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NUMERICAL SOLUTION OF UNSTEADY TWO - DIMENSIONAL HYDROMAGNETICS FLOW WITH HEAT AND MASS TRANSFER OF CASSON FLUID

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Abstract

The present investigation deals with the oscillatory flow of a Casson fluid subjected to heat and mass transfer along a porous oscillating channel in presence of an external magnetic field. Here we consider the flow through a channel in which the fluid is injected on one boundary of the channel with a constant velocity, while it is sucked off at the other boundary with the same velocity. Galerkins technique is used to find expressions for the velocity, temperature, concentration of mass, volumetric flow rate, shear stress, rate of heat, and mass transfer and found their numerical solutions. The effects of various parameters like Hartmann number, radiative parameter, Reynolds number, permeability parameter, Schimdth number on flow variables are discussed and shown graphically.

Keywords : Oscillating channel, radiative heat transfer, mass transfer, volumetric flow rate, shear stress, Casson fluid.

Nomenclatures

(x^*,y^*,z^*)	Space Coordinates	B_0	Magnetic induction
t^*	Time	D	Mass diffusibility
p^*	Fluid pressure	k	Permeability factor
u^*	Axial velocity	T	Fluid temperature
V	Velocity of suction/injection	q	Radiative heat flux
ω	Angular frequency	M	Hartmann number
ρ	Density of the fluid	N	Radiation parameter
μ	Dynamic viscosity	R_e	Reynolds number
ν	Kinematic viscosity	G_r	Grashoff number
σ	Electrical conductivity of the fluid	P_r	Prandtl number
κ	Thermal conductivity	S_c	Schimdt number,
g	Gravitational acceleration	C_p	Specific heat at constant
β^*	Coefficient of volume expansion pressure		
β^{**}	Coefficient of volume expansion with concentration		

Note : $\beta = 1+\frac{1}{\alpha_1}$ where α_1 is the Casson parameter.

I. Introduction

Studies related to the oscillatory fluid flow are increasingly important in recent times due to its numerous applications in many real-life problems.

The pulsatile flow of a fluid in a porous channel has been investigated by Wang (1997). Kumar and Narayana(2010) analyzed the pulsatile flow and its role on particle removable from surfaces. Makinde and Aziz (2010) discussed MHD mixed convection from a vertical plate embedded on a porous medium with convective boundary conditions. Mandal et.al (2012) studied the Pulsatile flow of shear dependent fluid in a stenoises artery. Malik et.al (2013) presented the pulsatile flow of Casson fluid in mild stenoises artery with periodic body accelerated and slip condition. Bitta et.al (2013) investigated the pulsatile flow of an incompressible micropolar fluid between permeable beds. Manyonge et.al(2013) analyzed the steady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field. Kirubha et.al (2014) reported the exact solution of unsteady MHD flow through parallel plates. Abou-Zeid et.al (2014) presented mathematical modeling for the pulsatile flow of a non-newtonian fluid with heat and mass transfer in a porous medium between two permeable parallel plates. Lin et.al (2014) discussed the flow enhancement in the pulsating flow of non-colloidal suspension in tubs. Kiema et.al (2015) investigated steady MHD poiseuille flow between porous plates. Venkateshwarlu and Padma(2015) discussed unsteady MHD free convective heat and mass transfer in a boundary layer flow past a vertical permeable plate with thermal radiation and chemical reaction. Samuel et.al (2016) analyzed MHD oscillatory flow through a porous channel saturated with a porous medium. Srinivas et.al(2017) reported a pulsatile flow of a non-Newtonian nanofluid in a porous space with thermal radiation. Kumar and Srinivas (2017) studied simulation effects of thermal radiation and chemical reaction on the hydromagnetic pulsatile flow of a Casson fluid in a porous space. Dhal et.al(2017) analyzed unsteady MHD flow between two parallel plates with a slip flow region and uniform suction at one plate.

Narender et.al (2018) investigated the radiation effects on the magnetic stagnation point flow of nanofluid and found that heat transfer rises with radiation parameters. Ganesh et.al(2018) studied MHD flow between two parallel plates in presence of magnetic field vertical flow generated by the driven pressure gradient. ChandraSekhar et.al (2019) discussed the MHD flow between two porous plates. Amjad et.al(2020) reported the impact of Lorentz force on the pulsatile flow of a nonNewtonian Casson fluid in a constricted channel using Darcy law. Noushima et.al(2020) highlighted mass transfer on hydromagnetic free convective non-newtonian flow. Noushima and Rafiuddin (2020) analyzed MHD free convective non-Newtonian flow with variable permeability.

The present study aims to extend Adhikhari and Misra (2011) to Casson (1959) memory fluid.

II. Formulation of the Problem

Consider unsteady incompressible two dimensional Casson flow-through channel y =0 and y= h(figure I) in presence of an external magnetic field with radiative heat and mass transfer. The fluid is being injected by one plate with constant velocity V and sucked off by the other plate with the same velocity. Then the continuity equation reduces to

$$\frac{\partial u^*}{\partial t^*} = 0$$

So that u* is the function of y* and t* only.

