Docol©c

Docol©c-Report
Digital signed

Reviewed document: **jmcms-2010040 ELZAKI TRANSFORM FOR MECHANICS PROBLEMS (UMAMAHESHWAR RAO) 23-8-2020.docx**
Processing date: **10.9.2020 18:28 CEST**

A total of 61 sentences were analysed. As a result **18** sentences (29.5%) were found in other documents.

These sentences are highlighted in the text by using different color shades according to the amount of similarity. The darker the highlighting color, the more words were found in another document. You may click the highlighted sentences in order to get further details about found reference documents. Learn more about this report and how to evaluate it.

The following graphic shows the distribution of found sentences within the checked document. The colored parts of the overview bar indicate those parts of the document in which sentences were found in other documents. The left boundary of the bar corresponds to the beginning of the document and the right boundary to the end of the document accordingly. By clicking into the overview bar you are directed to the corresponding position in the document.

Reference documents

The following list contains titles and addresses of documents in which similar sentences were found. With a click on the number of found sentences („ **x** Sentences") the corresponding sentences are highlighted in the document as well as in the navigation bar by a colored border and you are directed to the first position of the corresponding sentences in the document. Another click on „**x** Sentences" resets the highlighting.

2 Sentences were found in a text with the title: „ ***Advanced Engineering Mathematics | Statistical Hypothesis ...***", located at:
https://pt.scribd.com/document/260839956/Advanced-Engineering-Mathematics

2 Sentences were found in a text with the title: „ ***Schaum's Outline of Laplace Transforms***", located at:
https://www.scribd.com/document/211531563/Schaum-s-Outline-of-Laplace-Transforms

2 Sentences were found in a text with the title: „ ***Solved: A particle P of mass 2 grams moves on the x-axis ...***", located at:
https://study.com/academy/answer/a-particle-p-of-mass-2-grams-moves-on-the-x-axis-and-is-attracted-to-the-origin-o-with-a-force-numerically-equal-to-8x-if-it-is-initially-at-rest-at-x-10-find-its-position-at-any-subsequent-time-as.html

2 Sentences were found in a text with the title: „ ***IJREAMV04I0642088.pdf***", located at:
http://ijream.org/papers/IJREAMV04I0642088.pdf

2 Sentences were found in a text with the title: „ ***GJESR - 1.pdf***", located at:
https://www.gjesr.com/Issues PDF/Archive-2019/October-2019/1.pdf

2 Sentences were found in a text with the title: „ ***Applications of Mahgoub Transform to Mechanics, Electrical Circuit Problems***", located at:
https://www.ijsr.net/archive/v7i7/ART20183631.pdf

2 Sentences were found in a text with the title: „ ***Solved: 1) A Spring Hangs Vertically. A Weight Of 10 Lbs ...***", located at:
https://www.chegg.com/homework-help/questions-and-answers/1-spring-hangs-vertically-weight-10-lbs-attached-spring-stretches-2-inches-weight-replaced-q4559978

2 Sentences were found in a text with the title: „ ***Applications of Spring/Mass Systems***", located at:
http://faculty.valenciacollege.edu/pfernandez/des/Spring-Mass APP.doc

2 Sentences were found in a text with the title: „ ***Mathematics-Differential Equations Crash Course by Stephen ...***", located at:
https://issuu.com/stephennikita/docs/mathematics_-_differential_equations_crash_course_/60

2 Sentences were found in a text with the title: „ ***differential equations.61 - 54 DIFFERENTIAL EQUATIONS ...***", located at:
https://www.coursehero.com/file/13281398/differential-equations61/

2 Sentences were found in a text with the title: „ ***Solved: Solve The IVP By The Method Of Laplace Transforms ...***", located at:
https://www.chegg.com/homework-help/questions-and-answers/solve-ivp-method-laplace-transforms-y-2y-5y-c-sin-y-0-0-y-0-1-must-use-laplace-transforms--q30428291

▶ In 91 further documents exactly one sentence was found. (click to toggle view)

Subsequent the examined text extract:

ELZAKI TRANSFORM FOR MECHANICS PROBLEMS

Umamaheshwar Rao R and Yuvaraju Namala

Department of Science and Humanities, Sreenidhi Institute of Science and Technology, Ghatkesar, Hyderabad, 501301, Telangana, India.

ummrao@sreenidhi.edu.in*, yuvarajun@sreenidhi.edu.in

Abstract

The aim of this paper is to discuss the applicability of new kind of transform technique named as Elzaki transform in solving the higher order ordinary linear differential equations occurred in various categories of vibrations in the field of engineering mechanics.

