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UNSTEADY MHD FLOW OF GENERALIZED VISCO-ELASTIC OLDROYD FLUID UNDER TIME-DEPENDENT BODY FORCE THROUGH A POROUS CONCENTRIC CIRCULAR CYLINDRICAL DUCT

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Abstract

The aim of the present paper is the investigation of the unsteady unidirectional flow of an incompressible generalized visco-elastic Oldroyd type fluid between porous concentric cylindrical duct under the action of a transverse magnetic field with time dependent body force. Here we have calculated the velocity profile of a fluid element of the problem theoretically and graphically. From the analysis of this fluid motion, the dynamics of the ordinary viscous fluid is also discussed.

সংক্ষিপ্তসার

বর্তমান পত্রের উদ্দেশ্য হচ্ছে অস্থির একদৈশিক প্রবাহকে অনুসন্ধান করা যখন ইহা সময় নির্ভরশীল বস্তুজবল (Bodyforce) সহ তির্যক চৌম্বক ক্ষেত্রের কার্যের অধীন সছিদ্র এককেন্দ্রীক চোঙ্গাকার নলের মধ্য দিয়ে প্রবাহিত অসংনম্য সামান্যীকৃত সান্দ্র - স্থিতিস্থাপক ওলড্রয়ড় (Oldroyd) জাতীয় প্রবাহী পদার্থ। এখানে আমরা উক্ত সমস্যার প্রবাহী পদার্থের উপাদানগুলির গতিবেগ নক্‌শাটিকে তত্ত্বগতভাবে এবং লেখচিত্র আকারে নির্ণয় করেছি। এই প্রবাহী পদার্থের গতির বিশ্লেষণ থেকে সাধারণ সান্দ্র প্রবাহী পদার্থের গতিশীলতাকে আলোচনা করেছি।

1. Introduction:

There are fluids which exhibit the elastic property of solids and viscous property of fluids which are adequate in nature and relevant fluids generate visco-elastic fluid mechanics .These type of fluids are also called non-Newtonian fluids or visco-elastic fluid,the basic development of hydrodynamic flow has been presented in the work of Lamb (1945) and hydromagnetic problems have been considered by Cowling (1957), Carslaw and Jaeger (1949) studied the basic problem of viscous motion between two parallel plates under the action of initially applied body force. Das (1991) studied hydromagnetic flow of viscous conducting fluid through a circular cylinder. Sengupta and Roymahapatra (1971) studied visco-elastic Maxwell fluid through rectangular channel with transient pressure gradient. The hydromagnetic flow of two immiscible visco-elastic watler liquids between two inclined parallel plates has been investigated by Chakraborty and Sengupta (1991), Ghose and Sengupta (1993) studied the unsteady hydromagnetic flow of two immiscible visco-elastic oldroyd fluids between two parallel plates. In the present paper, we have considered the unsteady MHD flow of a generalized visco-elastic fluid of Oldroyd type through a porous medium in concentric cylindrical duct under time dependent body force.

2. Mathematical Analysis

We consider the cylinders $x'^2 + y'^2 = a^2$ and $x'^2 + y'^2 = b^2$ $(a>b)$ are the boundary walls and z' axis is the axis of cylinders, the direction of motion. Let us consider $w'(x', y', t')$ be the velocity of the fluid. Assuming the motion to be slow and neglecting pressure gradient.

The equation of motion of a visco-elastic oldroyd fluid through concentric cylindrical duct are given by

$$\left(1+\sum\lambda'_i\frac{\partial}{\partial t'_i}\right)\frac{\partial w'}{\partial t'}=\left(1+\sum\lambda'_i\frac{\partial}{\partial t'_i}\right)F(t')+\nu\left(1+\sum\mu'_i\frac{\partial}{\partial t'_i}\right)\nabla'^2 w'-\frac{\sigma B_0^2}{\rho}\left(1+\sum\lambda'_i\frac{\partial}{\partial t'_i}\right)w'-\frac{\nu}{k}\left(1+\sum\lambda'_i\frac{\partial}{\partial t'_i}\right)w' \quad (2.1)$$

where λ'_i are the stress relaxation time parameters, μ'_i the strain rate retardation time parameters, ν the kinematic coefficient of viscosity , k is the permeability of the medium , B_0 is the magnetic field perpendicular to the direction of flow, σ is the electrical conductivity of the fluid and $w'(x, y, t)$ is the velocity component of the fluid in z-direction.