The momentum equations are given by

$$\frac{\partial u^*}{\partial t^*} + V\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \beta\nu\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\nu}{k^*}u^* - \frac{\sigma B_0^2}{\rho}u^* + g\beta^*(T - T_0) + g\beta^{**}(c - c_0) \quad (1)$$

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} \quad (2)$$

While the energy equation is in the form

$$\frac{\partial T}{\partial t^*} + V\frac{\partial T}{\partial y^*} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^{*2}} - \frac{1}{\rho c_p}\frac{\partial q}{\partial y^*} \quad (3)$$

And also the concentration of mass equation reduces to

$$\frac{\partial C}{\partial t^*} + V\frac{\partial C}{\partial y^*} = D\frac{\partial^2 C}{\partial y^{*2}} \quad (4)$$

κ being the coefficient of thermal conductivity. The last term on the right-hand side of the eq.(3)arises owing to the radiation effect of the heat transfer

The corresponding boundary conditions are

$$u^* = U_0 e^{i\omega^* t^*}, T = T_w + e^{i\omega^* t^*}(T_w - T_0), C = C_w + e^{i\omega^* t^*}(C_w - C_0)$$

at y*=h (5)

$$u^* = U_0 e^{i\omega^* t^*}, T = T_0, \ C = C_0 \qquad \text{at } y^*=0 \quad (6)$$

In these equations, we have taken into account the temperature oscillation on the upper plate y*=h, while the lower plate y=0 is maintained at the fixed temperature T_0. For fluids like blood, the mean radiation absorption coefficient α <<1. In this case, the heat flux may be expressed as

$$\frac{\partial q}{\partial y^*} = 4\alpha^2(T - T_0) \quad (7)$$

Introduce the following non-dimensional variables

$$y = \frac{y^*}{h}, \quad x = \frac{x^*}{h}, \quad u = \frac{u^*}{V}, \quad K = \frac{k^*}{h^2}, \quad t = \frac{t^* V}{h}, \quad p = \frac{hp^*}{\rho\upsilon V}, \quad \omega = \frac{\omega^{*h}}{V},$$

$$\theta = \frac{T - T_0}{T_W - T_0}, \quad \emptyset = \frac{C - C_0}{C_w - C_0}, \quad R_e = \frac{Vh}{\nu}, \quad M^2 = \frac{\sigma h^2 B_0^2}{\rho\upsilon}, \quad P_r = \frac{Vh\rho c_p}{\kappa^`},$$

$$N^2 = \frac{4\alpha^2 h^2}{\kappa^`}, G_r = \frac{g\beta^*(T_w - T_0)h^2}{\upsilon V}, \quad G_c = \frac{g\beta^{**}(C_w - C_0)h^2}{\upsilon V} \quad (8)$$

Using dimensionless quantities from (8), governing equations together with the heat and mass transfer equation are re-written as

$$R_e\left\{\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}\right\}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-\left\{\frac{1}{K}+M^2\right\}u+G_r\theta+G_c\phi \quad (9)$$

$$0=-\frac{\partial p}{\partial y} \quad (10)$$

$$P_r\left\{\frac{\partial \theta}{\partial t}+\frac{\partial \theta}{\partial y}\right\}=\frac{\partial^2\theta}{\partial y^2}+N^2\theta \quad (11)$$

and

$$R_e\ S_c\{\frac{\partial \emptyset}{\partial t}+\frac{\partial \emptyset}{\partial y}\}=\frac{\partial^2 \emptyset}{\partial y^2} \quad (12)$$

While the boundary conditions will assume the form

$$u=U_0\,e^{i\omega t}\ ,\quad \theta=1+e^{i\omega t}\ ,\ \emptyset=1+e^{i\omega t}\qquad \text{at } y=1 \quad (13)$$

$$u=U_0\,e^{i\omega t}\ ,\ \theta=0\ ,\emptyset=0\qquad \text{at } y=0 \quad (14)$$

The definitions of different symbols used in the above equations are given in the list of nomenclatures.

III. The solution to the Problem

From (9) and (10), it follows that is a function of t alone. We consider

$$\frac{\partial p}{\partial x}=A+Be^{i\omega t} \quad (15)$$

A and B being undetermined constants. To solve the equations (9) and (11) subject to the boundary conditions (13)&(14), we write the velocity, temperature, and concentration in the form

$$u(y,t)=u_s(y)+u_p(y,t)$$
$$=u_s(y)+u_f(y)e^{i\omega t} \quad (16)$$

$$\theta(y,t)=\theta_s(y)+\theta_p(y,t)$$
$$=\theta_s(y)+\theta_f(y)e^{i\omega t} \quad (17)$$

$$\emptyset(y,t)=\emptyset_s(y)+\emptyset_p(y,t)$$
$$=\emptyset_s(y)+\emptyset_f(y)e^{i\omega t} \quad (18)$$

Where $u_s(y)$, $u_p(y,t)$, $\theta_s(y), \theta_p(y,t)$, $\emptyset_s(y)$, and $\emptyset_p(y,t)$ respectively represent the steady and unsteady parts of the velocity, temperature, and concentration. Substituting the above expressions in (9) and (11) and comparing the like terms, we have derived the equations that govern the corresponding steady and unsteady flow of heat and mass transfer of the problem under consideration. They are given below

