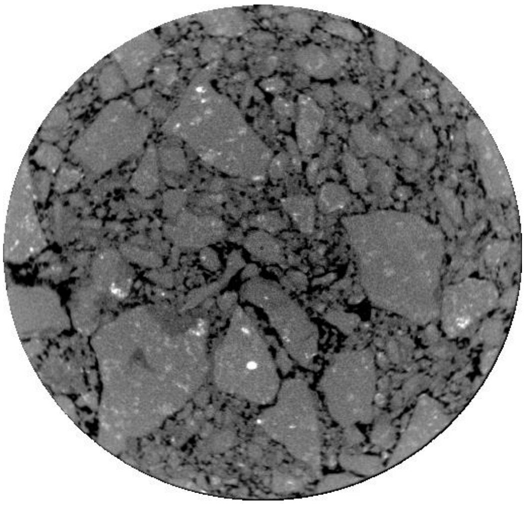


Fig. 1: Two-dimensional scans of the Laboratory Sample

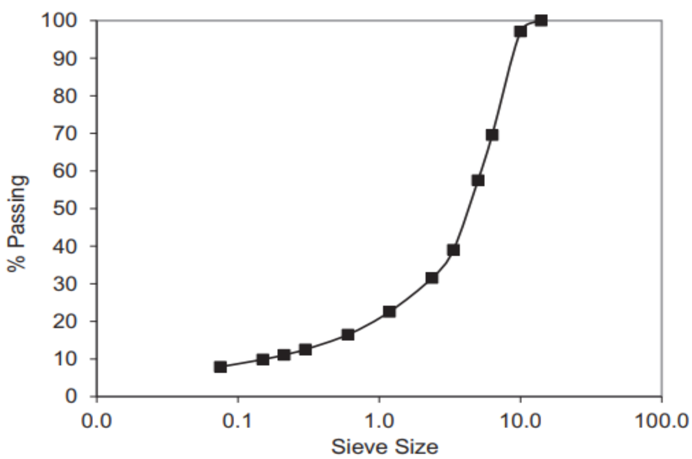


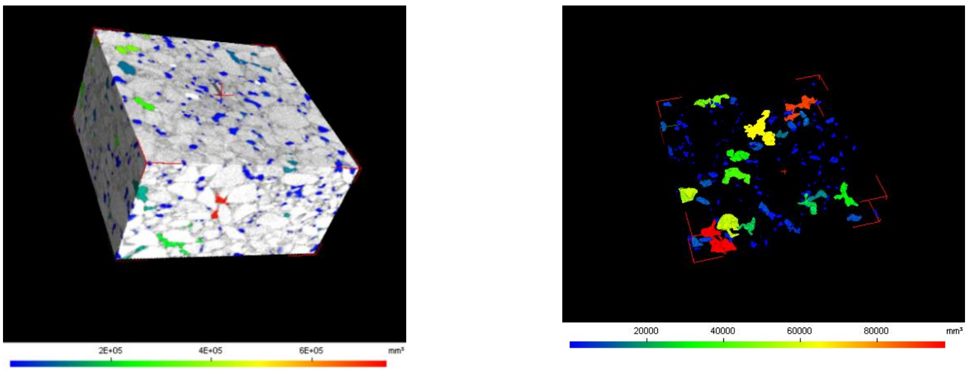
Fig. 2: Aggregate gradation curve

Therefore, to represent the true structure of aggregates and air voids, the two-dimensional scans do not represent the actual characteristics of a heterogeneous asphalt sample. In the Simpleware ScanIP image processing programme, the resulting two-dimensional scans have been imported. A three-dimensional representative image was created by stacking two-dimensional scans of the original Asphalt Concrete sample. A detailed discussion of the process of 3D modeling from 2D images is coming in the following section.

VII. Model development in Simpleware:

The development of such a 3D representative model from 2D images was done in Simpleware through the major steps including data acquisition, image processing, generation of mesh, and importing the generated mesh in such a format that is capable of using in the targeted finite element modeling software. In this study, the ABAQUS is used for the sack of finite element modeling (FEM).