We now introduce the following non-dimensional transformations:

$$w' = \frac{\upsilon}{a} W,\ t' = \frac{a^2}{\upsilon} t,\ \lambda'_i = \frac{a^2}{\upsilon} \lambda_i,\ \mu'_i = \frac{a^2}{\nu} \mu_i \text{ and } (x', y', z') = a(x, y, z).$$

Using the above transformations, the equation (2.1) reduces to following non-dimensional form:

$$\left(1+\sum \lambda_i \frac{\partial^i}{\partial t^i}\right)\left(\frac{\partial W}{\partial t}\right) = \left(1+\sum \lambda_i \frac{\partial^i}{\partial t^i}\right) F(t) + \left(1+\sum \mu_i \frac{\partial^i}{\partial t^i}\right) \nabla^2 W$$
$$-M^2\left(1+\sum \lambda_i \frac{\partial^i}{\partial t^i}\right) W - \frac{a^2}{k}\left(1+\sum \lambda_i \frac{\partial^i}{\partial t^i}\right) W \tag{2.2}$$

The boundary conditions of the problem in non –dimensional form are

W=0 on the surface $x^2 + y^2 = 1$ and (2.3)

W=0 on the surface $x^2 + y^2 = b^2/a^2$ (2.4)

3. Solution of the problem

We solved the problem in two cases.

Case I: When body force is of the form of transient in nature w.r.t. time t.

We consider a transient body forces $F_0 e^{-\omega t}$ is applied to the fluid and assume the velocity of visco-elastic fluid of the form

$$W = v(x, y) e^{-\omega t}, \tag{3.1}$$

The equation (2.2) with the help of (3.1) becomes

Or,

$$\nabla^2 v + p^2 v = -N \tag{3.2}$$

where

$$p^2 = (\omega - M^2 - \frac{a^2}{k}) - \frac{(1+\beta\omega)}{(1+\alpha\omega)} \text{ and } N = \frac{(1+\beta\omega)}{(1+\alpha\omega)} F_0.$$

Now the boundary conditions reduced to

$v = 0$ on the surface $x^2 + y^2 = 1$ (3.3)

and

$v = 0$ on the surface $x^2 + y^2 = b^2/a^2$. (3.4)

We construct a transformation of the form $s = x\cos\theta + y\sin\theta$ and using in the equation (3.2), we get

$$\frac{d^2v}{ds^2}+p^2v=-N \tag{3.5}$$

The boundary conditions are

$$\left.\begin{array}{l} v=0, \text{when } s=1 \\ v=0, \text{when } s=b/a \end{array}\right\} \tag{3.6}$$

Solving equation (3.5) under the boundary condition (3.6) we get

$$v=\frac{F_0}{M^2+a^2/k-\omega}\left[1-\frac{\cos p\left(s-(a+b)/2a\right)}{\cos p\left((a-b)/2a\right)}\right] \tag{3.7}$$

Thus the velocity of the fluid particle is given by

$$W=\frac{F_0e^{-\omega t}}{M^2+a^2/k-\omega}\left[1-\frac{\cos p\left(s-(a+b)/2a\right)}{\cos p\left((a-b)/2a\right)}\right] \tag{3.8}$$

We now consider the case where p is very small so that we approximate p as

$$\cos p\left(\frac{a-b}{2a}\right)=1-\frac{P^2\left((a-b)/2a\right)^2}{2!}$$

and

$$\cos p\left(\frac{a+b}{2a}-s\right)=1-\frac{P^2\left((a+b)/2a-s\right)^2}{2!}$$

Substituting these values in (3.8), we obtain

$$W=\frac{4aF_0(1+\beta\omega)}{(1+\alpha\omega)}\frac{(1+s)(as-b)}{\left\{8a^2-P^2(a-b)^2\right\}}e^{-\omega t} \tag{3.9}$$

Case II: When body force is in the form of periodic in nature w.r.t. time t.