Steady Case:

$$\beta\frac{\partial^2 u_s}{\partial y^2}-R_e\frac{\partial u_s}{\partial y}-\left\{\frac{1}{K}+M^2\right\}u_s=A-G_r\,\theta_s-G_c\phi_s \quad (19)$$

$$\frac{\partial^2\theta_s}{\partial y^2}-P_r\frac{\partial\theta_s}{\partial y}+N^2\theta_s=0 \quad (20)$$

and

$$\frac{\partial^2\phi_s}{\partial y^2}-R_e\,S_c\frac{\partial\phi_s}{\partial y}=0 \quad (21)$$

Along with the boundary conditions

$u_s = 0 \ , \ \theta_s = 1 \ , \ \phi_s = 1$ at y=h (22)

$u_s = 0 \ , \ \theta_s = 0 \ , \ \phi_s = 0$ at y=0 (23)

Unsteady Case:

$$\beta \frac{\partial^2 u_f}{\partial y^2} - R_e \frac{\partial u_f}{\partial y} - \left\{ R_e\, i\omega + \frac{1}{K} + M^2 \right\} u_f = B - G_r\, \theta_f - G_c \phi_f \tag{24}$$

$$\frac{\partial^2 \theta_f}{\partial y^2} - P_r \frac{\partial \theta_f}{\partial y} + \{N^2 - P_r i\omega\}\theta_f = 0 \tag{25}$$

And

$$\frac{\partial^2 \phi_f}{\partial y^2} - R_e\, S_c \frac{\partial \phi_f}{\partial y} - i\, R_e\, S_c \omega \phi_f = 0 \tag{26}$$

With the boundary conditions

$u_f = U_0, \quad \theta_f = 1 \ , \ \phi_f = 1$ at y=1 (27)

$u_f = U_0, \quad \theta_f = 0 \ , \ \phi_f = 0$ at y=0 (28)

Galerkins method of finite element yields the solution of (19), (20), and (21) subject to conditions (22) and (23), the steady components of the concentration of mass, temperature, and velocity are

$$\emptyset_s = m_1 y + m_2 y^2 \tag{29}$$

$$\theta_s = n_1 y + n_2 y^2 \tag{30}$$

$$u_s = \frac{1}{2(10\beta + m_1)}[10A - l_3 l_2 + 5 l_1][y^2 - y] \tag{31}$$

Similarly, Galerkins technique gives rise to the solution of (24), (25), and (26) subject to boundary conditions (27) and (28), the unsteady concentration of mass, temperature, and velocity are

$$\emptyset_f = y + m_3(y^2 - y) + i m_4 (y^2 - y) \tag{32}$$

$$\emptyset_p = \emptyset_f e^{i\omega t}$$

$$= [y + m_3\,(y^2 - y)]\cos \omega t - m_4\,(y^2 - y)\sin \omega t \tag{33}$$

$$\theta_f = y + n_3(y^2 - y) + i n_4 (y^2 - y) \tag{34}$$

$$\theta_p = \theta_f e^{i\omega t}$$

$$\theta_p = [y + n_3(y^2 - y)]\cos \omega t - n_4 (y^2 - y)\sin \omega t \tag{35}$$

$$u_f = u_0 + (l_7 + l_8)(y^2 - y) \tag{36}$$

$$u_p = u_f e^{i\omega t}$$

$$u_p = [u_0 + l_7(y^2 - y)]\cos \omega t - l_8 (y^2 - y)\sin \omega t \tag{37}$$

Where equation (33), (35), and (37) are extracted real parts of concentration of mass, axial temperature, and axial velocity respectively. The constants are not presented here for the sake of brevity.

In the case of oscillatory flow, the expression for the volumetric flow rate is

$$Q_p = \int_0^1 u_p \, dy$$

$$= \frac{1}{6}[l_8 \sin \omega t - l_7 cos\omega t] + u_0 \cos \omega t \quad (38)$$

The wall shear stress at the upper wall of the channel is

$$\tau_w = -\frac{\partial u_p}{\partial y}]_{y=1}$$

$$= l_8 \sin \omega t - l_7 \cos \omega t \quad (39)$$

heat transfer

$$N\theta_p = -\frac{\partial \theta_p}{\partial y}]_{y=1}$$

$$= (1 + n_3) \cos \omega t - n_4 sin\omega\omega t \quad (40)$$

and mass transfer

$$\text{Sh} = -\frac{\partial \phi_p}{\partial y}]_{y=1}$$

$$= m_4 \sin \omega t - (1 + m_3) \cos \omega t \quad (41)$$

IV. Results and Conclusion

In this section, we examined the nature of the variation of various physical quantities associated with the problem under consideration. For this purpose, we consider a particular case characterized by the following values of parameters involved in the analysis that has been presented in sections 2 and 3,

B=1.0, ω =1.0, G_r=1.0, 0≤P_r≤3.0, 1.0≤R_e≤8.0, 0.0≤U_0≤0.5, 0.0≤N≤3.2,

0.05≤K≤5.0,0≤ t ≤π.

