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## Subsequent the examined text extract:

### Uncertainties in Absolute GPS Positioning: A Statistical Error Characterization

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Abstract

Abstract

To enhance the Global Positioning System measurement accuracy, this paper proposes a novel approach for improving the accuracy of a Global Positioning System receiver position. The proposed method utilizes correntropy as an optimal criterion which is a local similarity measure unlike minimum mean square error (global similarity index) and also uses a fixed point iterative algorithm to update the posterior estimates. The proposed algorithm compared with the several Global Positioning System algorithms (Least Square Estimator, Kalman Filter, and Extended Kalman Filter) which are presented. Results of the theoretical study and simulation indicate that the efficiency of the proposed GPS navigation algorithm based on the criterion of correntropy outperforms that of the prevalent approaches (LSE, KF & EKF).

## Key words

Correntropy, Extended Kalman Filter (EKF), Fixed point algorithm, Global Positioning system (GPS), Kalman Filter (KF), Least Square Estimator (LSE).

## I Introduction

Global Positioning System (GPS) is a three-dimensional positioning system using many artificial satellites and has been used extensively in navigation systems, surveying, target tracking, etc. Specifically, it is so widely used in navigation systems that it has become an essential system nowadays. Nevertheless, there are various problems with GPS. For example, the positioning estimation cannot occur if the GPS receiver cannot receive the signals from more than three satellites [I]. Further, where barriers such as tall buildings block the radio wave, it is not possible to receive the signals from the satellite because of the straight path of the radio wave used in the GPS and the reflected wave. However, there is still a positioning error even if the GPS receiver can receive signals from more than three satellites. Nonetheless, as the above-mentioned difficulties cannot be overcome entirely with these solutions, many are aimed at improving the GPS itself. Such problems should be addressed by introducing an effective positioning algorithm to reliably estimate receiver position [II] while enhancing the system itself, such as increasing the number of satellites, which is impractical.

## II Least Square Estimator

Linearization is the prevalent way of solving the 4 nonlinear concurrent balances. However, generally, more than four satellites are noticeable at any point in time of the moment. If there are more than four satellites in perspective are more common in solving the user location [III]. In this scenario, there are first linearized arrangements of nonlinear equations and then more equations than unknowns. For the solution [II] [IV] you can use the commonly recognized least-square approximation algorithm.

## Navigation Solution with more than Four Satellites

At the point when multiple satellites are in view from a given recipient area, an increasingly well-known way to deal with tackle the user position is to utilize every one of the satellites [IV]. Then the navigation equation with more than 4 satellites can be given as in equation 1.

$$S_i = (a_i - a_u)^2 + (b_i - b_u)^2 + (c_i - c_u)^2 + E_u \quad (1)$$

Where,  $S_i$  is measured pseudo range between satellite antenna and GPS receiver

$a_u, b_u, c_u$  are the coordinates of user position

$a_i, b_i, c_i$  are the coordinates of satellite position

$E_u$  is receivers clock bias.

The navigational solution can be achieved and expressed in the form of linearization of Equation 1.

$$\Delta S_i = \beta \Delta A_u \quad (2)$$

Where,  $\Delta S_i$  is measured pseudo range error,  $\beta$  is observation matrix;  $\Delta A_u$  is receivers state vector error. Equation 2 could be extended as

$$\Delta P_1 \Delta P_2 \Delta P_3 \Delta P_4 : \Delta P_n = \beta_{11} \beta_{12} \beta_{13} \beta_{14} \beta_{21} \beta_{22} \beta_{23} \beta_{24} \beta_{31} \beta_{32} \beta_{33} \beta_{34} \beta_{41} \beta_{42} \beta_{43} \beta_{44} : \beta_{n1} \beta_{n2} \beta_{n3} \beta_{n4} \Delta a_u \Delta b_u \Delta c_u \Delta E_u \quad (3)$$

