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THE MINIMUM DEMAND METHOD – A NEW AND EFFICIENT INITIAL BASIC FEASIBLE SOLUTION METHOD FOR TRANSPORTATION PROBLEMS

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Abstract

It is one of the most important tasks to determine the optimal solution for large scale transportation problems in Operations research more efficiently, accurately and quickly. In this research, we develop a new and efficient initial basic feasible solution (IBFS) method for solving balanced and unbalanced transportation problems so that the cost associated with transporting a certain amount of products from sources to destinations is minimized while also satisfying constraints. The proposed method – the minimum demand method (MDM) – to find a starting (initial) solution for the transportation problems has been developed by taking minimum value in the demand row, and in case of a tie, the demand with the least cost in the corresponding column is selected. The performance evaluation of the proposed MDM is carried out with other benchmark methods in the literature, like the north-westcorner method (NWCM), least cost method (LCM), Vogel's approximation method (VAM) and revised distribution (RDI) method. The IBFSs obtained by the proposed MDM and existing NWCM, LCM, VAM and RDI has been compared against the optimal solutions acquired through the modified distribution (MODI) method on 12 balanced and unbalanced problems from literature, and the relative error distributions are presented for accuracy. The results obtained by the proposed MDM are better than NWCM, LCM, VAM and RDI. The proposed MDM gives initial basic feasible solutions that are the same as or very closer to the optimum solutions in all cases we have discussed. The comparison reveals that the proposed MDM reduces the number of tables and the number of iterations to reach at more accurate and reliable IBFS. The MDM will also save the total time period of performing tasks and reduce the number of steps in order to get the optimal solution.

Keywords: Transportation problem, initial basic feasible solution, Optimal solution, North-west-corner, Least cost, Vogel's approximation, Revised distribution, Modified distribution.

I. Introduction

The problems associated with decision making and cost-benefit studies usually lead to optimization problems, and the operations research (OR) is concerned with efficient handling and optimal way-outs for such problems while all associated constraints are met [XXIX]. For example, the problem of maximizing the power produced and minimizing the cost incurred by installing wind turbines in wind farms at optimal positions using a new dynastic optimization algorithm [XVI], tracking the maximum power point efficiently [IV] in the electric power systems, and the load flow studies [XI], etc. Besides solving most of the linear programming (LP) problems in OR, the usual Simplex method is not mostly used and considered most appropriate for solving certain specific types of problems: Transportations problems, Transshipment problems and Assignment problems. These problems are mainly allocation problems and can be solved using some special methods. The transportation problem is a special type of LP problem in which the products are transported from origins to destinations subject to the supply and demand conditions such that the total transportation cost is to be minimized. The methods of solving transportation problems comprise two phases. The first phase is to find a starting initial basic feasible solution, and then the second phase is to find an optimal solution iteratively. [XXIX] [XVIII]

There exist many methods for obtaining IBFS and optimal solutions to transportation problems. We chronologically discuss some important developments in transportation problems. A French mathematicians Monge formalized transportation in 1781 [XVIII]. Major progress has been made in the area during world war II by the Soviet/Russian economist and mathematician Leonid Kantorovich. Therefore, transportation problems are sometimes also known as Monge-Kantorovich transportation problems. Hitchcock in 1941 [VII] was the first who presented the origin of the transportation problem in a study presented by him concerning the distribution of a commodity from multiple sources to many destinations. His contribution was considered as the first significant role in the transportation problem's solution.

Kantorovich in 1942 alone and in 1949 together with Gavurian published helpful results in the quest of solving transportation problems [X]. A case study consisting of several shipping sources and a series of destinations on optimum utilization of transportation systems was contributed by Koopmans in 1947 [XII]. However, it could be achieved as a solution to complex problems optimally in 1951, after that George B. Dantzig gave the idea of LP to apply for the solution of transportation models, and thus the Simplex method was applied to solved transportation problems [VI].