Keywords: Elzaki Transform, Differential equations, Vibrations, Beams.

1. Introduction

1.1 Definition: The Elzaki transform is defined for function of exponential order, we consider function on the set A defined by A={ft:∃M,k1,k2>0,ft<Metkj,if t∈-1j×0,∞} , where the constant M must be a finite number, k1,k2 may be finite or infinite. The Elzaki transform of the function f (t) is defined as E{ft}=Tv=v0∞fte-tvdt, t>0 , v∈(-k1,k2).

1.2.1 Elzaki transform of standard functions:

f(t)	1	t	tn, n∈N tn, n>0	eat	teat	tn-1eatn-1!, n=1,2…
E[f(t)]	v2	v3	n!vn+2 τn+1vn+2	v21-av	v3(1-av)2	vn+1(1-av)n

f(t)	Sinat	cosat	sinhat	coshat
E[f(t)]=T(v)	av31+a2v2	v21+a2v2	av31-a2v2	v21-a2v2

1.2.2. Elzaki Transform of derivatives of function f(t): If E{f(t)} = T(v) then

(i) E{f'(t)}= 1vE{f(t)}-vf(0)

(ii) Ef''t= 1v2Eft-f0-vf'(0)

(iii) Efnt=1vnEft-k=0n-1v2-n+kfk0, n=1,2,3……

1.3. Ordinary linear differential equation with constant coefficients:

An equation of the form dnydtn+C1dn-1ydtn-1+C2dn-2ydtn-2+…+Cn-1dydt+Cny=Ft ___(1.3.1)

Where C1,C2… Cn are real constants and F(t) is a function of the independent variable t is called ordinary linear differential equation with constant coefficients of order n.

Elzaki transform to ordinary differential equations

The Elzaki transform may be used to solve higher order ordinary linear differential equations with constant coefficients and using given initial or boundary conditions. Let y0=a0 , y'0=a1 , y''0=a2 ,…… , yn-10=an-1 ___ (2.1)

be the given initial or boundary conditions where a0, a1, a2, ……, an-1 are constants. On taking the Elzaki transform of both sides of (1.3.1) and using conditions (2.1) we obtain an algebraic equation known as 'subsidiary equation' from which y(v) = E{y(t)} is determined. The required solution is the obtained by finding the inverse Elzaki transform of y(v).

2.1 Solve d2ydt2+3dydt+2y=0, y0=0, y'0=1.

Sol: The given equation can be written as

y''t+3y't+2y(t)=0, y0=0, y'0=1 ______ (2.1.1)

Taking the Elzaki transform to (2.1.1), we get E{y''t}+3E{y't}+2E{y(t)}=0

=> 1v2Eyt-y0-vy'0+31vEyt-vy0+2Ey(t)=0

=> 1v2+3v+2Eyt-0-v.1-0=0 => 1+3v+2v2v2Eyt=v

=> Eyt=v31+3v+2v2 =v3(1+2v)(1+v) =v21+v-v21+2v __________ (2.1.2)

Applying inverse Elzaki transform to (2.1.2), then y(t)=E-1{v21+v}-E-1{v21+2v} ∴ yt= e-t-e-2t is the solution.

2.2. Solve y''+y'=t2+2t, yt=4, y't=-2 **at t=0.**

Sol: Given y''t+y't=t2+2t, y0=4, y'0=-2 ________ (2.2.1)

Taking the Elzaki transform to (2.2.1), we have E{y''t}+E{y't}=E{t2}+2E{t}

=> 1v2Eyt-y0-vy'0+1vEyt-vy0=2v4+2v3

=> 1v2+1vEyt-4-v.-2-v.4=2v4+2v3

=> 1v2+1vEyt=2v4+2v3+2v+4 t => (1+vv2)Eyt=2v4+2v3+2v+4

=> Eyt=2v6+2v5+2v3+4v21+v =2v5v+1+2v2(v+2)1+v

∴E{yt}=2v5+2v2+2v21+v ____________ (2.2.2)

Taking inverse Elzaki transform of both sides of (2.2.2), we get

yt=2E-1{v5}+2E-1{v2}+2E-1{v21+v} =2t33!+21+2e-t

∴y(t)=t33+2+2e-t is the solution.