Where  $\beta = \beta_{11} \beta_{12} \beta_{13} \beta_{14} \beta_{21} \beta_{22} \beta_{23} \beta_{24} \beta_{31} \beta_{32} \beta_{33} \beta_{34} \beta_{41} \beta_{42} \beta_{43} \beta_{44} : \beta_{n1} \beta_{n2} \beta_{n3} \beta_{n4}$ ,  $\beta_{i1} = a_i - a_u$ ,  $\beta_{i2} = b_i - b_u$ ,  $\beta_{i3} = c_i - c_u$ ,  $\beta_{i4} = E_u$

Since the grid  $\beta$  is certifiably not a square lattice, it can't be reversed straight forwardly. Anyway, Equation 3 is as yet a linear equation. In several linear equations, the least square approximation can be used to find the solution when there are more equations than unknowns. To acquire the desired answer, you can use the pseudo-inverse of the  $\beta$  matrix. Therefore, Equation 3's navigation solution was

$$\Delta \mathbf{A}_u = \beta^T \beta^{-1} \beta^T \Delta \mathbf{S}_u \quad (4)$$

Using equation 4 we can obtain the values of  $\Delta \mathbf{a}_u$ ,  $\Delta \mathbf{b}_u$ ,  $\Delta \mathbf{c}_u$ , and  $\Delta \mathbf{E}_u$ . Generally speaking, the least square approximation provides a better alternative than only 4 satellites, as more information is used. The traditional way in which a GPS receiver is determined needs an iterative algorithm, which involves original estimates. The navigation solution in the recursive least squares approach originates from the estimated location of the user and is then predicted using that iterative approach to the actual position of the user.

### III Preliminaries

#### Cross-Correntropy

Cross-Correntropy [V][VI] is a pooled measure of similarity among a few arbitrary factors  $S, M$  with joint probability function ASM. Due to the two random variables  $S$  and  $M$ , the cross-correntropy is described by the distribution function  $ASM_{s,m}$  is

$$US, M = E n S, M = n s, m d ASM_{s,m} \quad (5)$$

Where,

$E$ : Expectation operator,  $n(\cdot, \cdot)$ : Definite Kernel of any continuous variable.

In this paper we have adopted the Gaussian Kernel ( $J\sigma$ ), so Eq.(5) becomes

$$ns, m = J\sigma(e) = \exp(-e^2 / 2\sigma^2) \quad (6)$$

Here, The error  $= s - m$ ,  $\sigma$ : kernel size ( $\sigma > 0$ )

By using a sample mean estimator, cross-correntropy can be estimated. In such cases,

$$US, M = 1/K \sum_{i=1}^K J\sigma(e_i) \quad (7)$$

Where  $e_i = s_i - m_i$  with  $s_i, m(i) \}_{i=1}^K$ ,

$K$ : Number of samples drawn from ASM.

Now we have extended the Gaussian Kernel Taylor Sequence

$$US, M = k=0 \infty (-1)^k \frac{e^{2k}}{2k!} E[(S-M)^{2k}] \quad (8)$$

#### Kalman Filter / Extended Kalman Filter

Replacing the nominal trajectory with the projected trajectory is a simple, efficient solution for the deviation problem. The Extended Kalman Filter [VII][VIII] is equivalent to a Kalman Filter except that linearization happens on the projected trajectory of the stream, rather than a conditional trajectory pre-computed. The Extended Kalman Filter provides a major method for working with nonlinear processes. Consider a nonlinear plant defined by the equations of nonlinear state and linear measurement:

$$\mathbf{s}_n = \mathbf{a}_n - 1 \mathbf{s}_{n-1} + \mathbf{p}_n - 1 \quad (9)$$

$$\mathbf{m}_n = \mathbf{B}_n \mathbf{s}_n + \mathbf{q}_n \quad (10)$$

Where,

$\mathbf{s}_n$ :  $i$ -dimensional state vector.

$\mathbf{m}(n)$ :  $j$ -dimensional measurement vector at instant 'n'.

$\mathbf{a}$ : Non linear system function

$\mathbf{B}$ : Observation matrix.

$p(n-1)$  : Process noise.