Charnes and Cooper in 1953 developed a new approach for an optimal solution from IBFS, which was termed as the Steppingstone method (SSM). They also contributed

in goal programming in 1961 [V] which was then extended by Ijiri in 1963 [VIII]. Shetty in 1959 [XXIII], presented an innovative method for solution of transportation problems taking nonlinear costs by measuring the event of a convex cost of production in addition to the transportation cost. Soland in 1971 [XXIV], presented a branch and bound method for solving transportation problem concave separable, called 'simplified algorithm". While Lee and Moore in 1972 [XV], used the goal programming model to solve transportation problems having multiple differing objectives. A similar approach was also used by Lawrence (1982) [XIV] for transport the chemical and pharmaceutical industries. The goal programming based solutions to transportation problems were also studied and used by Kwak and Schniederjans in 1985 [XIII], and Sharma in 1999 [XXII].

Recently, Pandian [XXI], Sudhakar [XXVIII] and Abdul Quddoos [II] contributed totally new ideas for the optimal solution of transportation problems. They attempted to provide a non-iterative direct optimal solution. The claim to have been able to solve some example transportation problems optimally was generalized by them as their contributions to have proposed new direct optimal solution methods for the transportation problems. The claims have been investigated recently, and it was found that some proposed methods in [XXI],[XXVIII],[II] were just better IBFS methods instead of being optimal solution methods on basis of some counter examples in [XXV] and [IX] recently in 2017 and 2019, respectively by us.

Hlayel and Alia in 2012 [I], proposed a method for IBFS using the best candidate strategy, and it was found to be in good comparison with other methods. Hasan in 2012 [XVII], also worked on direct methods, and claimed that the direct approaches namely zero-suffix and SAM methods for finding optimal solution directly for a transportation problem do not present optimal solution always, the concerns were later confirmed for other methods in [XXV] and [IX]. Aramuthakannan.and Kandasamy in 2013 [III], also worked on an optimal method for solving a high dimension problem using the RDI method directly.

Soomro et al. in 2014 [XXVI], compared several IBFS methods for solving transportation problems. The methods included NWCM, LCM and VAM whose algorithms are available in [XXIX]. Later in 2015 Soomro et al. [XXVII] worked on VAM to develop its modification by taking the penalty of each row and the penalty for each column. The penalty of each row was set equal to the difference between the two largest per unit transportation costs of that corresponding row, whereas the penalty of each column was set equal to the difference between the two smallest per unit transportation costs of that corresponding column, and the remaining steps were same as in the VAM. The results of modified VAM were better than NWCM and LCM, whereas equal to or better than the VAM.

The widely used method VAM was developed by Vogel in 1958 [XXIX], and is used to find an initial solution to a transportation problem by taking the penalty of each column and each row, the penalty of column and row is the difference between the minimum cost of that corresponding column and row. VAM is more efficient and one of the well-known transportation methods in the literature. Motivated form the recent studies concerning VAM and its modifications [XXVII], RDI method [III] and the thorough developments in the field of transportation problems, we have attempted to *Sanaullah Jamali et al*

propose a new IBFS method, referred here as the minimum demand method (MDM), to get more closer and accurate initial solution with reference to the optimal one, as compared to several existing IBFS methods: NWCM, LCM, VAM and RDI. The proposed MDM algorithm is described, and then its efficiency with regards to time, lesser computations, minimum use of tables and content to reach at a closer initial solution or in many cases the optimal solution is discussed in detail in the context of several test problems from the literature.

II. Material and methods

Here, we first discuss basic concepts related to transportation problems which are necessary for readers to follow the main contributions of this work. Followed by that, the algorithm of the proposed MDM is presented.

General Representation of Transportation Problems

The objective of the transportation problem is to reduce the shipping cost of commodities which are transported from origin to destination to meet the conditions of supply and demand requirement. The basic steps of the transportation method are given below:

- **Step 1**. Determine the initial solution of the transportation problems.
- **Step 2**. To check whether the solution is optimal or not, if optimal then stop, otherwise go to step 3.
- **Step 3**. Improve the solution by MODI or SSM.

