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Reference documents

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35 Sentences were found in a text with the title: „*Estimating the boundary condition in a 3D inverse ...*“<https://www.sciencedirect.com/science/article/pii/S0096300305009240>
article > **pii** > **S009630030500924021**“, located at:
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32 Sentences were found in a text with the title: „*Estimating the boundary condition in a 3D ... - DeepDyve*“<https://www.deepdyve.com/lp/elsevier/estimating-the-boundary-condition-in-a-3d-inverse-hyperbolic-heat-1pRq2MKqL0>
elsevier > **estimating...**“, located at:
<https://www.deepdyve.com/lp/elsevier/estimating-the-boundary-condition-in-a-3d-inverse-hyperbolic-heat-1pRq2MKqL0>

25 Sentences were found in a text with the title: „*An inverse non-Fourier heat conduction problem approach ...*“<https://www.sciencedirect.com/science/article/pii/S0307904X0300218X>
science > **article** > **pii** > **S0307904X0300218X17**“, located at:
<https://www.sciencedirect.com/science/article/pii/S0307904X0300218X>
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344939318 > **Hsu-2004**“, located at:
<https://www.scribd.com/document/344939318/Hsu-2004>

7 Sentences were found in a text with the title: „*10.1016/j.applthermaleng.2007.09.002 | DeepDyve*“, located at:
<https://www.deepdyve.com/lp/elsevier/the-inverse-estimation-of-the-thermal-behavior-and-the-viscosity-of-6NtdMyJJe7>

7 Sentences were found in a text with the title: „*10.1016/S1385-8947(01)00151-6 | DeepDyve*“<https://www.deepdyve.com/lp/elsevier/an-inverse-approach-for-estimation-of-the-surface-heat-flux-1hqQqF0OGi>
elsevier > **an-inverse...**“, located at:
<https://www.deepdyve.com/lp/elsevier/an-inverse-approach-for-estimation-of-the-surface-heat-flux-1hqQqF0OGi>

6 Sentences were found in a text with the title: „*An inverse problem approach for estimating the wall heat ...*“<https://www.sciencedirect.com/science/article/pii/S0307904X99000335>
science > **article** > **pii** > **S0307904X990003355**“, located at:
<https://www.sciencedirect.com/science/article/pii/S0307904X99000335>

6 Sentences were found in a text with the title: „*The inverse estimation of the thermal behavior and the ...*“<https://www.sciencedirect.com/science/article/pii/S1359431107002980>
science > **article** > **pii** > **S135943110700298013**“, located at:
<https://www.sciencedirect.com/science/article/pii/S1359431107002980>

4 Sentences were found in a text with the title: „*10.1016/S0017-9310(98)00128-8 | DeepDyve*“, located at:
<https://www.deepdyve.com/lp/elsevier/the-determination-of-two-heat-sources-in-an-inverse-heat-conduction-UtezFySuPV>

3 Sentences were found in a text with the title: „*An inverse approach for estimation of the surface heat ...*“<https://www.sciencedirect.com/science/article/pii/S1385894701001516>
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2 Sentences were found in a text with the title: „*Solving the two-dimensional inverse heat ... - ScienceDirect*“<https://www.sciencedirect.com/science/article/pii/S0017931097001257>
science > **article** > **pii** > **S001793109700125766**“, located at:
<https://www.sciencedirect.com/science/article/pii/S0017931097001257>

2 Sentences were found in a text with the title: „*A 2-D INVERSE METHOD FOR SIMULTANEOUS ESTIMATION OF ...16*“, located at:
<https://www.tandfonline.com/doi/abs/10.1080/10407789808914013>

2 Sentences were found in a text with the title: „*A three-dimensional inverse heat conduction problem ...*“, located at:
<http://iopscience.iop.org/article/10.1088/0022-3727/30/9/007>

2 Sentences were found in a text with the title: „*Simultaneously estimating the initial and boundary ...18*“, located at:
<https://www.sciencedirect.com/science/article/pii/S0017931097000951>