Two-dimensional scans of the laboratory prepared sample, in the form of stacks, are loaded into the ScanIP package of Simpleware software. Deformities in scans (2D stacks) in the form of brightness, contrast and noise have been excluded/rectified prior to analysis. The quality of the image and the slice interval were addressed during the analysis of three-dimensional images. The 2D scans are segmented according to the sequence of being followed during the X-ray CT scanning. Then Thresholding was implemented on the collected and segmented scans, in image processing, to distinguish components of the asphalt concrete. The method used to classify the components of the mixture is termed Thresholding. It transforms a component's representative greyscale into a value. This value is then used in the mixture to distinguish the component and then to represent all the constituents in the actual asphalt mixture in the image. The processes of segmentation and Thresholding is



represented in the figure 3.

Fig. 3: Model during process of Thresholding

A three-dimensional finite element mesh was generated from the grey scaled three-dimensional images in the software. Using the finite element (FE) module, a clear, decent quality volume mesh of the sample cylinder was developed. The 3D virtual model created comprises 210309 four-nodded tetrahedron elements. The exact geometry of the compacted asphalt specimens is reflected by the mesh. This reflects the real microstructure and the engineered air voids of the asphalt mixture as compared to an imagined mesh.

After completion of image processing and 3D modeling and meshing of the model, the meshed model is saved in such a format that can be supported by ABAQUS (finite element software). (.inp) is the format which is supported by ABAQUS, and hence saved the finalized model in the mentioned format for further analysis in finite element programme.

VIII. Relaxation Test:

An experimental technique for assessing the viscoelastic characteristics of materials is the stress relaxation test, which is widely used to determine the stress relaxation properties of viscoelastic materials like asphalt concrete and bitumen etc. The stress response is based on the history of loading.

According to the procedure of the stress relaxation test, a constant strain of 0.004 mm/mm was applied to the specimen prepared in the laboratory (Section 5) in the universal material testing machine. The required strain was achieved in approximately nine seconds. The strain was maintained constant as nearest possible to the value targeted, for a specific time period. This duration of time is called the relaxation time. The stress response (relaxation in stress) was observed during a relaxing time. For a viscoelastic material, owing to its properties, stress relaxation for a constant strain is recorded. At the time of applying strain, the stress started from zero and reached maximum at the time when the strain reached 0.004 mm/mm. After attaining the maximum value the stress was starting decreasing gradually with the passage of time. The relaxation time period was designed to be 30 minutes (1800 seconds), and it is the medium range of time in which the asphalt concrete specimen or pavement can recover a maximum of applied strain according to its capacity after removal of load. While the strain is 4% of the total height of the sample which sufficient for reasonable results. Figure 4 shows the specimen during the test.