We validate our results by comparing it with those reported by earlier researchers for Newtonian fluids. From figure 2, it is observed that if the Casson parameter α_1 is very large then Wang(1971) result is obtained and also Casson parameter α_1, reduces oscillatory velocity u_p. The system of equations that govern the Casson fluid between two parallel plates is solved numerically. The velocity, temperature, and concentration distributions are calculated for different values of R_e, M, N, U_0, K, S_c in figures 2-12.

The effects of physical parameters on the velocity distribution are shown in figures 2-6. In figure 2, we plotted oscillatory velocity up versus the coordinate y. And it is found that an increase in Reynolds number R_e suppresses oscillatory velocity u_p.

Figure 3 depicts the fluctuation of oscillatory velocity with radial distance at the instant of time for different values of N. As N increases, oscillatory velocity u_p increases up to N=3.

From figure 4, it is clear that the increase in Hartmann number M, the magnitude of the oscillatory velocity u_p decreases all the time. The permeability parameter K is diminishing the oscillatory velocity u_p, which can be seen from figure 5.

Figure 6 shows the effect of wall oscillation on the velocity. It is observed that as mean velocity U_0 increases, oscillatory velocity u_p also increases.

Figures 7-8 are oscillatory temperature distribution in the fluid. It is observed that the oscillatory temperature changes almost linearly with radiation parameter N and Prandtl number P_r.

The variation of volumetric flow rate Q_p with the radiation parameter N is shown in figure 9. Here volumetric flow rate Q_p increases with an increase in radiation parameter N.

Figure 10 depicts the wall shear stress $\tau_w$at the upper plate increases with the increase in radiation parameter N.

Variation of heat transfer $N\Theta_p$ is shown in figure 11. It is observed that as radiation parameter N increases heat transfer decreases.

The effect of Schmidth number S_c on mass transfer is shown in figure 12. It is clear that with the increase in Schmidth number S_c, mass transfer reduces.