2 Sentences were found in a text with the title: „*許寬東 - 樹德科技大學PWS伺服器*“, located at:
[https://www.pws.stu.edu.tw/ptshu/researchplan\(airight\).htm](https://www.pws.stu.edu.tw/ptshu/researchplan(airight).htm)

2 Sentences were found in a text with the title: „*S07 silva.pdf*“, located at:
http://www.me.ua.edu/inverse/Sicipe/Proceedings/Vol3/S07_silva.pdf

2 Sentences were found in a text with the title: „*A Three-Dimensional Inverse Problem of Estimating the ...*“, located at:
<https://asmedigitalcollection.asme.org/manufacturingscience/article/122/1/76/462005/A-Three-Dimensional-Inverse-Problem-of-Estimating>

2 Sentences were found in a text with the title: „*10.1.1.417.8262.pdf - download*“, located at:
<http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.417.8262&rep=rep1&type=pdf>

2 Sentences were found in a text with the title: „*J6-silva_et_al_ipse_revised_pss - J6r17.pdf*“, located at:
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2 Sentences were found in a text with the title: „*The estimation of the strength of the heat source in the ...*“, located at:
<https://www.sciencedirect.com/science/article/abs/pii/S0307904X06002629>
<https://www.sciencedirect.com/science/article/pii/S0307904X06002629>

2 Sentences were found in a text with the title: „*An inverse problem in estimating the space-dependent ...*“, located at:
https://www.researchgate.net/publication/257545916_An_inverse_problem_in_estimating_the_space-dependent_thermal_conductivity_of_a_functionally_graded_hollow_cylinder

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Subsequent the examined text extract:

DETERMINATION AND FORECASTING OF THE HEAT SOURCE FOR HYPERBOLIC HEAT CONDUCTION PROBLEM BY AN INVERSE METHOD AND GM(1,1) THEORY

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An inverse method combined with a grey system theory for estimating and forecasting the strength of heat source for the Non-Fourier heat conduction problem is presented in this paper. The matrix form of differential governing equation is constructed by finite-difference method and then a linear inverse approach based on linear least-squares method is used to estimate the strength of heat source. By using the estimated values, the further values of heat source are forecasted by the grey model GM(1,1). Three examples are performed to demonstrate the proposed method. The results show that the proposed method can be utilized successfully to estimate and forecast the strength of heat source for the Non-Fourier conduction problem.

Keywords: Inverse Problem; GM(1,1) Model; Linear Least-Squares; Non-Fourier Heat Conduction

Nomenclature

C	the coefficients of
C_p	heat capacity
T	dimensionless temperature temperature
k	thermal conductivity
A	matrix of function of thermal properties
B	coefficient matrix of θ
R	reverse matrix of inverse problem
X, Z	data sequences
a, b	grey input coefficient
c	propagation speed of thermal wave
x, y	dimensionless Cartesian coordinates Cartesian coordinates
t	dimensionless time coordinate time coordinate

Greek symbols

τ	relaxation time
ρ	density
	thermal diffusivity
ψ	heat source
θ	vector of initial and boundary conditions and heat source
	the coefficient vector of
σ	measurement error
λ	number random
	non-singular functions

Subscripts

i, j, k indices

Introduction

The grey system theory was first proposed by Deng [1], the fundamental concept is that all known information in the natural world is white, the unknown is black, and the uncertain-known and uncertain-unknown regions between black and white are grey. The grey system is primarily used to extract fundamental system qualities when there is insufficient information. It stresses system information supplements, adequate use of known white information, and execution of relational analysis and model construction for the system, so that the system is changed from grey to white. The grey theory is method for analysing, predicting, and rationalizing a decision under insufficient information from a system without sufficiently comprehensive data. This theory is primarily aimed at performing relational analysis and model construction for when a system model's information is uncertain and insufficient. Also, Prediction, Evaluation, and Decision are used to investigate and understand the system.