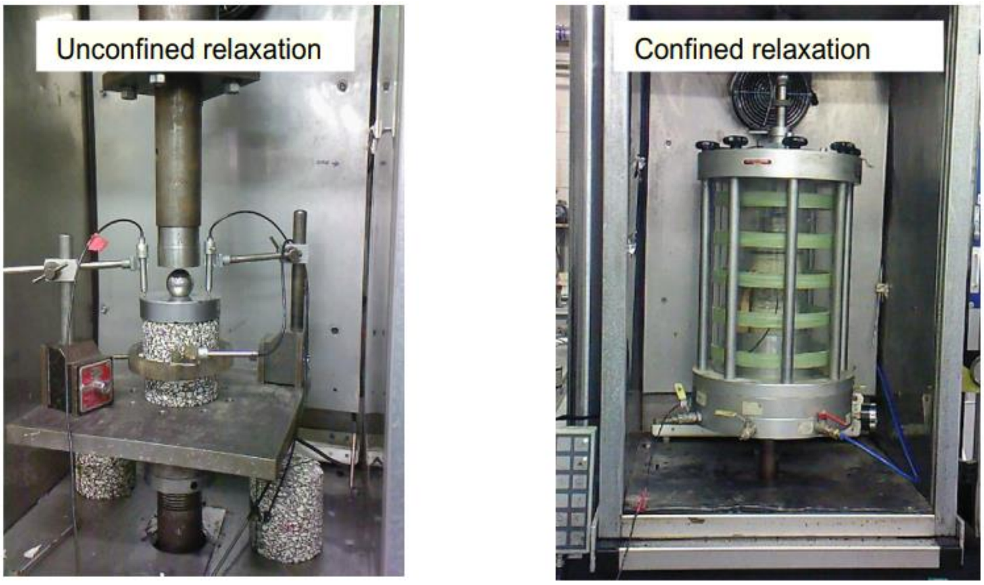


Fig. 4: Assembly of stress Relaxation test with specimen loaded

The test was conducted in the cycles of constant strain applications, i.e. after application of initial strain and allowing for 1800 second to recover stress, applied another increment of equal strain and held for relaxation time while undergoing relaxation in stress.

Stress values are fluctuating a little bit with time, but still in the pattern of decreasing. To fit the curve in the single pattern following a certain equation, the Fitting of Curve is performed in MATLAB and Excel. The curve Fitted in excel had a greater "R" value, therefore, used in the determination of viscoelastic parameters such as relaxation modulus, which were later used as an input in the finite element analysis software (ABAQUS).

IX. Finite Element Analysis in ABAQUS

In ABAQUS performed analysis for three conditions.

i. Model meshed in ABAQUS, applied a constant strain of 0.004 mm/mm for 30 minutes without applying side confinement pressure.

ii. Model meshed in ABAQUS, applied a constant strain of 0.004 mm/mm for 30 minutes along with applying 200 KPa side confinement pressure.

iii. Model meshed in Simpleware, applied a constant strain of 0.004 mm/mm for 30 minutes along with applying 200 KPa side confinement pressure.

The first two conditions are performed by adopting the same strategy as the detailed procedure is described in this section. The third condition is applied such that a mesh is created through Simpleware software and imported in Abaqus. These coming paragraphs elaborated the procedure to perform various modules in Abaqus.

A three dimensional deformable solid part was created, while the type was kept extrusion and approximate size of one meter. In the first two cases of model development, the part has the same above characteristics with less or more changes in the coming modules. The name of the model can also be given in this module, the name should be such that it gives a little information about the model. Enter continue, the window will open says "sketch the section for the solid extrusion" which requires the dimensions of the model. Keep in mind that once you start in a system of the unit for dimensions then keep the same unit throughout the development of the model same. We have a 100 mm x 100 mm model, so draw the length and width and click "done" after which put the depth of 100 mm in the popup window and click "ok". A 3D box (model) will be generated. This the start of the modeling.

The material properties of all of the three models created are the same, in which uniformly distributed mass density is kept 22100 kilograms per cubic meter. Elastic properties; isotropic Young Modulus of 2759 mega Pascal and 0.35 poison's ratio were given. Viscoelastic properties; relaxation test data were given in the time domain. The data was extracted in the form of relaxation modular ratio with time from the relaxation test explained in chapter three.

Three types of models were generated. In the first two types, both section and material properties were assigned to the part created. While in the 3rd type of model, in which the part and geometry were created in the Simpleware (image processing software) and imported to ABAQUS, only material properties and loading were assigned and there is no need to assign the section because the imported section is more realistic than the assumed section. Figure 5 show the realistic model given the material properties of viscoelasticity from the laboratory results.

