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TWO DIMENSIONAL LEGENDRE MOMENTS AND ITS APPLICATION IN CLASSIFICATION OF MEDICAL IMAGES

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Abstract

In this paper, we study the computational strategy for the implementation of orthogonal moments to two-dimensional images. Automatic and accurate classification of Magnetic Resonance Images is of importance for the interpretation and analysis of these images and for this purpose different techniques have been proposed. In this paper, we present Legendre Polynomial and two different classification-based methods for the classification of normal and abnormal MRI Images. In the first step, we apply Legendre polynomial to extract features from MRI images. In the second stage, two classifiers have been used which are employed to classify these images as normal and abnormal images. The proposed method was tested on tests with 75 images in which 15 images belong to the normal category images and the remaining 60 are abnormal images. The result derived from the confusion matrix test yielded a classification accuracy of 100.0% for these images.

Keywords: Legendre Polynomials, Shifted Legendre Polynomials, Classification, MRI Images, Image Processing

I. Introduction

Magnetic Resonance Imaging (MRI) is often the most widely used imaging technique whenever imaging of soft tissue human body is required. MRI provides expensive information for biomedical research and clinical diagnosis due to its high-quality images of the inner structures of different soft tissues. One of the most advantages of using MRI imaging is that it is a non-invasive technique [XVI]. The inspection of MRI images is becoming more and more difficult due to the shortage of expert radiologists and being cost expensive. Computer technology can be widely used

to overcome the abovementioned problem as its application in the medical decision support system is becoming more important for different medical applications, such as gastroenterology, cancer research, heart diseases, and brain tumors, etc. [V]. Normal and abnormal human brain classification images can be obtained from MRI; which is of importance for clinical and research studies [VII, XII]. Some of the recent results from [I, XI] shown that the classification of MRI images is possible via supervised [I] and unsupervised classification techniques [XI, VIII]. In this research, we have used simple logistic and Sequential Minimal Optimization for training Support Vector Machine (SMO(SVM)) for the classification of MRI images as either normal or abnormal images. For feature extraction, different researchers have used different mathematical techniques. Some of these techniques are based on Discrete Wavelet Transform (DWT) [XIII] and [XVI], two-level 2D Discrete Wavelet Transform (2D-DWT) [I, IV, XV], wavelet-entropy [XVI] and by using modified parameters of generalized autoregressive conditional heteroscedastic (GARCH) statistical model [VIII]. In the last decades, most of the researchers used Legendre polynomials in medical image and signal processing. Images can be correctly recovered from its Legendre moments invariants [XIV]. In image processing, Legendre Polynomials can be used for color image compression [II], construction of image from its blur invariant [III, XV]. [VII, IV] presents a method of how to calculate Legendre moments for the grayscale image. Legendre moments have high reconstruction abilities as compared to other sets of moments in the presence of noises [IX, VI]. [IX, XIV] efficiently apply one dimensional discrete Legendre polynomials for the classification of electrocardiograms signals. Due to the motivation of these results presented in the above reference; it is analyzed to used 2D Legendre polynomials in the classification of MRIs images. The main contribution of this research work is the integration of Legendre polynomials as a features extraction tool and two different classifiers to perform accurate MRI images classification as normal and abnormal images. Also, in this paper, we compared our results with similar studies found in the literature. The rest of the paper is organized as follows. In Section II short description of Legendre polynomials is presented. The method of feature extraction using Legendre polynomials is presented in Section C. Proposed scheme of classification is presented in Section III. In Section IV, we present some results and compare the proposed method with other image classification techniques. Finally, the conclusion of the proposed method is presented in Section V.

II. Methodology

In our hybrid technique, we have used a combination of well-known Legendre polynomials, simple logistic, and Sequential Minimal Optimization for training Support Vector Machine (SMO (SVM)). The proposed method is illustrated in Figure 1. In the following section, we will review the fundamentals of Legendre polynomials and how it can be used to extract features from MRI images. Assume that C[a,b] be the space of continuous function defined on the domain [a,b], so from the basic assumption and techniques of approximation theory, every $f(x) \in C[a,b]$ has a series representation as below.

$$f(x) = \sum_{i=0}^{\infty} c_i \varphi_i(x) \tag{1}$$

Where c_i is constant and $\varphi_i(x)$ is the component of the basis which are orthogonal and having the properties of boundness. Frequently used orthogonal polynomials which act as basis set are Legendre polynomials, Zernike polynomials, Jacobi polynomials, and Laguerre polynomials. In the proposed feature extraction scheme, we only considered Legendre polynomial. We present some basic facts and information related to Legendre polynomials.