The inverse source problem is the determination of the strength of the heat source from the temperature measured at a different point other than the source locations. It is practical in much design and manufacturing areas in which the strength of the heat source is undetermined. Common problems include the detection of the quantity of the energy generation in a computer chip, or in a microwave heating process, or in a chemical reaction process. The situations involving very low temperatures near absolute zero, very high temperature gradients, or extremely short times, heat is found to propagate at a finite speed. For these situations, the hyperbolic heat conduction (HHC) model is employed to account for the phenomena of finite heat propagation speed. Such non-Fourier conduction equations have been used successfully to examine rapid transient conduction processes in chemical process engineering [2], laser pulse heating [3], and laser annealing [4]. Guo and Xu [5] addressed the non-Fourier conduction in an IC chip with emphasis on the discrepancy from the Fourier conduction and its influence on thermal reliability. Many researchers have explored the HHC problem of electronic component cooling in the form of a 'direct problem'. However, when the heat flux or temperature at electronic components is very high or other special situations are concerned, direct measurement of surface conditions is impossible. Practically, 'inverse problems' are applied in these cases. Measurements of temperature are often made at one or more interior locations of the substrate. Then these measurements are curve-fitted to estimate physical quantities and surface conditions.

To date, there are only limited numbers of researches that work in the inverse source problem [9-11, 17, 21, 22]. Most analytical and numerical methods have only been used to deal with the Fourier's conduction problem. For the HHC problem, the previous studies [2-5] are restricted to the analysis of one-dimensional direct problems. Due to the complicated reflection and interaction of thermal waves, multi-dimensional hyperbolic heat conduction problems are much more difficult to solve than one-dimensional problems. To the authors' knowledge, the inverse source problem for dealing with multi-dimensional hyperbolic heat conduction problems has little studies [10, 11, 17, 23], and no literature to consider the strength of heat source about future time were happened. In order to complement this deficiency, at least in part, of the literature in this field, this study was intended to further recent studies of the methodologies [12, 13, 24] combined with Dynamics Grey system theory DGM (1,1) model for estimating and forecasting the strength of heat source about Non-Fourier heat conduction problem. The present approach is to rearrange the matrix forms of differential governing equation and estimated coefficients of unknown condition. Then, the linear least-squares method is adopted to find the solution. The estimating value used to forecasted the further data of heat source by dynamic grey system theory DGM (1,1) model.

Analysis

Description of the proposed model

A hyperbolic heat conduction equation with heat source, constant thermal properties, and dimensionless in the rectangular coordinate system can be presented as following [19, 20]:

$$(1)$$

For convenience of numerical analysis, the dimensionless parameters can be introduced as following:

$$(2)$$

where, $\alpha = k/(\rho C_p)$ is the thermal diffusivity, $c = (\alpha/\tau)^{0.5}$ is the propagation speed of the thermal wave, is the initial temperature, and is the ambient temperature. Therefore, the dimensionless form of Eq. (1) with the dimensionless variables of Eq. (2) can be written as following:

(3)

The initial and boundary conditions of Eq. (3) are given as following:

, (4)

... (5)

In Eq. (3), $\psi(x_s, y_s, t)$ is the point heat source, where, x_s and y_s are location of a specified points. This inverse problem is to identify the unknown heat source $\psi(x_s, y_s, t)$ from the temperature measurements taken at interior points of the board.

The direct solution

The finite-difference method is employed to discrete the above governing equation combined with the boundary conditions, and initial conditions can be expressed in the following form [19]:

(6)

where,

$\Delta x, \Delta y$ are the increments of the spatial coordinates, $\Delta \tau$ is the increment of the time domain, i is the i th grid along the x -coordinate, j is the j th grid along the y -coordinate, k is the k th grid along the time coordinate, T_{ijk} is the dimensionless temperature calculated at the grid point (i, j, k) , and is the dimensionless heat source at the grid point (i, j, k) .