$q(n)$  : Measurement noise.

$$E[p_{n-1}p_{n-1}^T] = P_{n-1} \quad E[q_nq_n^T] = Q_n \quad (11)$$

Similar to the KF, the EKF includes two steps also, namely prediction and correction. The only change needed is to substitute partial derivatives in assessments. In other terms, partial derivatives are measured along a line that has been modified with the calculations of the filter; which depends on the measurements and thus the filter gain sequence depends on the sample measurement sequence.

### Step 1: Predict

The mean and covariance matrix which are predicted, given by

$$s_{n-1} = a_{n-1} s_{n-1-1} \quad (12)$$

$$C_{n-1} = A_{n-1} C_{n-1-1} A_{n-1}^T + P_{n-1} \quad (13)$$

Where  $A$  is a Jacobian matrix of  $a$ , and is described as follow:

$$A_{n-1} = \partial a(n-1, s_{n-1-1}) / \partial s_{n-1-1}$$

### Step 2: Update

The following steps must be considered in order to determine EKF gain

$$N_n = C_{n-1} B_n (B_n C_{n-1} B_n^T + Q_n)^{-1} \quad (14)$$

The new state is equivalent to the expected state plus EKF gain

$$s_n = s_{n-1} + N_n m_n - B_n s_{n-1} \quad (15)$$

In addition to that, computed covariance is updated is shown in Eq. (16)

$$C_n = I - N_n B_n C_{n-1} I - N_n B_n + N_n Q_n N_n^T \quad (16)$$

## IV Cross - Correntropy Kalman Filter

The Extended Kalman Filter works admirably under Gaussian noises. However, its output can mainly breakdown under non-Gaussian noises, particularly, when impulsive noises disturb the basic system. EKF's underlying purpose is to be implemented according to the MMSE criterion [VI] [IX], which collects only second-order error signal statistics and is vulnerable to significant deviations. In this article, a new Kalman filter is implemented using the cross-correntropy criterion that can be better performed in non-Gaussian noisy systems [V], with cross-correntropy being adapted to second and higher order error statistics [IX].

### Algorithm Derivation

In the previous section we have the linear model

$$s_{n-1} m(n) = I B_n s_n + u_n \quad (17)$$

Where,  $I$  represents matrix of Identity.

$$\begin{aligned} \text{And } u_n &= -(s_n - s_{n-1}) q(n), \text{ with } E[u_n u_n^T] = C_{n-1} - 100 Q(n) \\ &= D_{cnn-1} D_{cTnn} - 100 D_{qn} D_{qT}(n) = D_n D_n^T \end{aligned} \quad (18)$$

Where  $D(n)$  denotes cholesky decomposited matrix form of  $E[u(n)u(n)^T]$

By multiplying with  $D^{-1}I$  in Eq. (17) on both sides, we get

$$F_n = V_n s_n + e_n \quad (19)$$

Here,  $F_n = D^{-1}(n) s_{n-1} m(n)$ ,  $V_n = D^{-1}I B_n$ ,

$e_n = D^{-1}u_n$ , since  $E[e_n e_n^T] = I$ ,

where,  $e(n)$  : white residual error.

The following cross-correntropy cost function is proposed:

$$Z L_s n = 1 L_i = 1 L J \sigma_{fin} - v_{insn} \quad (20)$$

Where  $fin$  :  $i^{th}$  element of  $F(n)$ ,  $vin$  :  $i^{th}$  row of  $V(n)$ ,  $L = a + b$  is  $F(n)$  dimension.

The optimal estimate for  $s(n)$  is given under the cross-correntropy criterion

$$\mathbf{s}_n = \arg\max_{\mathbf{s}_n} \mathbf{Z}^T \mathbf{s}_n = \arg\max_{\mathbf{s}_n} \mathbf{1}^T \mathbf{J} \mathbf{e}_n \quad (21)$$

Where  $\mathbf{e}_n$  :  $i^{\text{th}}$  element of  $\mathbf{e}(n)$ :

$$\mathbf{e}_n = \mathbf{f}_n - \mathbf{v}_n \mathbf{s}_n \quad (22)$$

It enables the optimal solution by resolving

$$\frac{\partial \mathbf{Z}^T(\mathbf{s}_n)}{\partial \mathbf{s}_n} = \mathbf{1}^T \mathbf{J} \mathbf{e}_n \mathbf{v}_n^T (\mathbf{n} \mathbf{f}_n - \mathbf{v}_n \mathbf{s}_n) = 0 \quad (23)$$