A. Legendre Polynomials

We present some basic mathematical aspects of Legendre polynomials. Mathematically Legendre polynomials can be express by the following equation.

$$P_{i+1}(x) = \frac{2i+1}{i+1} x P_i(x) - \frac{i}{i+1}(x)$$
 (2)

where $i = 1, 2, \dots, n$ and for $P_0(x) = 1$ and $P_1(x) = x$. The transformation $t = \frac{\tau(x+1)}{2}$ makes these polynomials applicable within the domain [0,1] and shifted Legendre polynomials can be obtained as.

$$P_p^T(t) = \sum_{k=0}^p \beth_{p,k} t^k_{,i=0,1,2}$$
(3)

where

$$(p,k) = \frac{(-1)^{p+k}(p+k)!}{(p-k)!T^{k(k!)^2}} \tag{4}$$

The orthogonality relation of Legendre is defined by.

$$\int_{0}^{1} P_{p}(t) P_{q}(t) dx = \begin{cases} \frac{2}{2p+1} if p = q \\ 0 if p \neq q \end{cases}$$
 (5)

Any continuous signals in the domain interval [0, I] can be represented in the form of an infinite series of Legendre polynomials as below.

$$f(t) = \sum_{p=0}^{m} c_p P_p(t).$$
 (6)

Where the coefficients can be calculated by the relation as.

$$c_p(2l+1) = \int_0^1 f(t)P_p(t)dt \tag{7}$$

As $m \to \infty$ the series representation of the function becomes equal to the actual function.

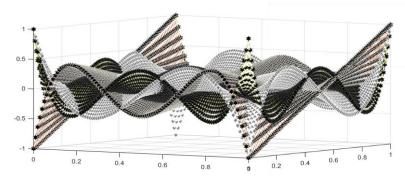


Fig. 1: 2D Legendre polynomials at indices $P_{23}(x,t)P_{26}(x,t)P_{33}(x,t)$

B. Two Dimension Legendre Moments

Any image is a 2D array of numerical values. Therefore, first, we will introduce the concept of 2D Legendre polynomials and develop a mathematical scheme for 2D Legendre polynomials. Legendre polynomials for 2D images can be obtained by taking the product of two basic sets. Let's have two basis sets $B_m(x)$ and $B_m(t)$ in two different variables. We can calculate the basis set by taking the product of $B_m(x) \times B_m(t)$, i.e. $B_m(x,t) = B_m(x) \times B_m(t)$. We can also represent the basis set $B_m(x,t)$ using the following relation.

$$P_{n+q}(x,t) = P_n^T(x) P_n^T(t)$$
(8)

At scale level m, we have $(m+1)^2$ terms in the resultant basis set. Any image having the domain $C([0,1] \times [0,1])$ can be represented in terms of Legendre polynomials.

$$f(x,t) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} c_{pq} P_p(x) P_q(t)$$
 (9)

Where coefficients can be calculated as.

$$C_{pq} = (2p+1)(2q+1)\int_0^1 \int_0^1 (t,x)P_p(t)P_q(x)dtdx.$$
 (10)

Fig 1 shows Legendre polynomials at different indices values. It is very clear from Figures 2, 3 that 2D Legendre polynomials up to level 45 are smooth and bounded in the domain [0,I].

C. Feature Extraction from 2D images

In many image processing applications, feature extraction is the most important step since each of these features represents a digital image in a very lower dimension. A digital image is an array of pixels. The centers of each pixel are (x_i, t_i) . The intensity function represents the intensity of each pixel of an image.