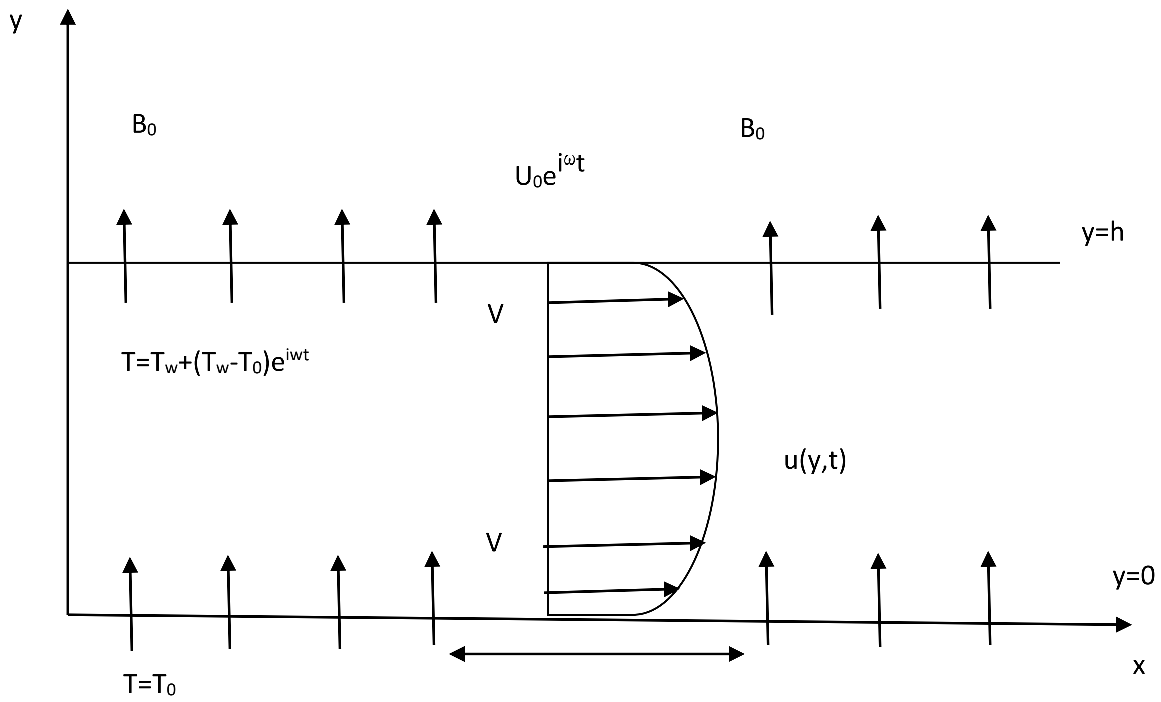

Fig. 1 : Fluid flow between two parallel plates oscillating in their own planes

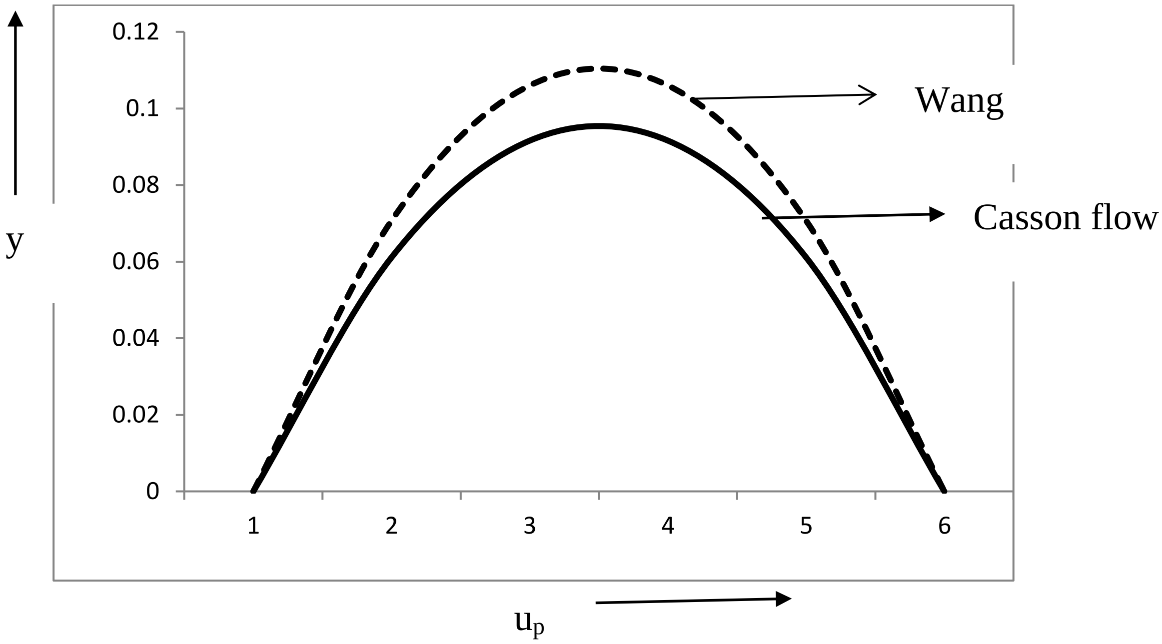

Fig. 2: Study of Wang and our flow

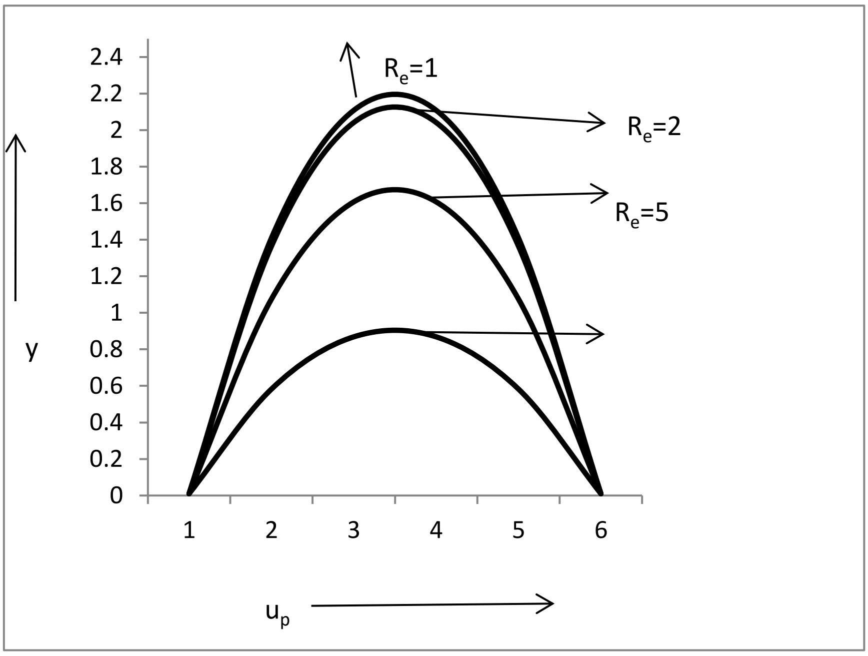

Fig. 3: Distribution of oscillatory flow velocity for different values of Re with M=1.0, U_0=0.01, N=1.0, P_r=0.71, K=0.05, G_r=1.0, G_c=1.0, S_c=0.1, ωt=π/4, α_1=0.5

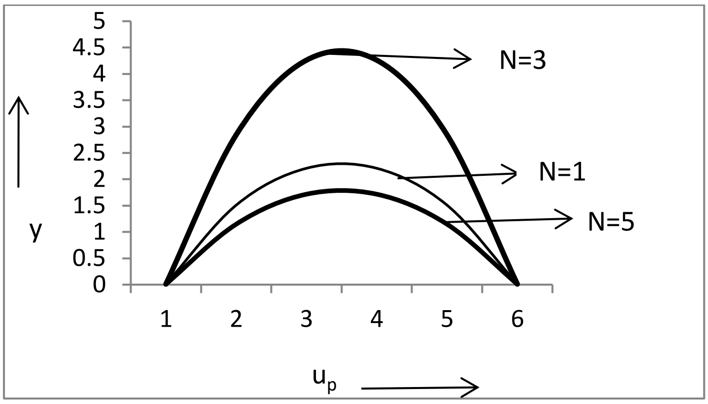

Fig. 4: Distribution of oscillatory flow velocity for different values of N with M=1.0,U_0=0.01,R_e=1.0,P_r=0.71,K=0.05,G_r=1.0,G_c=1.0,S_c=0.1, ωt=π/4, α_1=0.5