2.3. Solve D4-1y=1 , y=Dy=D2y=D3y=0, **at t=0, where** D=d dt.

Sol: The given equation can be written as

y''''-y=1, y0=y'0=y''0=y'''0=0 ________ (2.3.1)

Taking the Elzaki transform to (2.3.1), we have Ey''''t-Eyt=E{1}

=> 1v4Eyt-1v2y0-1vy'0-y''0-vy'''0-Eyt=v2

=> 1v4-1Eyt-0-0-0-0=v2

=> Eyt=v61-v4 =-v2+v21-v4 =-v2+v2(1-v2)(1+v2)

=-v2+ 12v21-v2+12v21+v2 ___________ (2.3.2)

Taking inverse Elzaki transform of both sides of (2.3.2), we get

yt=-E-1{v2}+ 12E-1{v21-v2}+12E-1{v21+v2}

∴ y(t)=-1+12cosht+12cost is the solution.

2.4. Solve d3ydt3+dydt=2, y 0=3, y'0=1, y''0=-2.

Sol: The given equation can be written as y'''t+y't=2, y0=3, y'0=1, y''0=-2 ______ (2.4.1)

Taking the Elzaki transform to (2.4.1), we have E{y'''t}+E{y't}=2E{1}

=> 1v3Eyt-1vy0-y'0-vy''0+1vEyt-vy0=2v2

=> 1v3+1vEyt-3v-1+2v-3v=2v2 => 1+v2v3Eyt=2v2+v+1+3v

=> Eyt=2v5+v4+v3+3v21+v2 =2v3+v2+2v2-v31+v2 __________ (2.4.2)

Taking inverse Elzaki transform of both sides (2.4.2), we get

y(t)=2E-1{v3}+E-1{v2}+2E-1{v21+v2}-E-1{v31+v2}

∴ y(t) =2t+1+2cost-sint is the required solution.

3. Applications to Mechanics:

The Elzaki transform can also be used to solve the problems in mechanics.

3.1. Example: A particle P of mass 2 grams moves on the X-axis and is attracted towards origin O with a force numerically equal to 8X. If it is initially at rest at X=10, find its position at any subsequent time assuming

a) No other force acts

b) A damping force numerically equal to 8 times the instantaneous velocity acts.

Sol: a) From Newton's law, the equation of motion of the particle is

2d2Xdt2=-8X (or) d2Xdt2+4X=0 ________ (3.1.1)

with the initial conditions X(0) = 10 and X'0=0.

Taking the Elzaki transform of both sides of (3.1.1), we have

E{d2Xdt2}+4E{X}=0 => 1v2EXt-X0-vX'0+4EXt=0

=> (1v2+4)EXt-10-0=0 => EXt=10v21+4v2 _______ (3.1.2)

Taking inverse Elzaki transform of both sides of (3.1.2), we have

Xt= 10E-1(v21+4v2) ∴ Xt=10 cos2t

b) In this case the equation of motion of particle is

2d2Xdt2=-8X-8dXdt (or) d2Xdt2+4dXd t+4X=0 ________ (3.1.3)

with the initial conditions X(0) = 10 and X'0=0.

Taking the Elzaki transform of both sides of (3.1.3), we have

E{d2Xdt2}+4E{dXdt}+4E{X}=0

=> 1v2EXt-X0-vX'0+41vEXt-vX0+4E{Xt}=0

=> 1v2+4v+4EXt-10-0-40v=0 => 1+4v+4v2v2E{Xt}=10+40v

=> EXt=10v2(1+2v)2+40v3(1+2v)2 =10v21+2v-20v3(1+2v)2+40v3(1+2v)2

=10v21+2v+20v3(1+2v)2 ___________ (3.1.4)

Taking inverse Elzaki transform of both sides of (3.1.4), we get

X(t)=10E-1{v21+2v}+20E-1{v31+2v2}

∴ Xt=10 e-2t+20 te-2t

3.2. Example: A mass m moves along the X-axis under the influence of a force which is proportional to its instantaneous speed and in a direction opposite to the direction of motion. Assuming that at t = 0 the particle is located at x = a and moving to the right with speed V0, find the position where the mass comes to rest.