The intensity of each pixel in any image can be represented by the image intensity function. Let image intensity function for the discrete set of points $(x_i, t_i) \in [0,1] \times [0,1]$ is $f(x_i, t_i)$ and $\Delta_{x(i+1)}$ and $\Delta_{t(i+1)}$ are sampling intervals in the *x-axis* and *t*

-axis. The intervals Δx_i and Δy_i are fixed at some constant values $\Delta X_i = \frac{1}{M}$, and $\Delta Y_i = \frac{1}{N}$, respectively. Therefore, the points (x_i, y_i) calculated as

$$x_i = -1 + (i - \frac{1}{2})\Delta x \tag{11}$$

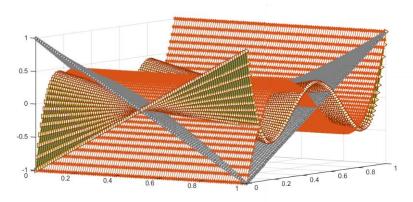


Fig. 2: 2D Legendre polynomials of different indices $P_{34}(x,t)P_{36}(x,t)P_{44}(x,t)$

$$t_i = -1 + (j - \frac{1}{2})\Delta y \tag{12}$$

with i = 1,2, 3....N and j = 1,2,3....M. Image function can be written as.

$$f(x_i t_i) = \sum_{p=0}^{\infty} \sum_{p=0}^{\infty} L_{pq} P_p(t) P_q(x)$$
(13)

where we can calculate the coefficients L_{pq} as.

$$L_{pq} = \frac{(2p+1)(2q+1)}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} P_p(x_i) P_q(t_j) f(x_{i,t_i})$$
(14)

D. Image Reconstruction Using Legendre Moments

Since, Legendre polynomial on the interval [0, 1] forms a complete basis set. So, any image function f(x,y) can be written in the form of an infinite series of Legendre polynomials.

$$f(x,y) = \sum_{p=0}^{\infty} \sum_{p=0}^{\infty} L_{pq} P_p(x) P_q(y)$$
 (15)

If the order of Legendre moments is Max, then the image intensity function f(x,y) in equation 15 can be approximated as.

$$f(x,y) = \sum_{p=0}^{Max} \sum_{q=0}^{p} L_{(p-q,p)} P_{(p-q)}(x) P_q(y)$$
(16)

The number of moments required using the above relation is calculated.

$$N_{Total} = \frac{(Max+1)(Max+2)}{2} \tag{17}$$

We introduce the new scale which relates the index n with p and q by using the following relation

$$n = M * (p) + q + 1 \tag{18}$$

Where *n* represents the length moments vector. We have considered different artificial images and again reconstruct it from the moments. The reconstructed images are shown in Figure 4. Different test images are reconstructed using different scale levels i.e., 10, 15, 25, 40. It is clear from Figure 4 that, if the scale level increase, the resultant reconstructed image became clear. It should be noted that only up to level 45, optimal results can be achieved. Increasing the scale level from 45, the polynomials become discontinues, and they lose the property of boundness. In the proposed methods, we used scale level 45 to extract features from different images. The number of features using scale level can be calculated by *scale*×*scale*, i.e. 45×45. From this relation, if we use the scale level 45, then a total of 2025 extracted coefficients are used as features for the proposed features. The detailed moments for these different four test images are shown in Figure 5. It can be observed that each image is splinted into its bunches of different moments.

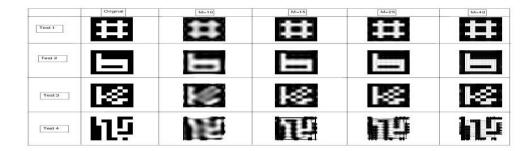


Fig. 4: Reconstruction of different objects with a different scale.

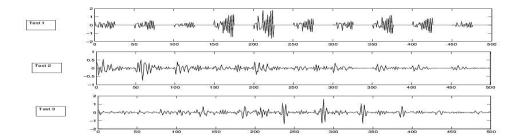


Fig. 5: 2D Legendre moments for the three test objects using scale level M = 25.

III. Classification

The grouping of different images into a different group of classes is called classification. To classify images two different techniques can be used. These techniques are either supervised or unsupervised technique. Supervised techniques are those classifiers in which the classifier is trained with both normal and abnormal data sets. Artificial Neural Network (ANN) and K-nearest neighbors (KNN) are mostly used the example of the supervised technique. In the unsupervised technique of classification, the classifier itself selects both types of data, i.e. normal and abnormal

data. Self-optimization map (SOM) and fuzzy c-means are widely used unsupervised techniques. In this paper, two classifiers namely simple logistic and SMO(SVM) are employed to carry out the classification of these images.