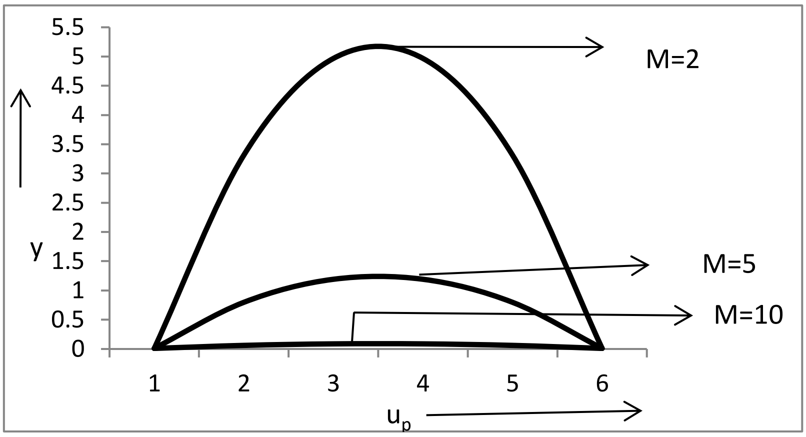

Fig. 5: Distribution of oscillatory flow velocity for different values of M with N=1.0, U_0=0.01, R_e=1.0, P_r=0.71, K=0.05, G_r=1.0, G_c=1.0, S_c=0.1, ωt=π/4, α_1=0.5

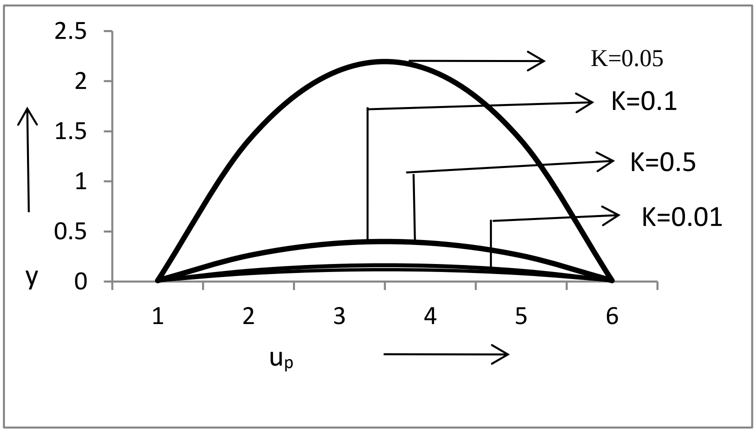

Fig. 6: Distribution of oscillatory flow velocity for different values of K with N=1.0, U_0=0.01, R_e=1.0, P_r=0.71, M=1.0, G_r=1.0, G_c=1.0, S_c=0.1, ωt=π/4, α_1=0.5

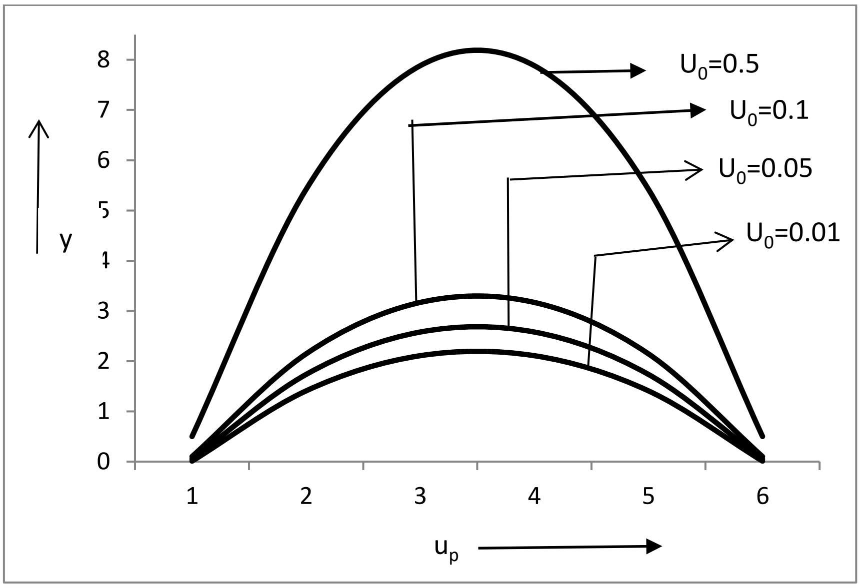

Fig. 7: Distribution of oscillatory flow velocity for different values of U_0 with $N=1.0$, $K=0.01$, $R_e=1.0$, $P_r=0.71$, $M=1.0$, $G_r=1.0$, $G_c=1.0$, $S_c=0.1$, $\omega t=\pi/4$, $\alpha_1=0.5$