In this case, we assume that the body force $F_0e^{j\omega t}$ is applied to the fluid and consider the velocity of visco-elastic fluid is of the form

$$W=v(x,y)e^{j\omega t} \tag{3.10}$$

where

$$j=\sqrt{-1}.$$

The equation (2.2) with the help of (3.10) becomes

$$\nabla^2 v - p_1^2 v = -N_1 \tag{3.11}$$

where

$$p_1^2 = \left(M^2 + \frac{a^2}{k} + j\omega\right)\frac{(1-j\beta\omega)}{1-j\alpha\omega} = \mathrm{Re}^{i\theta} = A + iB \text{ and } N_1 = F_0\frac{1-j\beta\omega}{1-j\alpha\omega}.$$

A solution of (3.11) with the boundary conditions (3.6) is

$$v = \frac{F_0}{\left(M^2 + a^2/k + j\omega\right)}\left[1 - \frac{\cosh p_1\left((a+b)/2a - s\right)}{\cosh p_1\left((a-b)/2a\right)}\right] \tag{3.12}$$

Hence the solution of (3.10) is

$$W = \frac{F_0 e^{j\omega t}}{\left(M^2 + a^2/k + j\omega\right)}\left[1 - \frac{\cosh p_1\left((a+b)/2a - s\right)}{\cosh p_1\left((a-b)/2a\right)}\right]$$

$$= \frac{F_0 e^{j(\omega t-\phi)}}{R_1}\left[\left[\left\{1 - \frac{\left\{\cosh(1-s)A\cos\left(s-\frac{b}{a}\right)B + \cosh\left(s-\frac{b}{a}\right)A\cos(1-s)B\right\}}{\cosh\left(\frac{a-b}{a}\right)A + \cos\left(\frac{a-b}{a}\right)B}\right\}\cos(\omega t-\phi)\right.\right.$$

$$\left. + \left\{\frac{\sinh(1-s)A\sin\left(s-\frac{b}{a}\right)B + \sinh\left(s-\frac{b}{a}\right)A\sin(1-s)B}{\cosh\left(\frac{a-b}{a}\right)A + \cos\left(\frac{a-b}{a}\right)B}\right\}\sin(\omega t-\phi)\right]$$

Taking the real part only, we obtain purely viscous fluid which is given below:

$$W = \frac{F_0}{R_1}\left[\left[\left\{1 - \frac{\left\{\cosh(1-s)A\cos\left(s-\frac{b}{a}\right)B + \cosh\left(s-\frac{b}{a}\right)A\cos(1-s)B\right\}}{\cosh\left(\frac{a-b}{a}\right)A + \cos\left(\frac{a-b}{a}\right)B}\right\}\cos(\omega t-\phi)\right.\right.$$

$$\left. + \left\{\frac{\sinh(1-s)A\sin\left(s-\frac{b}{a}\right)B + \sinh\left(s-\frac{b}{a}\right)A\sin(1-s)B}{\cosh\left(\frac{a-b}{a}\right)A + \cos\left(\frac{a-b}{a}\right)B}\right\}\sin(\omega t-\phi)\right] \tag{3.13}$$

where

$$R_1^2 = \left(M^2 + a^2/k\right)^2 + \omega^2 \text{ and } R^2 = A^2 + B^2$$

$$A = \frac{\left(M^2 + a^2/k\right)\left(1 + \alpha\beta\omega^2\right) - (\alpha - \beta)\omega^2}{\left(1 + \alpha\beta\omega^2\right)}, \quad B = \frac{\left(1 + \alpha\beta\omega^2\right)\omega + \left(M^2 + a^2/k\right)(\alpha - \beta)\omega}{\left(1 + \alpha^2\omega^2\right)}$$

and

$$\phi = \tan^{-1}\frac{\omega}{\left(M^2 + a^2/k\right)}.$$

4. Numerical calculation and discussions

The numerical calculation of velocity profile W is performed for various values of M in both the cases with taking $\lambda_1 = \alpha = 0.05$, $\mu_1 = \beta = 0.025$, $\omega = 0$, $k = 0.5$, $b = 1$, $F_0 = 1$, $a = 2$ and $s = 0.75$.

In the first case i.e. when the body force is of the form transient in nature, the velocity distribution in the equation (3.9) is solved numerically. In Table-1 the value of $W \times 10^{-3}$ is given for several values of t and M and in Figure-1 the values of W is plotted against t for three values of M. It is seen from both the table and the figure that velocity decreases continuously due to increase of time and with increasing M the velocity decreases.