A. Sequential Minimal Optimization (SMO) (Support Vector Machine)

In the proposed method, we have used Sequential Minimal Optimization (SMO), one of the most popular algorithms for training Support Vector Machine (SVM) [III]. SVM is a supervised learning technique that is widely used for the classification of MRI images. The input vectors are mapped into dimensional feature space. In SVM, a hyperplane is constructed. The Hyperplane on each side of the separating class with the largest separation margin between the training points of the two classes is chosen. A model is then created and trained with input data, which predicts the class of a new sample.

B. Simple Logistic Classifier

To check the classification performance of the proposed scheme, a multi-class classifier called simple logistics is used. Simple logistic is based on logistic regression and is mainly used for binary classification. The logistic regression probabilistic model which is mainly used for binary classification. There are two types of the variable used in this classifier. Independent variables are input to the classifier model whereas dependent variables are the output of the classifier. The logistic function is used to find out the probability of the class of the independent variables.

• **Logistic function:** This Logistic function is used to find the probability of which independent variables belong to which class. The logistic function has either zero or one value. Mathematically, it can be expressed as,

$$P(t) = \frac{e^t}{e^t + 1} = \frac{1}{1 + e^{-1}} \tag{19}$$

In the above equation, P(t) is the logistic function, e represents exponential function and t represents a function for the independent variable, if x represents different variables and t represents its linear function, then the logistic function can be calculated as.

$$C(t) = \frac{1}{1 + e^{-\beta_0 + \beta_{1x}}} \tag{20}$$

Where C(t) is the probable outcome of each a case, β_1 is the product of the regression coefficient, β_0 = intercept of linear regression. If there are more independent variables, then β_1 can be calculated.

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \dots \tag{21}$$

• **Logit:** Logit is the inverse of the logistic function and mathematically can be represented as.

$$g(x) = \ln \frac{f(x)}{1 + f(x)} = \beta_0 + \beta_1 +. \tag{22}$$

IV. Results of Proposed System

In this section, we will discuss the performance evaluation methods which can be used to evaluate the proposed technique. First, we will present some experimental results, and examine the classification performance of the proposed methods. To evaluate the performance of the proposed method, we have used confusion matrix, accuracy, sensitivity, specificity, positive predictivity. These four matrices are used to measure classification performance [XI]. Mathematically they are represented as.

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN} \tag{23}$$

$$Sensitivity(Sen) = \frac{TP}{TP + FN}$$
 (24)

$$Specificity(Spe) = \frac{TP}{TP + FP}$$
 (25)

$$Positive rpredictivity = \frac{TP}{TN+TP}$$
 (26)

Where the term True Positive (TP) which represents correctly classified positive cases, False Positive (FP) represents incorrectly classified negative cases, True Negative represents correctly classified negative cases and False Negative (FN) represents incorrectly classified positive cases.

A. Dataset and Experimental Setup

The data set in our proposed scheme consist of T2-weighted MR brain images of 256×256 plain resolution. These images ware downloaded from the Harvard Medical School website¹. We have randomly selected 75 images in which 15 belong to the class of images while 60 images belong to the category of abnormal images. These abnormal brains consist of the following diseases: (a) Glioma, (b) Lyme encephalopathy, (c) Neo Plastic Diseases, and (d) Herpes encephalitis. A sample of different abnormal images is shown in Figure 6. The proposed algorithm is implemented using *MATLAB* for the extraction of the feature of coefficients. These coefficients are treated as primary features for the classification of MR images. We have used 70% of data for training and the remaining 30% for testing purposes. These extracted features are classified by using *Weka* 3.8.

B. Classification Performance of Support Vector Machine and Simple Logistic Classifier

In this section, we evaluate the classification performance of the proposed method. Table 1 shows the confusion matrix for the classification performance of two classifiers. Confusion Matrix shows the total number of correctly and incorrectly predictions of the classification scheme. Based on different values provided in the confusion matrix, overall accuracy, sensitivity, specificity and Positive predictivity of 100% can be achieved for both the classifiers. In the confusion matrix Table 1, it can be observed that both the classifier correctly classifier 7 normal MRI brain images and 15 abnormal MRI images. Figure 7 shows the classification result obtained using *Weka*

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3.8 indicating the mentioned results. Similarly, both classifiers did not classify even a single image instance incorrectly. In Table 2 the experimental results of the proposed classifiers are compared. The classification rate for the two different image classes is presented in terms of percentages. Experimental results show that the classification accuracy of 100% can be achieved for both the classifiers 7 8.