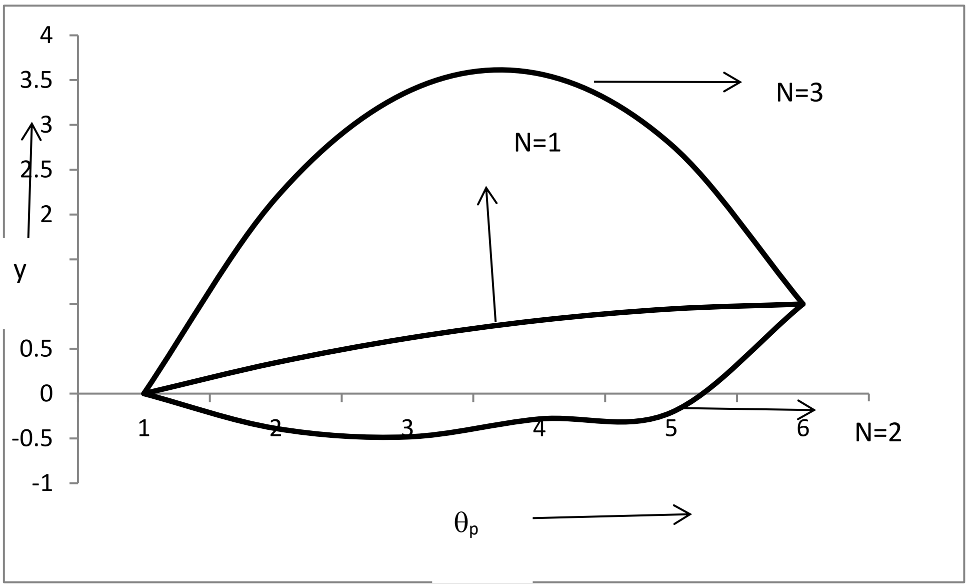

Fig. 8: Distribution of temperature for different values of N with $K=0.05$, $R_e=1.0$, $P_r=0.71$, $M=1.0$, $G_r=1.0$, $G_c=1.0$, $S_c=0.1$, $\omega t=\pi/4$, $\alpha_1=0.5$

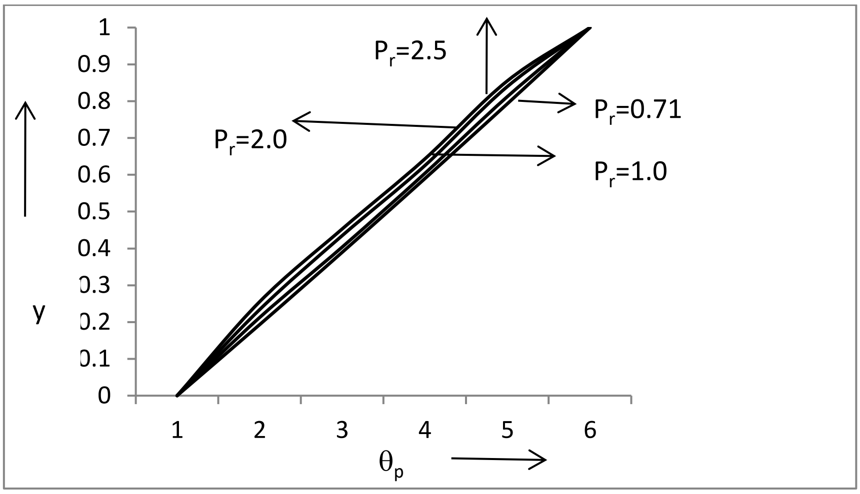

Fig. 9: Distribution of temperature for different values of P_r with K=0.05,R_e=1.0, P_r=0.71, M=1.0, N=1.0, G_r=1.0, G_c=1.0, S_c=0.1, ωt=π/4, α_1=0.5