Sol: Here the equation of motion of the particle is

md2xdt2=-μdxdt (or) md2xdt2+μdxdt=0 _______ (3.2.1)

with initial conditions x(0) = a and x'0=V0.

Taking the Elzaki transform to (3.2.1), we have mE{d2xdt2}+μE{dxdt}=0

=> m1v2Ext-x0-vx'0+μ[1vExt-vx0=0

=> mv2+μvExt-ma-mvV0-μav=0

=> m+μvv2Ext=ma+v(mV0+μa)

=> Ext=mav2m+μv+(mV0+μa)v3m+μv

=av21+μmv+(mV0+μam)v31+μmv

=av21+μmv+mV0+μam[mμv2-mμv21+μmv]

=mV0μ+av2+(a-mV0+μaμ)v21+μmv

=mV0μ+av2+-mV0μv21+μmv __________ (3.2.2)

Taking inverse Elzaki transform of both sides of (3.2.2), we get

xt=mV0μ+aE-1{v2}+-mV0μE-1{v21+μmv}

∴ x(t)=mV0μ+a-mV0μe-μmt __________ (3.2.3)

Now if dxdt=V0e-µmt=0 then from (3.2.3), x(t)=mV0µ+a. Hence the mass m comes to rest at a distance mV0µ+a from the center O.

3.3. Example: A spring hangs vertically. A weight of 10 lb. is attached and the spring stretches 2 inches. The weight is replaced with a 60 lb. weight and allowed to come to rest in equilibrium. It is then pulled down 6 inches and released with velocity zero. Find the formula for the resulting motion.

Sol: From Newton's second law md2sdt2=-ks ___________ (3.3.1)

where m is the mass, and –ks is the force exerted by the spring. The force constant k is defined as k = [(force applied) / (distance stretched)]

Thus, k = [(10 lb) / (2 inches)] = [(10 lb) / {(1/6 ft)] = 60 lb/ft.The mass of the 60 lb. weight is m=wg=[60 lbg] where g= the acceleration due to gravity. Substituting these values in (3.3.1), we get 60gd2sdt2=-60s (or) d2sdt2+gs=0 _______ (3.3.2)

With the initial conditions are s0=-6 inches=-12 ft ; v0=s'0=0

Taking the Elzaki transform of both sides of (3.3.2), we have E{d2sdt2} +gE{s}=0 => 1v2Est-s0-vs'0+gEs=0

=> 1v2+g Est+12-0=0 => Es(t)= -12 v21+gv2 _______ (3.3.3)

Taking inverse Elzaki transform of both sides of (3.3.3), we get

s(t)= -12 E-1{v21+gv2} =-12cosg t or - 12cosh(-gt) ft

Which is the required solution.

3.4. Example: A 10-kilogram mass is attached to a spring which is thereby stretched 0.7 meters from its natural length. The mass is started in motion from the equilibrium position with an initial velocity of 1 meter/sec in the upward direction. Find the resulting motion if the force due to air resistance is -90dxdt newtons.

Sol: The motion is damped (air resistance) and free (no external force). From Newton's second law, md2xdt2=-90dxdt-kx __________ (3.4.1)

Where m = the mass (10 kg), and k is the spring constant. Since k = [{(9.8) (10)}/ 0.7] = 140 newtons/meter. (Note that a mass of 10 kg exerts a force or weight of (9.8) *(10) = 98 newtons). Substituting these values in (3.4.1), we have

10d2xdt2=-90dxdt-140x (or) d2xdt2+9dxdt+14x=0 _________ (3.4.2)

With the initial conditionals x0=0 and x'0=-1 (the initial velocity is in the negative x-direction).

Taking the Elzaki transform to (3.4.2), E{d2xdt2}+9E{dxdt}+14E{x}=0

=> 1v2Ext-x0-vx'0+9 1vExt-vx0 +14Ext=0

=> 1v2+9v+14 Ext-0+v-0=0 => (1+9v+14v2v2)Ext=-v

=> Ext=-v3(1+2v)(1+7v) =-15 v21+2v+15 v21+7v ____________ (3.4.3)

Applying inverse transform to (3.4.3), we get

xt=-15 E-1{v21+2v}+15 E-1{v21+7v}= -15 e-2t+15 e-7tis the required solution.