This proceeds quickly

$$\mathbf{s}_n = \mathbf{1}^T \mathbf{J} \mathbf{e}_n \mathbf{v}_n^T \mathbf{n} \mathbf{v}_n - 1 \times \mathbf{1}^T \mathbf{J} \mathbf{e}_n \mathbf{v}_n^T \mathbf{n} \mathbf{f}_n \quad (24)$$

Since  $\mathbf{e}_n = \mathbf{f}_n - \mathbf{v}_n \mathbf{s}_n$

Eq. (24) represents the fixed-point equation  $[\mathbf{I}\mathbf{X}][\mathbf{X}]$  of  $\mathbf{s}_n$  and this can be rewritten as

$$\mathbf{s}_n = \mathbf{g}_n \mathbf{s}_n \quad (25)$$

With

$$\mathbf{g}_n = \mathbf{1}^T \mathbf{J} \mathbf{e}_n \mathbf{v}_n^T \mathbf{n} \mathbf{v}_n - 1 \times \mathbf{1}^T \mathbf{J} \mathbf{e}_n \mathbf{v}_n^T \mathbf{n} \mathbf{f}_n$$

Fixed-Point algorithm can be simply obtained as follows

$$\mathbf{s}(n)_{t+1} = \mathbf{g}_n \mathbf{s}_n \quad (26)$$

At fixed-point iteration 't',  $\mathbf{s}(n)_t$  denotes the required solution.

The Eq. (24) is a fixed-point equation can be further represented as

$$\mathbf{s}_n = \mathbf{V}^T \mathbf{n} \mathbf{H}_n \mathbf{V}_n - 1 \mathbf{V}^T \mathbf{n} \mathbf{H}_n \mathbf{F}_n \quad (27)$$

Where  $\mathbf{H}_n = \mathbf{H}_x(n) \mathbf{0} \mathbf{0} \mathbf{H}_y(n)$

With

$$\mathbf{H}_x = \text{diag} \mathbf{J} \mathbf{e}_1 \mathbf{n}, \dots, \mathbf{J} \mathbf{e}_a \mathbf{n}, \mathbf{H}_y = \text{diag} \mathbf{J} \mathbf{e}_a + 1 \mathbf{n}, \dots, \mathbf{J} \mathbf{e}_a + b \mathbf{n}$$

Eq. (27) may also in another way be represented as

$$\mathbf{s}_n = \mathbf{s}_{n-1} + \mathbf{N}_n \mathbf{m}_n - \mathbf{n}_n \mathbf{s}_{n-1} \quad (28)$$

Here,

$$\mathbf{N}_n = \mathbf{C}_{n-1} \mathbf{B}^T \mathbf{n} \mathbf{B}_n \mathbf{C}_{n-1} \mathbf{B}^T \mathbf{n} + \mathbf{Q}_n - 1 \mathbf{C}_{n-1} = \mathbf{D}_{cnn-1} \mathbf{H}_x - 1 \mathbf{n} \mathbf{D}_c \mathbf{T}_{nn-1} \mathbf{Q}_n = \mathbf{D}_{qn} \mathbf{H}_y - 1 \mathbf{n} \mathbf{D}_q \mathbf{T}_n \quad (29)$$

### Steps in computing the Cross-Correntropy Kalman Filter

Actually  $\mathbf{N}_n$  depends on  $\mathbf{C}_{n-1}$  and  $\mathbf{Q}(n)$ , normally both are correlated to  $\mathbf{s}_n$  through  $\mathbf{H}_x$  and  $\mathbf{H}_y$ , respectively. Eq. (28) relies on above mentioned estimation  $\mathbf{s}_{n-1}$  which can be calculated by Eq.(12) with the updated estimate  $\mathbf{s}_n - 1 \mathbf{n} - 1$ . With the above mentioned mathematical equations, we can recap the proposed cross-correntropy algorithm is shown in the below steps.