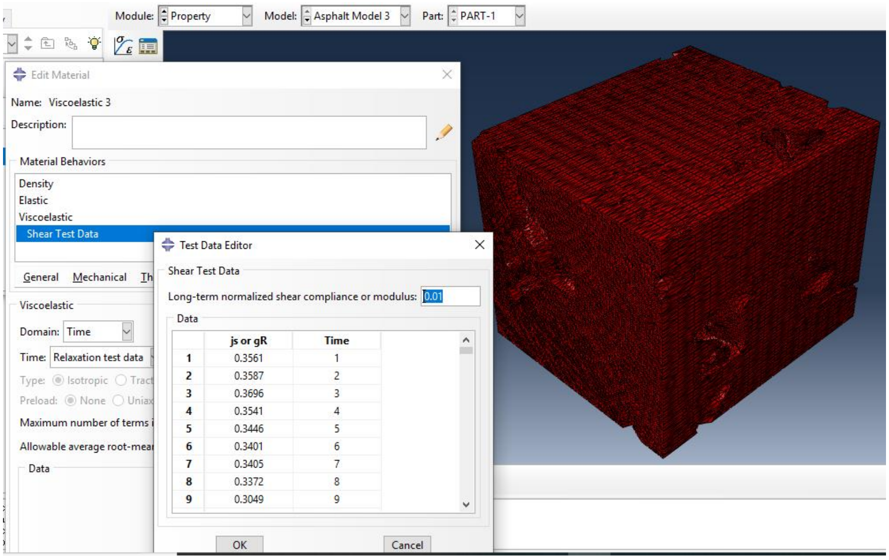


Fig. 5: Property Module applied to Model 3 in ABAQUS

Mostly the General procedure type is adopted and it is also by default, hence, the General procedure type is used in this study as well. Furthermore, the General procedure provides a dozen of analysis types where one can select the analysis of their interest. Dynamic Explicit General analysis was choose for this research because the nature of load in the case of traffic is not purely static but moving loads of traffic.

So the Step module for all three cases is the same, because whatever the conditions are, but a single analysis procedure was adopted for simulating the results, which is the relaxation stress of asphalt specimen under the condition of constant strain.

The Load Module is different for all three models used in this study. We used two types of loading.

Model One: for the first model two boundary conditions were used, which were the fixed base and the uniform strain of 0.004 mm/mm applied on the top of the model in z-z direction, according to the procedure explained above.

Model Two and Three: for the 2nd and 3rd model three boundary conditions were used. The base of the model was kept fixed by applying a boundary condition of the Mechanical (Symmetry) category. At the top of the model, a uniform strain of 0.004 mm/mm strain was applied for the whole analysis time i.e. for 1800 seconds. And the last boundary condition applied to both the mentioned models was confinement pressure of 200 KPa. This is the difference between the three models. Side confinement pressure shows that a specific piece of road (specimen) is not free from all sides but confined by other fellow material in the actual field condition. Figure 6 shows the Boundary conditions applied to Model 3. The difference between model 2 and model 3 is meshing. The meshing of the Model 2 was performed in the ABAQUS software and keep the size as nearer to the actual as possible. While the meshing for the Model 3 was performed in special image processing software called Simpleware, which is capable of prepare a 3D representation of the actual sample prepared in the field/lab. For the refined results of the Asphalt Concrete specimen, the above-mentioned methodology was adopted.

Meshing divides the model into finite elements of the given size. The parts and assemblies that were created in ABAQUS, can be meshed with the tools in the Mesh Module.

In this research, the "Approximate global size" for both Model 1 and Model 2 were kept

0.005, while Model 3 was meshed already before importing in the ABAQUS. Tick the box

for applying curvature control and the rest of the setting was kept by default.

Model 3, which was generated as a result of combining 2D scans into a 3D model in an image processing software "Simpleware", was meshed in Simpleware rather than in ABAQUS. The meshing quality of the 3rd model was finer than the first two models and reflect the actual geometry and appearance of the sample being prepared in the laboratory.

The Job of the Model is first created, given the relevant name and submit for full analysis. This procedure was adopted for Model 2 and Model 3 as well.