The cost function is usually represented as Z, the decision variables as x_{ij} 's, the cost on transporting a unit of product as c_{ij} 's with S_i sources and D_j destinations for i =1,2,..., m and j = 1,2,..., n. The mathematical model is described as in equations (1)-(3).

$$Minimize Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$
 (1)

Subject to
$$\sum_{j=1}^{n} x_{ij} = a_i$$
 (2)

$$\sum_{i=1}^{m} x_{ij} = b_j \tag{3}$$

$$x_{ij} \ge 0 \tag{4}$$

The non-negativity restrictions on the decision variables are described in (4), and a_i and b_j , respectively are the supplies and demands from the corresponding sources and destinations. The network diagram if the general transportation problem relating to all sources with all destinations is shown in Fig. 1.

The transportation problem (1)-(4) is said to be balanced if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{5}$$

And unbalanced if

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \tag{6}$$

Moreover, when $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$, we add dummy destination with zero cost demand in transportation table. Otherwise, if $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$, we add dummy sources with zero cost supply in the transportation table. For a balanced problem, the transportation array is shown in Fig. 2.

The Proposed Minimum Demand Method

The idea of the proposed method is motivated by the VAM, modified VAM and RDI methods [XIV],[XIII],[XXII]. Unlike in the VAM where penalties are computed for rows and columns, and RDI where demands are computed for rows and columns, we restrict the proposed MDM method to the minimum demand value in the demand row only. Thus, the computational cost of the proposed MDM is much lower than VAM, modified VAM, RDI [XIV],[XIII],[XXII] and other methods: NWCM and LCM [XXIX]. The algorithm of the proposed methods is stated as.

Given the transportation array, and its final balanced form by adding dummy sources/destinations, if necessary, then.

Step 1. Start with the minimum value in the demand row. If tie occurs, then select the demand with least cost.

Step 2. Allocate possible units to least cost cell in the demand column.

Step 3. If the demand in the column is satisfied, move to the next minimum value in the Demand row.

Step 4. Repeat Steps (ii) and (iii) until supply and demand conditions are satisfied.

We claim that the proposed MDM, in general, is not an optimal solution method, in fact a way better IBFS method than the existing NWCM, LCM, VAM and RDI methods. While it will be shown through an exhaustive comparison of the performance of the proposed MDM with other methods in section III that the initial solutions attained by the MDM are mostly optimal and in one case much closer to the optimal one as compared to other methods under discussion.

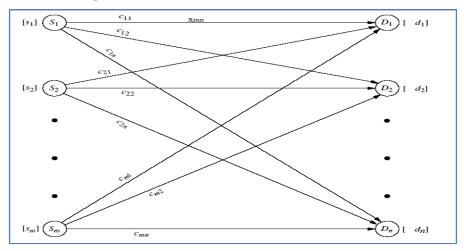


Fig. 1. Network representation of the transportation problem

DESTINATIONS												
				D1	D2 Dn					Supply a _i		
l			C ₁₁		C ₁₂			Cın				
l	s	S ₁		X 11		X12			X _{In}	a1		
l	0		C ₂₁		C22			C _{2n}				
П	บ	S ₂		X 21		X22			X _{2n}	a2		
П	R											
l	c		C _{m1}		C _{m2}			Cmn				
l	E	-										
l	s	-		X _{m1}		X _{m2}		x _{mn} a _m		a m		
		Sm										
	Demand		•		1			bn		Balanced model		
	bj		b ₁		b ₂					$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$		

Fig. 2. Array representation of a balanced transportation problem

One of the important features showing its simple adaptability, ease of use and implementation and very low execution time to reach the solution lies in the fact that in the proposed MDM algorithm, we look for only one minimum value in the demand row at a time, whereas in the VAM and RDI we simultaneously search in both demand and supply arrays. In RDI method, we search for the minimum throughout the values in supply and demand arrays, and in the VAM we first calculate the penalties and then have to look for the penalty of two smallest numbers of each row and each column. Thus, the implementation of the proposed MDM requires half computational overhead as compared to the widely used VAM and RDI.