Using the recursive form, a matrix equation can be expressed as following:

(7)

where the matrix A is the function of thermal properties, and the scale of the position and time; the components of T are the dimensionless temperatures in discretized points; the components of Θ are the functions of the boundary, initial conditions and heat source, which M is the total number of nodes along the x -coordinates, N is the total number of nodes along the y -coordinates, L is the total number of nodes along the time-coordinates.

The direct problem presented here concerns the determination of the dimensionless temperatures at the nodes when all boundary conditions, initial conditions, and other thermal properties are known. The direct problem of Eq. (7) can then be solved by the Gaussian elimination method, the run method or other methods. In this paper, the Gaussian elimination method was used.

The inverse problem

This inverse problem is to identify the heat source $\psi(x, y, t)$ from the temperature measurements taken at interior points of the board. The heat source $\psi(x, y, t)$ is represented in the following series form for the problem domain [7, 8]:

(8)

where, a_i are non-singular functions and C_i are the coefficients of.

For an inverse problem, A can be constructed according to the known physical model and numerical methods, and T can be measured by thermocouples. The coefficients of $\psi(x_s, y_s, t)$ need to be determined. The coefficients of $\psi(x_s, y_s, t)$ from Θ will transfer the direct formulation to the following inverse form:

(9)

where, B is the coefficient matrix of, and is the coefficient vector of. can be solved by using the least-squares error method as follows:

(10)

where, is the reverse matrix of the inverse problems.

Eq. (10) is assumed to represent the measurements of all discretized points in the problem. In most cases, not all of the points are needed to be measured. Therefore, only part of matrix B and T , and part of vector Θ correspond to the measuring positions need to be constructed. In general, when a large portion of the matrices and vector is selected, the number of transducers or measuring points is large. Therefore, the cost of computation and experiment increases. However, a larger number of measuring points yield more accurate estimated results.

In Eq. (10), the inverse problem is solved by using the linear least-squares error method so that the problem is solved in a linear domain to avoid the iterative process. When the rank of the reverse matrix is equal to the number of the unknown variables, then: 1) If the matrix of Eq. (10) is consistent, a solution exists and is unique; 2) If the matrix equation is inconsistent, a unique least-squares solution can be approximated.

Grey forecast

The GM (1,1) is a basic model of the Grey system theory which is widely applied to many fields with high accuracy of forecasting[1]. The Grey forecasts are done by using GM (1,1) models as a foundation on

existing data. In actuality, it is meant to find out the future dynamic situation of various elements within a certain number series. First, we need to perform Accumulated Generating Operation (AGO) for system information to serve as internal information for model construction and weaken the randomness of the original number series. If the original series is non-negative, then it would exhibit an ascending law after the generating operation (grey exponential law). The generation function established by AGO is the foundation for system model construction and forecast. Therefore, the grey system theory stipulates that for all systems that can be broadly defined as energy systems and are compatible with exponential law calculations, their generation functions can be replaced by the following formula:

$$(11)$$

where, X_0 is the initial sequence, a is the developing coefficient, b is the grey input coefficient, and Z_1 is the background value sequence.

The Z_1 can be calculated as following:

$$, k=2, 3, \dots, n \quad (12)$$

X_1 is the first AGO (1-AGO), that can be calculated as following:

$$(13)$$

If the original data series is $X^{(0)} = (X^{(0)}(1), X^{(0)}(2), \dots, X^{(0)}(n))$ then using AGO calculations can turn the $X^{(0)}$ series into a new series, $X^{(1)}$, as shown below:

$$(14)$$

For the newly acquired series, $x^{(1)}$, the differential formula for the GM (1,1) model construction is:

$$(15)$$

$$(16)$$

The a, b coefficients can be derived through the method of least squares, as shown in the formula below:

$$(17)$$

where, the D matrix is:

$$(18)$$

and

$$(19)$$

By introducing vector into the differential formula, the derived formula is thus:

$$(20)$$

Because, therefore:

$$(21)$$

In order to reduce flaw and enhance precision, the dynamic grey model DGM(1,1) was presented to improve the prediction precision of GM (1,1). In the DGM(1,1), the next data sequence is constructed by eliminating the first data and adding the new forecasted data in the original series. Thus, the DGM(1,1) are founded on the belief of newer information being superior to older information to dynamically adjust changes in the development trends of the series. Eventually, DGM(1,1) can be set up as following [25]:

New data series is:

$$X^{(0)} = (X^{(0)}(2), \dots, X^{(0)}(n+1)) \quad (22)$$

and new AGO series is

$$(23)$$

Finally, we get new model, called DGM(1,1), as following:

$$(24)$$

Results and Discussion

The inverse problem defined by Eqs. (1)-(5) is used in the following examples to verify the accuracy, efficiency, and versatility of the proposed method for estimating the strength of the heat source $\mu(x, y, t)$. Each of the dimensionless spatial intervals $0.0 \leq x \leq 1.0$ and $0.0 \leq y \leq 1.0$ and they are divided into 8 intervals for all examples of the direct problem. Twenty steps of dimensionless time are taken. The iteration step corresponds to a mesh size of $\Delta x = 0.125$ and $\Delta y = 0.125$, and $\Delta t = 0.1$.

The exact dimensionless temperature and the heat source $\mu(x, y, t)$ used in the following examples are preselected so that these functions satisfy Eqs. (1)-(5). The accuracy of the proposed method is assessed by comparing the estimated results with the preselected values of the heat source $\mu(x, y, t)$. Meanwhile, the temperature distributions at some specified positions where the thermocouples are assumed to be installed are generated from the preselected exact dimensionless temperature. In a real engineering application, the

temperature measurements are always subject to a certain degree of measurement error. Hence, the simulated temperature value used in the inverse method, is given by:

$$(25)$$

Where λ is the exact temperature calculated from the direct problem, λ_m is the measured temperature at the grid points, λ_e is temperature measurement error, and λ_r is the random error. The value of λ_e is calculated by the IMSL subroutine DRNNOR [26] and lies within the 99% confidence range. The measurement errors are caused by the interpolation of the measuring instruments, uncertainty due to calibration, and fluctuation in sensor reading during measurement.

In the simulation problem, the accuracy of the estimation of unknowns from the knowledge of the temperature at measurement points depends strongly on the accuracy of the measurements. As will be seen in the examples, the estimated solutions without measurement error () converge to the solutions solved by the finite-difference method for all examples. Furthermore, the solutions are unique through the proposed verifying method. Using the above inverse method to have the estimating value, then taking this value to forecast the further date about the strength of the heat source. Detailed descriptions for the problem are presented as follows:

Example 1:

The unknown heat source $\psi(x_s, y_s, t)$ is taken at $x_s = y_s = 0.375$ as following:

$$(26)$$

The estimated value using two measurement points of $(0.25, 0.25)$ and $(0.5, 0.5)$ without considering measurement errors are presented in Fig. 1. The inverse values from the present study are a little difference with the direct solutions. In the Fourier heat conduction problem without measurement errors that has a better estimated values. Due to the complicated reflection and interaction of the thermal waves, it is difference from the inverse Fourier heat conduction problem.

Fig. 1 also shows a comparison of the estimated heat source $\psi(x_s, y_s, t)$ for measurement errors of 0.1%, 1.0%, and 10.0% with two measurement points (of positions located at $(x, y) = (0.25, 0.25)$ and $(0.5, 0.5)$). The result shows that the magnitudes of the discrepancies in heat source $\psi(x_s, y_s, t)$ are directly proportional to the size of measurement errors. The average discrepancies between the estimated quality of $\psi(x_s, y_s, t)$ and the exact value are 3.54%, 8.09% and 10.97% for the cases of $\epsilon = 0.1\%$, 1.0% and 10.0% , respectively.