3.5. Example: A mass of 1/4 slug is attached to a spring of force constant k = 1 lb./ft. The mass is set in motion by initially displacing it 2 ft. in the downward direction, and giving it an initial velocity of 2 ft./sec. in the upward direction. Find the subsequent motion of the mass if the force due to air resistance is -1(dx/dt) lb.

Sol: From Newton's second law, md2xdt2=-adxdt-kx ____________ (3.5.1)

where -adxdt is the force of air resistance and -kx is the spring force. Substituting the values in (3.5.1), 14d2xdt2=-1dxdt-1.x (or) d2xdt2+4dxdt+4x=0 __________ (3.5.2)

With the initial conditions x0=2 and x'0=-2. Taking the Elzaki transform to (3.5.2),we get E{d2xdt2}+4E{dxdt}+4E{x}=0

=> 1v2Ext-x0-vx'0+4 1vExt-vx0 +4Ext=0

=> 1v2+4v+4 Ext-2+2v-8v=0 => (1+4v+4v2v2)Ext=2+6v

=> Ext=2v2(1+2v)2+6v3(1+2v)2 =2v21+2v-4v3(1+2v)2+6v3(1+2v)2 =2v21+2v+2v3(1+2v)2

Taking inverse Elzaki transform to above, we get

xt=2E-1{v21+2v}+2E-1{v31+2v2} =2(1+t)e-2tis the required solution.

3.6. Example : Find the solution of the suspended spring if the vibration is free and undamped.

Sol: By a suspended spring we mean a spring hanging vertically with top end fixed and bottom end handling a mass m. Initially, the spring and mass are in equilibrium position. Then at t=0, the spring is stretched by moving the mass at x0, some positive distance below the equilibrium position. It is then released from rest. The initial conditions are thus x0=x0 and v0=x'0=0 __________ (3.6.1)

Where x(t) is the position of the mass at t, and vt=dx(t)dt is its velocity. If the vibration is free, we mean there is no external force, Ft. If the vibration is undamped, we mean there is no air or medium resistance force. Thus, the only force acting on our system is the force of the spring which by Hooke's law is –kx. By Newton's second law, our differential equation is

md2xdt2=-kx (or) d2xdt2+kmx=0 ___________ (3.6.2)

Taking the Elzaki transform to (3.6.2), we have E{d2xdt2}+kmE{x}=0

=> 1v2Ext-x0-vx'0+kmEx(t)=0

=> 1v2+km Ext-x0-0=0 [since using conditions (3.6.1)]

=> m+kv2mv2 Ext=x0 => Ext=x0mv2m+kv2 ____________ (3.6.3)

Now applying inverse Elzaki transform to (3.6.3), we get xt=x0 E-1{v21+kmv2} ∴x(t)=x0 cos(tkm) , x0 is called the amplitude of the motion, and kmis the circular frequency. The natural frequency is defined as f=12π km and the period is T=1f=2π mk .

4. Applications to Beams:

Let a beam whose ends are x=0 and x=l be coincident with x-axis. Let a vertical load, given by W(x) per unit length, act transversely on the beam. Then

X

Y

x=l

x=0

O

the axis of the beam has a transverse deflection y(x) at the point x which satisfies the differential equation

d4ydx4=W(x)EI , 0<x<l

The quantity EI is called the flexural rigidity of the beam where E is Young's modulus of elasticity for the beam and I is the moment of inertia of a cross-section of the beam about axis.

Boundary conditions:

If beam is clamped, built-in or fixed end then y=y'=0

If beam is hinged or simply supported end then y=y''=0

If beam is free end then y=y'''=0

4.1. Example: A beam which is hinged at its ends x=0 and x=l carries a uniform load W0 per unit length. Find the deflection at any point.