We demonstrate the implementation and performance of the proposed MDM versus other methods from literature in the next section III.

III. Numerical Problems, Results and Discussion

The proposed IBFS method – the MDM – is compared in section IV with the NWCM, LCM, VAM and RDI methods. The algorithms of NWCM, LCM and VAM are available in [XXIX], [XXVI] and [XXVII], and of the RDI method in [IX] and [III], and are skipped here for brevity. The optimal solutions are sought using the MODI method, and we refer the readers to [XXIX], [XXVI] and [XXVII] for its algorithm. The optimal solutions by the MODI method are considered as a reference to comment on the accuracy of the initial solutions obtained by the IBFS methods under discussion including the proposed MDM. The formula to compute the relative error of the initial solution acquired through an IBFS method *X* versus the MODI optimal solution is used as in equation (7).

$$RE_X = \left| \frac{Inital_X - Optimal_{MODI}}{Optimal_{MODI}} \right| \tag{7}$$

where X = NWCM, LCM, VAM, RDI method, and proposed MDM.

The test transportation problems 1-6 and 7-12 used for the performance evaluation of the proposed MDM and other IBFS methods are taken from [XIX], [XXVI], [II], [XXVI], [II], [XXVI], [III], [XXVII], [III], [XXVII], and are summarized in Figs. 3 and 4, respectively. Besides the size of the problem as a number of 'sources by destinations' and the nature of the problem 'balanced/unbalanced' is also mentioned in Figs. 3-4 for each of the problems 1-12.

For better understanding and implementation details of the proposed MDM, we mention here its step-by-step implementation for Problems 1-2, which are a balanced and an unbalanced transportation problems, respectively.