Table-1 The values of $W \times 10^{-3}$ for several values of t and M with transient body force.

t	$W \times 10^{-3}$		
	M=0	M=5	M=10
0.0	0.0275	0.0163	0.0079
0.1	0.0101	0.0060	0.0027
0.2	0.0037	0.0022	0.0010
0.3	0.0014	0.0008	0.004
0.4	0.0005	0.0003	0.001
0.5	0.0002	0.0000	0.000
0.6	0.000	0.0000	0.000
0.7	0.000	0.0000	0.000
0.8	0.000	0.0000	0.000
0.9	0.000	0.0000	0.000
1.0	0.000	0.0000	0.000

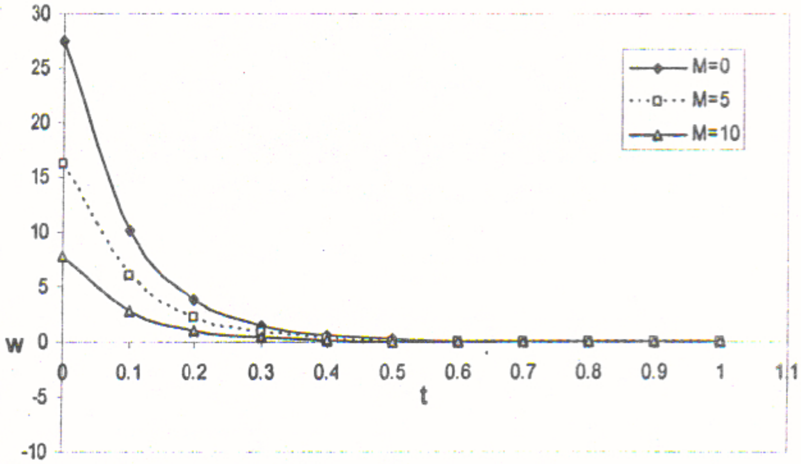

Figure-1 The velocity distribution W against for several values of t and M with transient body force.

Now in the second case i.e. when the body force is of periodic in nature, the velocity W in the equation (3.13) is solved numerically. In Table2 the value of $W\times10^{-2}$ is given for several values of t and M and in Figure2 the values of W is presented against t for two values of M. From Table2 and Figure2 clearly shows that the velocity starts with a finite positive value and becomes oscillatory type with increasing t for both value of M. But, due to increase in M the modulus of oscillation is reduced.

Table-2 The values of $W\times10^{-2}$ for several values of t and M with periodic body force.

t	$W\times10^{-2}$	
	M=0	M=4
0.0	6.044	3.547
0.1	8.095	3.210
0.2	2.710	-1.973
0.3	-5.165	-3.292
0.4	-8.209	3.841
0.5	-3.808	-0.475
0.6	4.179	2.571
0.7	8.326	3.687
0.8	4.820	1.016
0.9	-3.110	-2.588
1.0	-8.188	-3.815

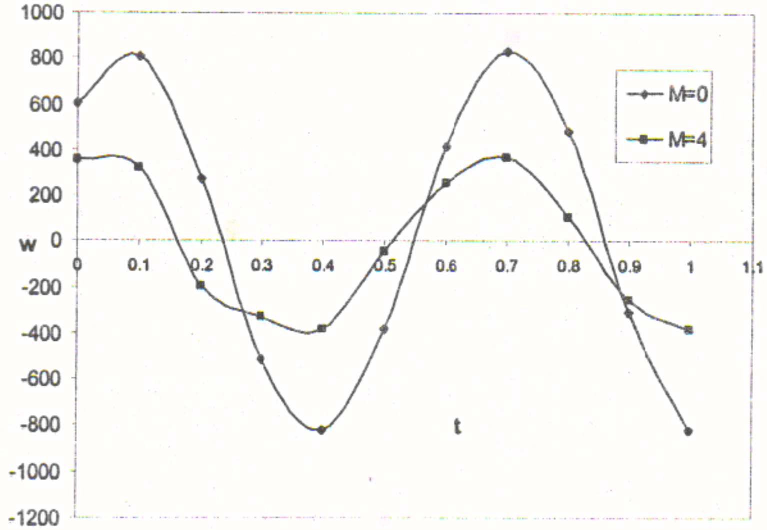

Figure-2 The velocity distribution W against for several values of t and M with transient body force

5. Conclusions

An investigation is made on unsteady unidirectional flow of an incompressible generalized visco-elastic Oldroyd type fluid between porous concentric cylindrical duct with transverse magnetic field and time dependent body force. From the analysis it reveals that in case of transient body force the velocity deceases with increasing time and magnetic field and on other hand, in case of periodic body force though the velocity is oscillatory type, the oscillation range decrease with increase in magnetic field.

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