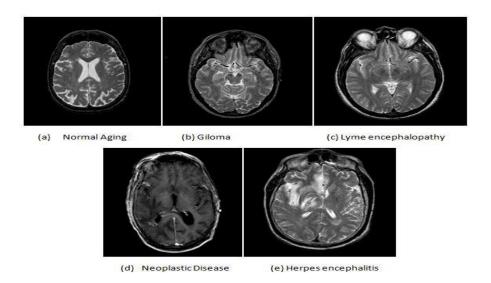


Fig. 6: Different brain images, (a) Normal brain images, (b) Glioma, (c) Lyme encephalopathy, (d) Neo Plastic Diseases, and (e) Herpes encephalitis

Table 1: Confusion Matrix of Simple logistic and SVM classifier

Classes	SMO(SVM) classifier		Simple logistic classifier		
Normal	7	0	7	0	
Abnormal	0	15	0	15	

Table 2: Classification rates for the used classifiers

Technique	Acc/%	Sen/%	Spe/%	PPr/%
Legendre Polynomial + Simple logistic	100	100	100	100
Legendre Polynomial + SMO(SVM)	100	100	100	100

In Fig 9, the star represents normal images while the cross represents abnormal images. It can be shown in Fig 9, that normal and abnormal images are clustered in different

J. Mech. Cont.& Math. Sci., Vol.-15, No.-9, September (2020) pp 355-367 positions. Also, normal images have a very different pattern as compare to abnormal images.

Fig. 7: Classification performance of Sequential Minimal Algorithm for Training Support Vector Machine

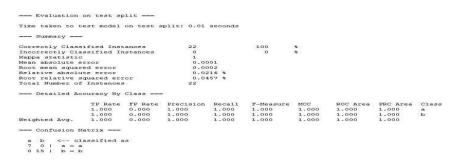


Fig. 8: Classification performance of Simple Logistic Classifier

The results show that our proposed method obtains high accurate results on both Simple Logistic and SMO(SVM) classifiers. Moreover, we have also compared our results with recent results using the same MRI datasets, which are shown in Table 3. Our proposed method earns the highest classification accuracy as compared to others found in the literature.

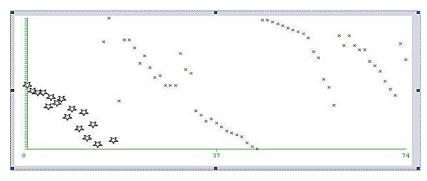


Fig. 9: Moments of normal and abnormal images

Table 3: Comparison results of the proposed method with other MRI images classification methods

Approaches	Methods of feature extraction	Classifier	Accuracy %	
El-Dhashan et al (2009)	3-level DWT	KNN + FFBPN	(98.7% & 97.5%)	
Zhang et al. (2012)	DWT	PCA + KSVM	(99.38 %)	
S. Chaplot <i>et al.</i> (2006)	DWT	SOM + SVM	(94 % & 98 %)	
H. Kalbkhani et al (2013)	DWT	DWT KNN + SVM		
A. Padma et al. (2014)	DWT	SVM + PNN	(97.32 %)	
Das et al. (2013)	RT	LS-SVM	(99.39 %)	
X. Zhou et al. (2015)	2D DWT	Naive Bayes	(92.6 %)	
Proposed Method	Legendre Polynomials	Simple logistic + SVM	(100 %)	

V. Conclusions

In this paper, we present a new method of feature extraction for the classification of normal and abnormal brain MRI images. The proposed scheme is based on well-known Legendre polynomials and two different classifiers i.e. Simple logistic and sequential minimal algorithms for training support vector machine as a classifier. The proposed approach achieved very efficient results in the classification of two different classes of brain images with high accuracy, specificity, sensitivity, and high positive predictivity rates. It can be assumed that the proposed system can be used in developing a computer-aided application for the classification of MRI images. The proposed method is limited by the fact that it can be only applied to a 2-dimensional slice of MR image of grayscale only. The proposed algorithm can be extended to the classification of the different color images.

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Conflicts of Interest: The authors declare that they have no conflicts of interest.

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