Sol: The deflection y(x) at the point x satisfies the differential equation

d4ydx4=W0EI , 0<x<l _______ (4.1.1)

with the boundary conditions y0=0, y''0=0 _______ (4.1.2)

yl=0, y''l=0 _______ (4.1.3)

Taking the Elzaki transform to (4.1.1), we have E{ d4ydx4}=W0EIE{1}

=> 1v4Eyx-1v2y0-1vy'0-y''0-vy'''0=W0EI v2

=> 1v4Eyx-0-Av-0-Bv=W0EI v2 , where y'0=A and y'''0=B

=> Eyx=Av3+Bv5+ W0EI v6 ___________ (4.1.4)

Taking inverse Elzaki transform to (4.1.4), we get

y(x)=AE-1{v3}+BE-1{v5}+ W0EIE-1{ v6}

∴ y(x)=Ax+Bx36+W0EI x424 ____________ (4.1.5)

Differentiating (4.1.5) w.r.to x twice, we have y'x=A+Bx22+W0EI x36 ____ (4.1.6)

and y''x=Bx+W0EI x22 ___________ (4.1.7)

Using the condition (4.1.3), we have yl=Al+Bl36+W0EI l424=0 __________ (4.1.8)

and y''l=Bl+W0EI l22=0 ______________ (4.1.9)

solving equations (4.1.8) and (4.1.9), we get A=W0l324EI and B=-W0l2EI

From (4.1.5), the required deflection is

y(x)=W0l324EIx-W0l2EIx36+W0EI x424 =W024EI(l3x-2lx3+x4) =W024EIx(l-x)(l2+lx-x2).

4.2. Example: A beam which is clamped at its ends x=0 and x=l carries a uniform load W0 per unit length. Show that the deflection at any point is yx=W0x2(l-x)224EI.

Sol: The deflection y(x) at the point x satisfies the differential equation

d4ydx4=W0EI , 0<x<l ______ (4.2.1)

with the boundary conditions y0=0, y'0=0 _______ (4.2.2)

yl=0, y'l=0 _______ (4.2.3)

Taking the Elzaki transform to (4.2.1), we have E{ d4ydx4}=W0EIE{1}

=> 1v4Eyx-1v 2y0-1vy'0-y''0-vy'''0=W0EI v2

=> 1v4Eyx-0-0-A-Bv=W0EI v2 , where y''0=A and y'''0=B

=> Eyx=Av4+Bv5+W0EI v6 ________ (4.2.4)

Taking inverse Elzaki transform of both sides of (4.2.4), we get

y(x)=AE-1{v4}+BE-1{v5}+W0EI E-1{v6} =Ax22!+Bx33!+W0EI x44!

y(x)=Ax22+Bx36+W0EI x424 _________ (4.2.5)

Now, differentiating (4.2.5) w.r.to x, we get y'x=Ax+Bx22+W0x36EI ______ (4.2.6)

Using the condition (4.2.3), we have yl=Al22+Bl36+W0EI l424=0 ________ (4.2.7)

and y'l=Al+Bl22+W0l36EI=0 ________ (4.2.8)

Solving equations (4.2.7) and (4.2.8), we get A=W0l212EI and B=-W0l2EI. From (4.2.5), we have yx= W0l212EI x22-W0l2EI x36+W0EI x424 = W0x224EI(l2-2lx+x2) = W0x2(l-x)224EI.

Conclusion: We can apply this Elzaki Transform technique to get the analytical solutions of Engineering Mechanics problems.

References

Tarig Elzaki M, Salih Elzaki M, “The New Integral Transform Elzaki Transform” , Global Journal of pure and Applied Mathematics, pp 57-64, (2011).

Tarig Elzaki M, Salih Elzaki M, “On the connections between Laplace and Elzaki Transforms” Advances in Theoritical and Applied Mathematics, 6(1), 13-18,(2011).

Naresh,P, Umamaheshwar Rao R, Eswarlal,T., “Elzaki Transform for Two Tank Mixing Problems”, International Journal of Chemical Sciences, Vol.5, Issue-4, pp: 1-15, (2017) .

J.Zhang, J .“A.Sumudu Based Alogorithm for Solving Differential Equations”, Comp. Sci.J.Moldova, 15(3), 303-313, (2007)

Alan Jeffery, “Advanced Engineering Mathematics” , Academic Press, (2002).

R.Umamaheshwar Rao, P.Naresh, “Elzaki Transform for Exponential Growth and Decay” Inernational Journal of Mathematics and its Applications, Vol.5, 3-C, pp: 305-308, (2017).