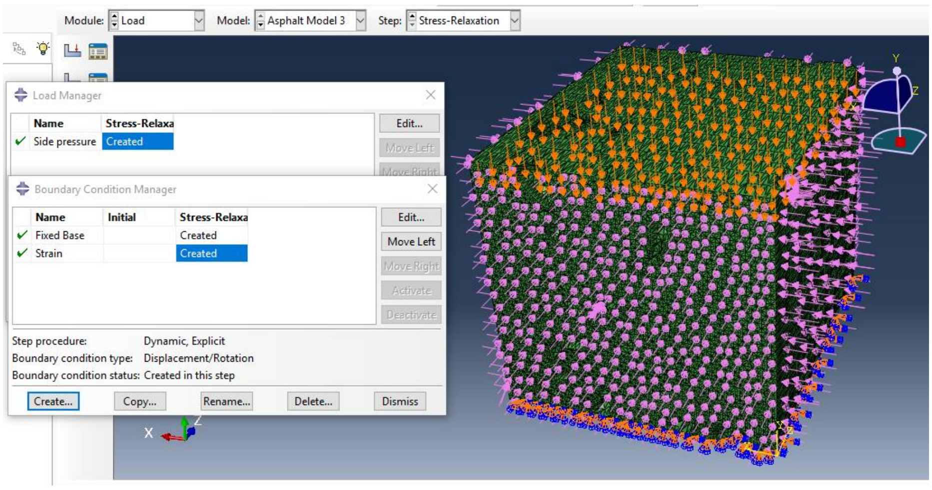


Fig. 6: Applying Load boundary conditions to Model 3.

X. Finite Element Analysis Results and Validation of Model with Laboratory test

After finite element analysis of different Models in the ABAQUS, the Stress Relaxation curve of the model 1 and 2 are presented in figure 7 and 8. The pattern of the test has been followed, but from figure 9, where the comparison of the Model 1 and 2 are depicted, it is obvious that both of the models have considerable difference with the laboratory test.