#### Step 1:

Initialize kernel size ' $\sigma$ ' and ' $\epsilon$ ' which is a small positive number. Take ' $n$ ' as 1 and set  $\mathbf{s}_0, \mathbf{C}_0$  which represents the initial estimate and covariance matrix respectively.

#### Step 2:

Compute  $\mathbf{s}_{n-1}$  and  $\mathbf{C}_{n-1}$  by using Eq. (12) and (13) and also determine  $\mathbf{D}_{cnn-1}$  with the cholesky decomposition.

#### Step 3:

Assume ' $t$ ' as 1 and  $\mathbf{s}_{nt} = \mathbf{s}_{n-1}$ ,

Where  $\mathbf{s}_{nt}$  : Estimated state at the fixed-point iteration t;

#### Step 4:

Compute  $\mathbf{s}_{nt}$  by using the Equations from (30) to (36)

$$\mathbf{s}_{nt} = \mathbf{s}_{n-1} + \mathbf{N}_n (\mathbf{m}_n - \mathbf{B}(n) \mathbf{s}_{n-1}) \quad (30)$$

With

$$\mathbf{N}_n = \mathbf{C}_{n-1} \mathbf{B}^T \mathbf{n} (\mathbf{B}_n \mathbf{C}_{n-1} \mathbf{B}^T \mathbf{n} + \mathbf{Q}(n))^{-1} \quad (31)$$

$$\mathbf{C}_{n-1} = \mathbf{D}_{cnn-1} \mathbf{H}_x - 1 \mathbf{n} \mathbf{D}_c \mathbf{T}_{nn-1} \quad (32)$$

$$\mathbf{Q}_n = \mathbf{D}_{qn} \mathbf{H}_y - 1 \mathbf{n} \mathbf{D}_q \mathbf{T}_n \quad (33)$$

$$\mathbf{H}_x = \text{diag} \mathbf{J} \mathbf{e}_1 \mathbf{n}, \dots, \mathbf{J} \mathbf{e}_a \mathbf{n} \quad (34)$$

$$\mathbf{H}_y = \text{diag} \mathbf{J} \mathbf{e}_a + 1 \mathbf{n}, \dots, \mathbf{J} \mathbf{e}_a + b \mathbf{n} \quad (35)$$

$$\text{ein} = \text{fin} - \text{vinsnt} - 1 \quad (36)$$

#### Step 5:

If (37) holds, put  $\text{snn} = \text{snnt}$  and continue to step (6) in case of comparing the estimations of current step and last step. if not,  $t+1 \rightarrow t$  and go back to step (4).

$$\text{snnt} - \text{snnt} - 1 \leq \varepsilon \quad (37)$$

#### Step 6:

By using Eq. (38) update the matrix of corrected covariance,  $n+1 \rightarrow n$  and go back to step (2).

$$\text{Cnn} = \text{I} - \text{NnBn} \text{Cnn} - 1 \text{ I} - \text{NnBnT} + \text{NnQ}(n) \text{NTn} \quad (38)$$

In the implementation of the CCKF algorithm mainly we involved the equations (12), (13), (30)-(36) and (38).

## V Simulation Results & Discussion

This paper proposes a new approach called the Cross-Correntropy Kalman Filter (CCKF) based on the cross-correntropy criterion and the fixed-point iterative algorithm for the problem of GPS based position estimation. The actual RINEX pseudo-range measurement dataset has been collected on 1st January 2014 by the GPS receiver positioned at IISc, Bangalore, south zone of the Indian subcontinent (Lat/Lon: 13.010 N/77.56 0 E). The data is sampled at an interval of 30 seconds. For the simulation, a total of 2640 epochs of data (i.e. 22 Hrs x 120 epochs/hour) is analyzed and post-processed in the MATLAB environment.