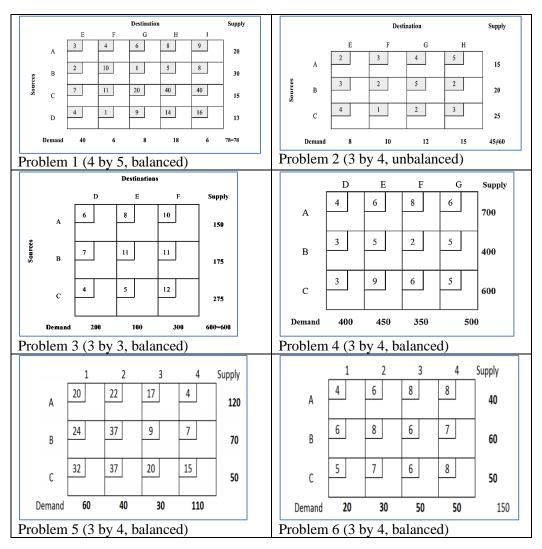


Fig. 3. Test transportation problems 1-6 for performance evaluation

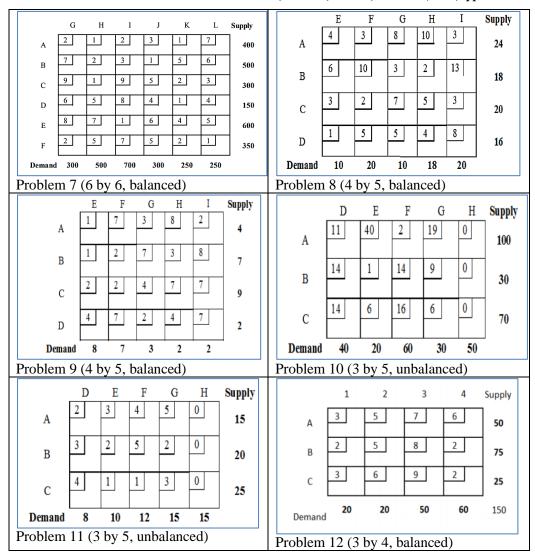


Fig. 4. Test transportation problems 7-12 for performance evaluation

The transportation array definitions of Problems 1-2 are given in Fig. 3. The step-by-step implementation of the MDM is described in Fig. 5 for Problem 1 (4 by 5, balanced) and in Fig. 6 for Problem 2 (3 by 4, unbalanced). The final allocation table with assigned units of products from a particular set of sources to the destinations so that the minimum transportation cost (1) has arrived while meeting the constraints (2)-(4). Using values in equation (1) from the final allocation tables as in Figs. 5-6 (h), it appears that the minimum transportation cost found by the proposed MDM is 281 and 80 units, respectively. The remaining test problems 3-12 using a similar detailed procedure as described in Figs. 5-6 for Problems 1-2. In Table 1, we list the IBFSs to the optimal minimum transportation cost from test Problems 1-12 obtained by all methods including proposed MDM and the optimal MODI methods. It can be

seen that the initial solutions obtained by the proposed MDM are mostly the optimal solutions, except in test problem 12.

On the other hand, the IBFSs attained by the ancient NWCM and the so popular VAM methods were not optimal in any of the problems 1-12, while the VAM was better than NWCM and LCM in most of the problems. The LCM initial was found to be optimal only in problem 4. The RDI method comparatively was the second best after the proposed MDM in all transportation problems, but not always optimal as claimed in the corresponding base study [III]. The Fig. 7 represents the relative errors for all methods for Examples 1-12 computed using (7) with the MODI optimal solution as a reference. It is evident from Fig. 7 that the proposed MDM exhibits zero relative errors in problems 1-11 and much smaller error in problem 12 as compared with other methods. It is also important to note that such a high consistency rate of obtaining an optimal or nearly optimal solution of the transportation problems by the proposed MDM method is at first place achieved with lower computational cost per step of eliminating the sources and destinations as compared to other methods as well. In Fig. 8 the number of successes in obtaining exactly the optimal solution versus the cases in which non-optimal but feasible solutions were achieved by all methods has been compared. The statistics in Fig. 8 demonstrate that the proposed MDM attains the optimal solution more often, even we claim the proposed MDM not as an optimal solution method, rather quite a better alternative to the traditional IBFS methods used widely in the literature. In the future, the proposed MDM will certainly find its place in related studies to save the computational efforts without compromising its accuracy to efficiently obtain the minimum IBFSs to the transportation problems which are exactly or nearly optimal.

IV. Conclusion

A new and efficient IBFS method for solving the transportation problems was proposed in this study. An exhaustive comparative analysis of the results acquired by the proposed MDM was performed with other four benchmark methods: NWCM, LCM, VAM and RDI for finding an initial basic feasible solution of twelve transportation models from the literature. It is important to note that in the proposed MDM, we have used minimum value in demand row which takes short time in the calculation and helps in reducing the number of iterations and tables towards the optimal solution. The proposed MDM is easy to apply for balanced and unbalanced transportation problems. From the exhaustive comparison, it appeared that the proposed MDM results were found to be better than all other methods used in the comparison, and for all test problems. Out of the 12 test transportation problems, the success rate of the proposed MDM towards getting the optimal solution directly was approximately 92%, whereas the same was 58%, 0%, 8% and 0% for the RDI, VAM, LCM and the NWCM under similar conditions.