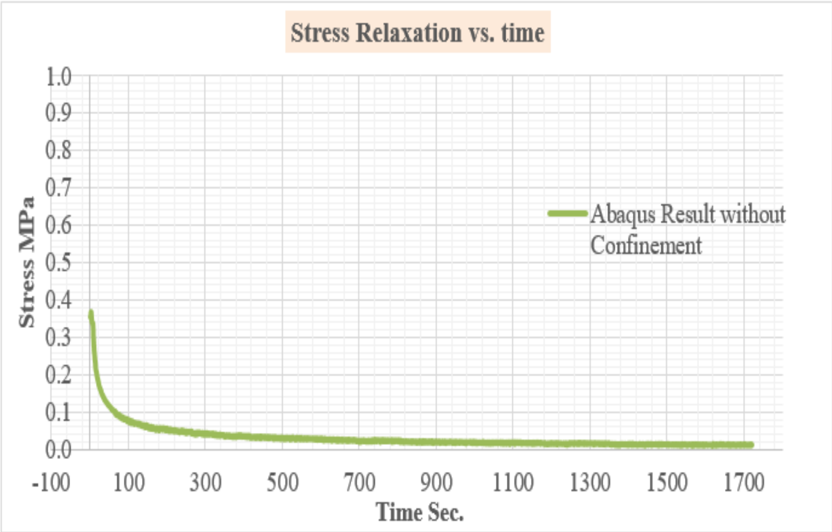


Fig. 7: Abaqus Analysis for Model 1

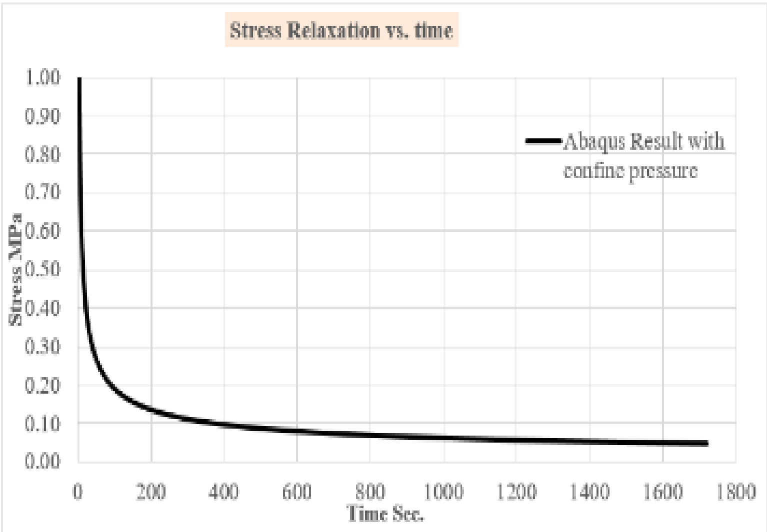


Fig. 8: Abaqus Analysis for Model 2

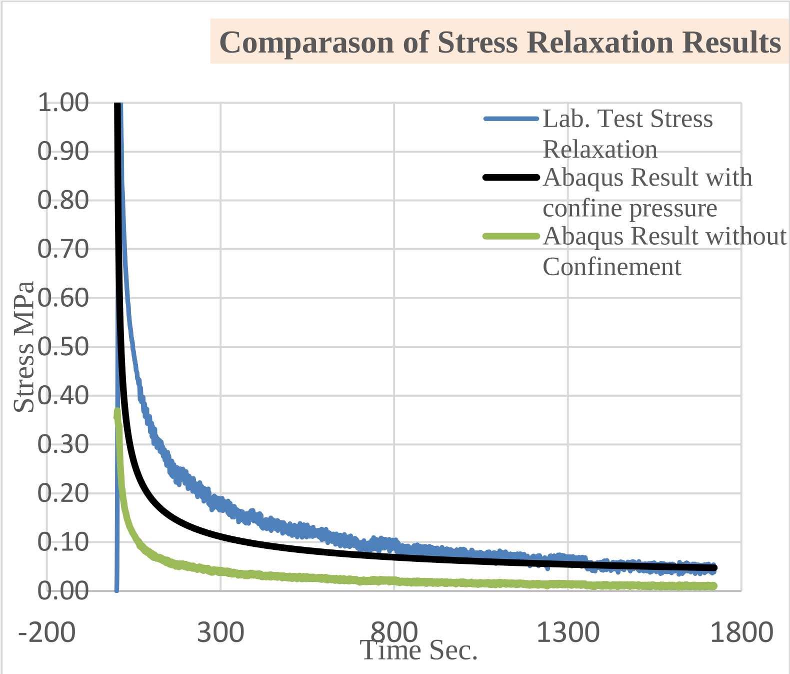


Fig. 9: Comparison of Model 1 and Model 2 with test results

The 3rd Model which is created in the Simpleware (an image processing software), loaded and analyzed in the ABAQUS, gave approximately similar results to the laboratory test results for the recovery of stress under constant strain with the passage of time. The results of Model 3 and its comparison with practical laboratory tests are shown in Figure 10.

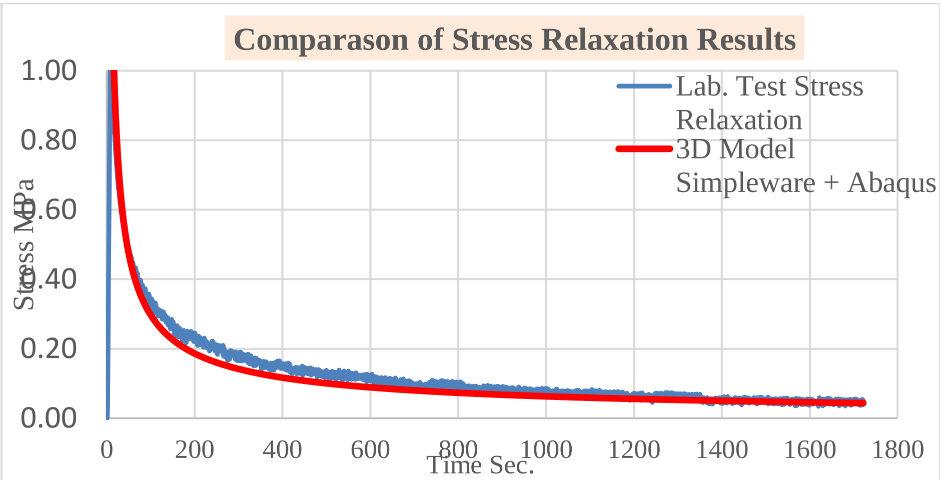


Fig. 10: Comparison of Model 3 with test results

The similarity of the three models with the actual properties of the Asphalt laboratory sample is presented in the Figure 11.