Table 1 and Fig. 2 shows a comparison of the estimated and forecasted heat source $\psi(x_s, y_s, t)$ for measurement errors of 10.0% with two measurement points (of positions located at $(x, y) = (0.25, 0.25)$ and $(0.5, 0.5)$). In Table 1, the DGM (1,1) forecasting values which mainly uses four estimated data (0.887, 1.506, 0.997, and 1.023) entries in the same series (usually 4 points) to construct a GM (1,1) model, and then predict the forecasting value of the next data (0.74) entry, then abandon the first data (0.887) entry from the original series, add a fifth data (0.886) entry to construct a second series, forecast the value of the sixth data (0.87) entry, etc., until the last data entry in the original data. The result shows that the forecasting value (0.18) of the future time ($t=2.1$) can be obtained. The advantage of applying this method in inverse analysis is that we can forecast the further unknown output data, then to predict or to control generating equipment reliability, the device failure occurs can be avoided. The average discrepancies between the estimated quality, forecasted strength of $\psi(x_s, y_s, t)$ and the exact value are 10.97%, and 17.68% for the error = 10.0%, respectively.

Example 2:

The second example is the same as the first example, except that the unknown heat source $\psi(x_s, y_s, t)$ are embedded at $x_s = y_s = 0.5$ in different forms as following:

$$(27)$$

Fig. 3 shows a comparison of the estimated quality of $\psi(x_s, y_s, t)$ for three cases of measurement errors, i.e. = 0.1%, 1.0% and 5.0% with one measurement point $(x, y) = (0.375, 0.375)$ and dimensionless time interval of $t = 0.01 \sim 2.0$ in this example. It was found that the oscillation for heat source $\psi(x_s, y_s, t)$ response became more severe as the measurement error increases. The average discrepancies between the estimated quality of $\psi(x_s, y_s, t)$ and the exact value are 0.011%, 9.56% and 32.76% for the cases of $\epsilon = 0.1\%$, 1.0% and 5.0% , respectively.

Table 2 and Fig. 4 shows a comparison of the estimated and forecasted heat source $\psi(x_s, y_s, t)$ for measurement errors of 10.0% with $N=3$; the measurement points are located at $(x, y) = (0.375, 0.375)$, $(0.75, 0.25)$ and $(0.25, 0.75)$. In Table 2, the DGM (1,1) forecasting values which mainly uses four estimated data (0.887, 1.506, 0.997, and 1.023) entries in the same series (usually 4 points) to construct a GM (1,1) model, and then predict the forecasting value of the next data (0.74) entry, then abandon the first data (0.887) entry from the original series, add a fifth data (0.886) entry to construct a second series, forecast the value of the sixth data (0.87) entry, etc., until the last data entry in the original data. The result shows that the forecasting value (0.18) of the future time ($t=2.1$) can be obtained. The average discrepancies between the estimated quality, forecasted strength of $\psi(x_s, y_s, t)$ and the exact value are 10.97%, and 17.68% for the error = 10.0%, respectively.

Example 3:

The third example is the same as the first example, except that heat source $\psi(x_s, y_s, t)$ are expressed in the following form:

$$(28)$$

Fig. 5 shows a comparison of the estimated quality of $\psi(x_s, y_s, t)$ for three cases of measurement errors, i.e. = 0.1%, 0.5% and 1.0% with one measurement point $((x, y) = (0.375, 0.375))$ and dimensionless time interval of $t=0.01 \sim 2.0$ in this example. It was found that the oscillation for heat source $\psi(x_s, y_s, t)$ response became more severe as the measurement error increases. The average discrepancies between the estimated quality of $\psi(x_s, y_s, t)$ and the exact value are 2.13%, 11.43% and 19.76% for the cases of = 0.1%, 0.5% and 1.0%, respectively.