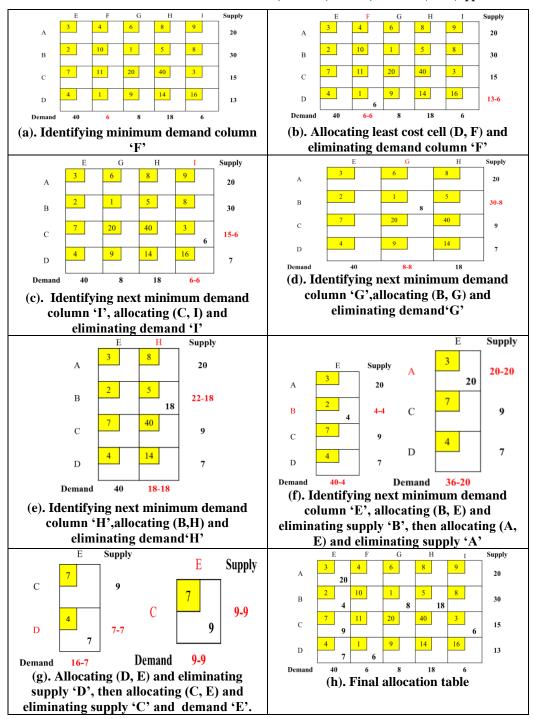


Fig. 5. Implementation details of the proposed MDM for balanced transportation problem 1

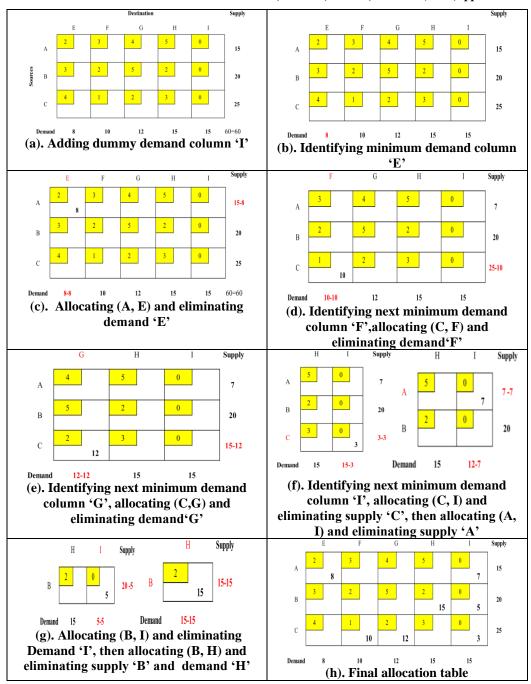


Fig. 6. Implementation details of the proposed MDM for unbalanced transportation problem 2

Table 1: Comparison of solutions obtained by IBFS methods and the optimal MODI method

Test problem	NWCM	LCM	VAM	RDI	MDM	MODI
1.	878	604	317	377	<mark>281</mark>	281
2.	143	91	91	83	80	80
3.	5925	4550	5125	5025	<mark>4525</mark>	4525
4.	7750	<mark>7350</mark>	7750	<mark>7350</mark>	<mark>7350</mark>	7350
5.	3680	3670	3520	<mark>3460</mark>	<mark>3460</mark>	3460
6.	980	960	960	990	<mark>920</mark>	920
7.	8700	4100	4400	3500	<mark>3000</mark>	3000
8.	396	208	220	204	<mark>204</mark>	204
9.	68	51	43	41	<mark>41</mark>	41
10.	1810	1020	880	<mark>760</mark>	<mark>760</mark>	760
11.	143	79	79	71	<mark>68</mark>	68
12.	670	650	650	780	630	610

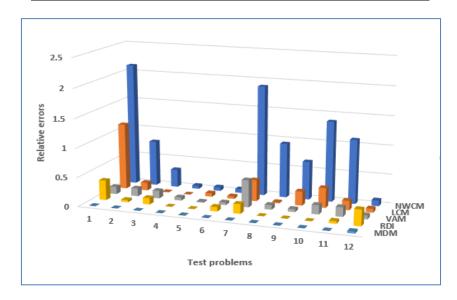


Fig. 7. Comparison of relative errors in the initial solutions by all methods versus optimal MODI

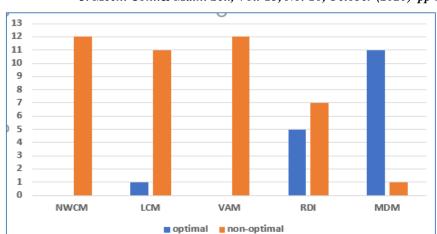


Fig. 8. Comparison of number of times exactly the optimal/non-optimal solution was obtained

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Conflict of Interest:

No conflict of interest regarding this article

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