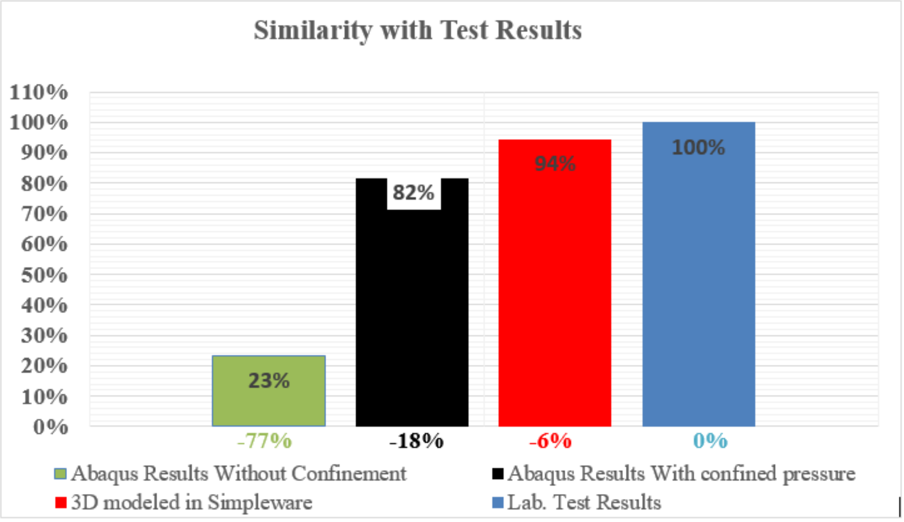


Fig. 11: Similarity of different Models with Lab results

XI. Conclusion

Below are the conclusions that can be drawn from the research based on the studies and the test results.

- A micromechanical modeling framework based on the FEM was developed to predict the response of the AC material under constant strain.
- The results show that the applications of stress/strain on the asphalt specimen without considering the side confinement pressure cannot exhibit the actual stress level.
- The model (ABAQUS model without confined pressure) simulation results show that the stress is relaxed (decreased) very quickly after gaining the peak value for a specific strain (0.004 mm/mm in this case).
- The model (ABAQUS model without confined pressure) simulation results show only 23% similarity with the test results, which clearly underestimate the stress in the relaxation test.
- The model (ABAQUS model with confined pressure) simulation results show 82% similarity with the test results, which predict the behavior of Asphalt concrete with an error of 18% (shows reduced stress).
- In comparison with the viscoelastic FE model, the elastic FE model without considering the effect of side pressure and actual material structure significantly underestimated pavement response, which would not be conservative for design purposes.
- The virtual model created from the 2D scans of the actual specimen made a 3D model of asphalt specimen in image segmentation and processing package, meshed the model so fine that reflect the actual geometry and exported to FE software.

- The meshed model imported from Simpleware is given the material properties calculated from the practical test (Stress relaxation test), performed dynamic analysis.
- For material like AC, a three-dimensional model through Computed Tomography is proved to be efficient by the FE analysis in Abaqus.
- The figures indicate that the experimental curves were close to the model curves for the 3D model prepared in Simpleware and analyzed in ABAQUS.
- The 3D model prepared in Simpleware, analyzed in ABAQUS shows 96% agreement with the practical results of the test.
- The 4% error is due to the interface between the aggregate and the asphalt mortar was not considered in this study.
- Normally the results with an error of less than 10% are considered to be good results.

XII. Recommendation

Based on the computational micromechanical FE analysis conducted and the results obtained, the following recommendations can be suggested:

- While this study modeled the material from the Relaxation test data, future work can utilize other tests like creep recovery and frequency-based data to model the Asphalt concrete.
- The research can be further enlarging by applying different tests simultaneously and using different materials.
- This research assumes the homogeneous response of material while executing the analysis because the test gives the overall result of the specimen, in the future, the heterogeneous properties may be employed.

Conflict of Interest

There was no relevant conflict of interest regarding this paper.

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