The position of the receiver is estimated with C / A measurements using the Least Square Estimator, Kalman Filter, Extended Kalman Filter, and Cross Correntropy Kalman Filter algorithms. (Omitted relativistic and satellite clock offset biases). The true location coordinates for the receiver are  $X=1337936.309\text{m}$ ,  $Y=6070317.116\text{m}$ , and  $Z=1427876.908\text{m}$ . These values correspond to the GPS receiver position surveyed at IISc, Bangalore, India. The estimated receiver position among four algorithms (LSE, KF, EKF, CCKF) compared with the receiver true position is shown in Fig. 1, whereas from Fig.2 to Fig. 4 represents the comparison of estimated x, y, z position error obtained using four algorithms along with the numerical values shown in Table 1 respectively. In addition to the above plots, a comparison of mean position error also presented in Fig.5.

Fig. 1 Receiver true position vs estimated position with four algorithms  
Fig. 2 Comparison of X – position mean error

Fig. 3 Comparison of Y – position mean error  
Fig. 4 Comparison of Z – position mean error

Fig. 5 Comparison of mean position error

**Table 1:** Mean position error comparison among LSE, KF, EKF and CCKF

GPS Time (Hours)	X- position (meters)				Y-position (meters)				Z-position (meters)			
	LSE	KF	EKF	CCKF	LSE	KF	EKF	CCKF	LSE	KF	EKF	CCKF
00-02	38.3	37.0	31.1	28.0	21.8	17.6	21.7	10.8	7.7	4.4	2.3	7.6
02-04	41.8	43.1	23.1	31.5	22.5	21.0	19.7	11.4	4.2	3.9	0.1	4.2
04-06	38.9	40.7	15.1	28.6	28.0	19.2	23.4	12.7	6.7	7.1	6.1	4.3
06-08	34.7	35.5	19.6	23.4	27.1	22.0	17.5	13.4	6.7	6.7	15.5	4.4
08-10	29.1	29.8	28.1	17.7	33.6	34.2	18.7	20.1	7.9	7.2	7.1	4.5

10-12	28.6	27.5	32.2	18.5	40.4	47.4	18.1	25.6	7.1	8.6	6.3	2.6
12-14	32.9	36.5	27.7	23.7	38.7	37.4	16.0	24.3	6.3	6.0	9.0	1.6
14-16	35.3	41.0	26.8	27.2	31.4	23.2	16.2	16.2	3.9	2.3	8.2	3.3
16-18	35.4	36.9	29.3	25.5	25.9	21.0	15.6	13.8	3.3	3.6	5.4	2.6
18-20	30.4	31.5	30.9	19.8	24.2	21.0	23.6	10.2	3.5	4.4	4.6	2.0
20-22	31.1	31.6	32.8	21.3	26.4	16.9	24.1	11.4	7.7	4.5	4.9	6.5
<b>Mean</b>	<b>34.2</b>	<b>35.6</b>	<b>27.0</b>	<b>24.1</b>	<b>29.1</b>	<b>25.5</b>	<b>19.5</b>	<b>15.4</b>	<b>5.9</b>	<b>5.4</b>	<b>19.5</b>	<b>4.0</b>
<b>Standard deviation</b>	<b>4.3</b>	<b>5.0</b>	<b>5.6</b>	<b>4.5</b>	<b>6.2</b>	<b>9.7</b>	<b>3.2</b>	<b>5.5</b>	<b>1.8</b>	<b>1.9</b>	<b>4.0</b>	<b>1.8</b>
<b>Variance</b>	<b>18.3</b>	<b>24.7</b>	<b>31.2</b>	<b>20.1</b>	<b>38.8</b>	<b>94.9</b>	<b>10.4</b>	<b>30.2</b>	<b>3.2</b>	<b>3.6</b>	<b>15.7</b>	<b>3.3</b>