Table 2 and Fig. 4 shows a comparison of the estimated and forecasted heat source $\psi(x_s, y_s, t)$ for measurement errors of 0.5% with $N=3$; the measurement points are located at $(x, y) = (0.375, 0.375)$, $(0.75, 0.25)$ and $(0.25, 0.75)$. In Table 2, the DGM (1,1) forecasting values which mainly uses four estimated data (0.5, 0.695, 0.729, and 0.946) entries in the same series (usually 4 points) to construct a GM (1,1) model, and then predict the forecasting value of the next data (1.09) entry, then abandon the first data (0.5) entry from the original series, add a fifth data (0.967) entry to construct a second series, forecast the value of the sixth data (1.13) entry, etc., until the last data entry in the original data. The result shows that the forecasting value (1.12) of the future time ($t=2.1$) can be obtained. The average discrepancies between the estimated quality, forecasted strength of $\psi(x_s, y_s, t)$ and the exact value are 5.59%, and 17.46% for the error = 0.5%, respectively.

In a practical engineering application, as the number of transducers or measuring points becomes larger, the cost of computation and experiment increases. However, a larger number of measuring points yield more accurate estimated results.

The direct problems are mathematically classified as well-posed, that is, their solutions satisfy the requirements of existence, uniqueness and stability with respect to the input data [22, 23]. In general, inverse problems are mathematically classified as ill-posed, that is, their solutions may not satisfy the requirements of existence, uniqueness and stability under small perturbations in the input data [22, 23]. In the first example, the estimated values using two measurement point ($N=2$) and excluding measurement errors result in a very good approximation regardless of the measured positions. When the measurement errors are concerned, the magnitude of the discrepancies in the heat source $\psi(x_s, y_s, t)$ is directly proportional to the size of measurement error. In the second example, due to the complicated reflection and interaction of thermal waves, the numerical solutions exhibit severe oscillations in the vicinity of the jump discontinuity. This phenomenon is known as the Gibbs phenomenon, which is of practical importance since it causes difficulty to obtain a convergent solution in the neighbourhood of the jump discontinuity [7]. The phenomenon reflects the fact that the inverse non-Fourier heat conduction problem is different from the inverse Fourier heat conduction problem.

One shortcoming of this method is that it is needed to solve a system of simultaneous equations. Besides, the inverse matrix \mathbf{R} [Eq. (8)] is very sensitive to temperature measurement error, which renders it ill-conditioned and unstable. The sensitivity depends on the type of problem being solved (i.e., the governing equation and its boundary conditions), the position at which the temperature is measured and the measurement error. The present results confirm that the inverse values are extremely sensitive to the measurement error, sensor locations, the number of sensors and the range of measurement time interval, as mentioned by Beck et al. [16] and Hensel [21]. The advantage of applying this method in inverse analysis is that we can forecast the further unknown output data, then to predict or to control generating equipment reliability, the device failure occurs can be avoided.

Conclusion

An efficient algorithm has been introduced for estimating the strength of heat source $\psi(x, y, t)$ in the inverse non-Fourier heat conduction problem. A direct inverse formulation is constructed using the reverse matrix, which is derived from the governing equations as well as initial and boundary conditions. Three examples have been examined to show the robustness of the proposed method. The results of example 1 show that the exact solution can be found with only two points while the measurement error is neglected. When the measurement error is concerned, the magnitudes of the discrepancies for the heat source $\psi(x_s, y_s, t)$ are directly proportional to the size of the measurement error. This phenomenon is due to the complicated reflection and interaction of thermal waves. A special feature of this method is that the value of the unknown heat source $\psi(x_s, y_s, t)$ can be estimated directly by solving the inverse problem in a linear domain with only one calculation process. This implies that the present model offers a greater deal of flexibility. The advantage of applying GM(1,1) model in inverse analysis is that we can forecast the further unknown output data, then to predict or to control generating equipment reliability, the device failure occurs can be avoided. After all, the results confirm that the proposed method is effective for other kinds of inverse problems such as heat flux estimation, boundary estimation, and initial estimation in the one- or multi-dimensional transient non-Fourier heat conduction problems.

Conflicts of Interest

The authors declare no conflict of interest.