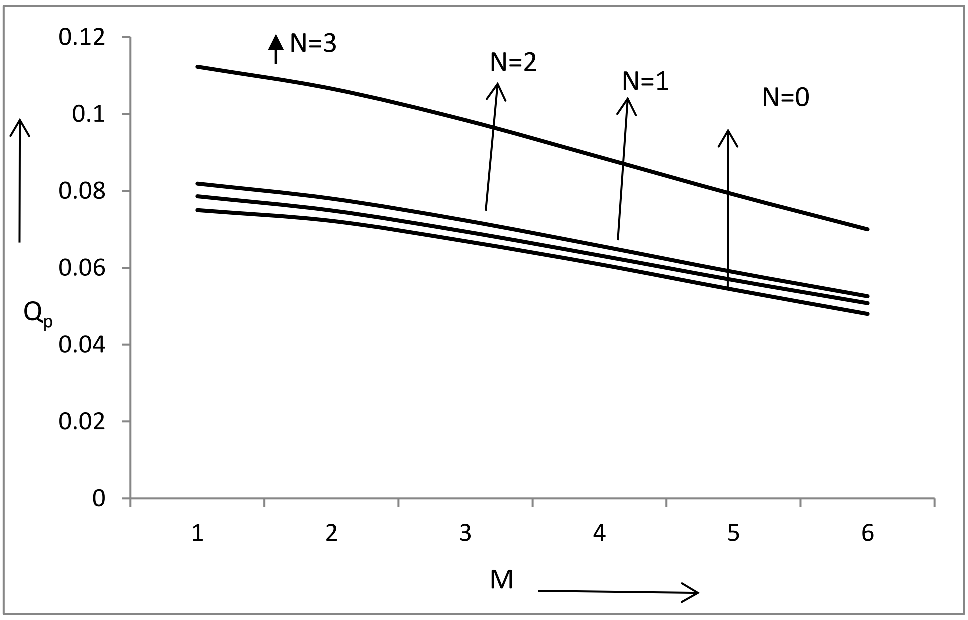

Fig. 10: Variation of Volumetric rate of flow with different values of N

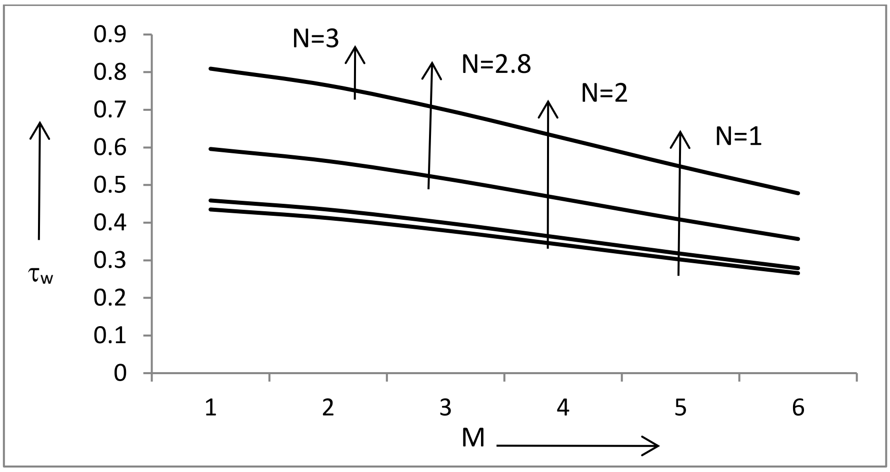

Fig. 11: Variation of wall shear stress τ_w with different values of N

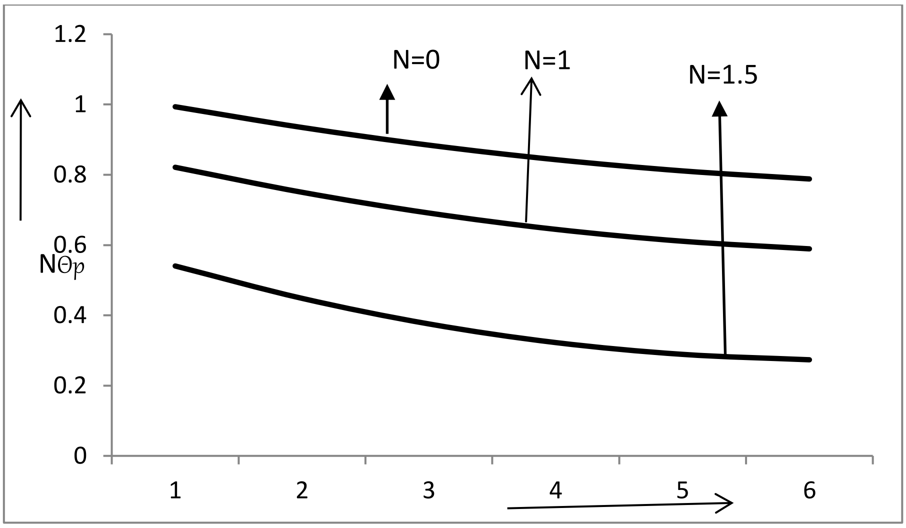

Fig. 12: Variation of heat transfer with different values of N

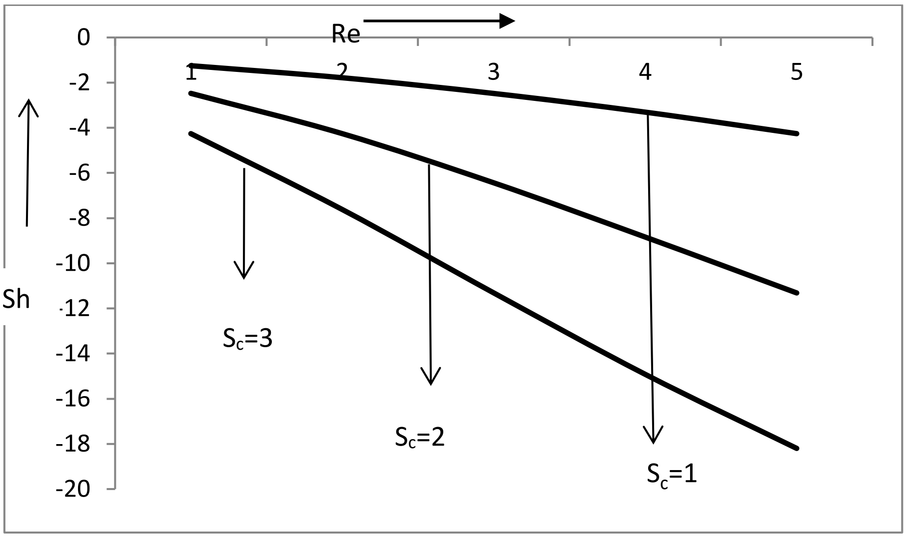

Fig. 13: Variation of mass distribution with different values of S_c

Conflict of Interest :

No conflict of interest regarding this article.

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