From Fig. 5 and Table 1, it is observed that the mean position error is improved due to the CCKF algorithm when compared to LSE, KF, and EKF. Also, Table 1 provides information regarding four algorithms 3 error pdf's over IISc, Bangalore GPS receiver data. These parameters are used to compute the position accuracy measures in 2-D and 3-D surface which are shown in Table 2. Accuracy metrics are the statistical approaches used to characterize the efficiency of the algorithm for GPS receiver position estimation. The results of CCKF show that accuracy and precision are better than those of the prevailing approaches (LSE, KF, & EKF). The parameters for error analysis are shown in the above table and are calculated for the entire data range. Finally, CCKF algorithm performance is compared to the other three algorithms (LSE, KF, & EKF), from the results, CCKF is observed to surpass the LSE, KF, EKF algorithms by providing high accuracy and low variance in position estimation. The Statistical Accuracy Measures (SAM) [II] [XI][XII] of the estimated receiver position is used to characterize the position accuracy with a single value. These measures define accuracy for the position estimation with a single value contrary to the statistical moments of the mean and deviation allocated separately w.r.t x, y, z coordinate positions. Table 2 provides a list that the most commonly used accuracy measures of the 2-D and 3-D GPS receiver position.

**Table 2:** Comparison of Position Accuracy Measure among LSE, KF, EKF and CKF

Dimensions	Accuracy measure	Probability	LSE	KF	EKF	CCKF
		%	(meters)	(meters)	(meters)	(meters)
2	Circular error probable (CEP)	50	10.2	12.9	9.1	8.7
2	1-dev root mean square error (1DRMS)	63	12.4	15.8	11.0	10.5
2	2 -dev root mean square error (2DRMS)	95	24.8	31.7	21.9	20.9
2	95% Circular error probable (CEP <sub>95</sub> (R95))	95	21.2	26.8	18.9	18.2
3	Spherical error probable (SEP)	50	10.6	13.2	11.4	9.0
3	Mean radial spherical error (MRSE)	61	12.9	16.4	12.9	10.9
3	90% spherical accuracy standard (90% SAS)	90	17.3	21.6	18.6	14.7

From Table 2, it can be found that 50% of estimated horizontal point positions (X, Y) are within 10.2m, 12.9m, 9.1m, 8.7m for LSE, KF, EKF, and CCKF respectively. Similarly, 50% of the estimated 3-D point positions are within 10.6m, 13.2m, 11.4m, and 9.0m respectively for LSE, KF, EKF & CCKF algorithms. In addition to CEP and SEP values, other numerical values of the statistical measures DRMS, 2DRMS, MRSE, SAS are presented in Table 2. The estimated position error due to the four algorithms (LSE, KF, EKF, and CCKF) is shown in Fig. 2 to Fig. 4. All these errors have logged over 22 hours and two-dimensional surface scatter plot that uses only horizontal position errors (X, Y) is generated. Individually, the position error scatter plots of four algorithms represented from Fig. 6 to Fig. 9.

Fig. 6 Horizontal scatter plot with LSE Fig. 7 Horizontal scatter plot with KF

Fig. 8 Horizontal scatter plot with EKF Fig. 9 Horizontal scatter plot with CCKF

Fig. 6 to Fig. 9 reveals that the CEP, R95, and 2DRMS circles of CCKF is smaller than the circles of the remaining three methods (LSE, KF, EKF) and the CCKF receiver position is nearer to the actual receiver position.

## VI Conclusion

A new Kalman filter based on the criterion of correntropy is proposed in this paper and tested with real-time GPS data. The results of the simulation show that the proposed filter outperforms that of the prevalent approaches (LSE, KF & EKF). The performance of four algorithms (LSE, KF, EKF, and CCKF) is assessed in terms of accuracy and precision. Hence from the results, it is concluded that the position estimated with CCKF is more accurate than LSE, KF & EKF. Interestingly, the mean position error recorded as a consequence of the CCKF, EKF, KF & LSE algorithms is  $X=24.1m$ ,  $Y=15.4m$ ,  $Z=4.0m$ ,  $X=27.01m$ ,  $Y=19.5m$ ,  $Z=5.6m$ ,  $X=35.6m$ ,  $Y=25.5m$ ,  $Z=5.4m$ , and  $X=34.2m$ ,  $Y=29.1m$ ,  $Z=5.9m$  respectively. It is also evident that the CCKF results in estimates of the low variance position are compared to the LSE, KF, EKF methods. Based on the results obtained in this paper, it can be concluded that the proposed CCKF algorithm can be used in precise positioning GPS applications, such as CAT I aircraft landing and geodesy.

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