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Table captions

Table 1. Estimates and Forecasts of the strength about heat source for example 1 with 10% measurement errors

Table 2: Estimates and Forecasts of the strength about heat source for example 2 with 10% measurement errors

Table 3: Estimates and Forecasts of the strength about heat source for example 3 with 0.5% measurement errors

Table 1. Estimates and Forecasts of the strength about heat source for example 1 with 10% measurement errors

Time(t)	$\Psi(x, y, t)$		
	Exact	Estimate	Forecasting
0.1	1.0	0.887	
0.2	1.0	1.506	
0.3	1.0	0.997	
0.4	1.0	1.023	
0.5	1.0	0.886	0.74
0.6	1.0	0.841	0.87
0.7	1.0	1.272	0.75
0.8	1.0	1.342	1.48
0.9	1.0	1.032	1.71
1.0	1.0	0.963	1.0
1.1	0.0	-0.056	0.77
1.2	0.0	0.113	0.18
1.3	0.0	0.007	0.0
1.4	0.0	-0.068	0.04
1.5	0.0	0.014	0.0
1.6	0.0	-0.053	-0.03
1.7	0.0	-0.055	0.03
1.8	0.0	-0.0004	-0.17
1.9	0.0	0.055	-0.04
2.0	0.0	0.068	0.0
2.1			0.18

Table 2: Estimates and Forecasts of the strength about heat source for example 2 with 10% measurement errors

Time(t)	$\Psi(x, y, t)$		
	Exact	Estimate	Forecasting

0.1	0.1	0.165	
0.2	0.2	0.314	
0.3	0.3	0.175	
0.4	0.4	0.472	
0.5	0.5	0.611	0.56
0.6	0.6	0.787	1.02
0.7	0.7	0.806	1.0
0.8	0.8	0.903	0.93
0.9	0.9	0.933	0.96
1.0	1.0	0.961	1.01
1.1	0.9	0.932	0.99
1.2	0.8	0.935	0.80
1.3	0.7	0.838	0.75
1.4	0.6	0.616	0.71
1.5	0.5	0.571	0.53
1.6	0.4	0.429	0.44
1.7	0.3	0.364	0.38
1.8	0.2	0.225	0.28
1.9	0.1	0.01	0.18
2.0	0.0	0.002	0.05
2.1			0.00

Table 3: Estimates and Forecasts of the strength about heat source for example 3 with 0.5% measurement errors

Time(t)	$\Psi(x, y, t)$		
	Exact	Estimate	Forecasting
0.1	0.5	0.5	
0.1	0.691	0.695	
0.2	0.854	0.729	

0.3	0.962	0.946	
0.4	1.0	0.967	1.09
0.5	0.962	1.023	1.13
0.6	0.854	0.756	1.03
0.7	0.691	0.642	0.81
0.8	0.5	0.513	0.48
0.9	0.309	0.279	0.44
1.0	0.146	0.148	0.22
1.1	0.038	0.003	0.04
1.2	0.0	0.0	0.02
1.3	0.038	0.004	0.01
1.4	0.146	0.159	0.01
1.5	0.309	0.316	0.29
1.6	0.5	0.498	0.51
1.7	0.691	0.682	0.81
1.8	0.854	0.813	0.97
1.9	0.962	0.963	1.03
2.0	1.0	1.021	1.14
2.1			1.12

Figure captions

Figure 1. The estimation of the strength of heat source for example 1 with measurement error and used two measurement points.

Figure 2. The estimation and the forecasting of the strength of heat source for example 1 with measurement error 10%.

Figure 3. The estimation of the strength of heat source for example 2 with measurement error and used one measurement point.

Figure 4. The estimation and the forecasting of the strength of heat source for example 2 with measurement error 10%.

Figure 5. The estimation of the strength of heat source for example 3 with measurement error and used one measurement point.

Figure 6. The estimation and the forecasting of the strength of heat source for example 3 with measurement error 0.5%.

