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# CERTAIN EQUATION OF CENTRALIZERS ON SEMIPRIME INVERSE SEMIRING

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## Abstract

Let $S$ *will represent a semiprime inverse semiring with* $k$*-torsion free and has identity element, where* $k \in \{pq(p+q+2), p+q\}$. *An additive mapping* $G$ *from* $S$ *in to itself satisfying* $G(x^{p+q+1}) + x^p G'(x) x^q = 0$ *fulfilled for all* $x \in S$ *and* $p, q \geq 1$ *be distinct integers, forces* $G$ *to be centralizer. Moreover one more result involving centralizer has also been discussed.*

**Keywords :** Semiprime Semiring, Inverse Semiring, Commutator, Centralizer, Left (right) Centralizer.

## I. Introduction

$(S, +, .)$ Represents a semiring with '+' and '.' are binary operations such that $(S, +)$ and $(S, .)$ are semigroups with additively commutative and absorbing zero (see [II]) (i.e., $u + 0 = u = 0 + u$, $u.0 = 0.u = 0$ for all $u \in S$) and multiplication is distributes over addition. The concept of additive inverse semiring was introduced by Karvellas in [V] if for every $u \in S$ incase a unique $u' \in S$ exists, such that $u + u' + u = u$ and $u' + u + u' = u'$, where $u'$ is said to be the pseudo inverse of $u$. Also proved that for all $u, v \in S, (uv)' = u'v = uv'$, $(u')' = u$ and $u'v' = uv$. Inverse semiring $S$ satisfying the $A_2$ condition of [I] then for all $u \in S$, $u + u'$ lies in center $Z(S)$. This classical result of semiring is studied as MA semiring [IV]. The examples of MA semirings are Commutative inverse semirings and distributive lattices. We refer [IV, VII, VIII] to study more examples and fundamental concepts concerning inverse semirings. For any $u, v \in S$, the notation $[u, v] = uv + v'u = vu + vu'$ denotes the commutator of $u$ and $v$. Recall that $S$ is prime if u$Sv = 0$ implies $u = 0$ or $v = 0$ for all $u, v \in S$, and semiprime if $uSu = 0$ with $u \in S$ implies $u = 0$. $S$ is said to be $n$ -torsion free if $nu = 0$

with $u \in S$ implies $u = 0$. Following [X] $G$ is an additive mapping of $S$ in to itself is said to be a left centralizer in case $G(xy) = G(x)y$. Moreover if $G$ from $S$ in to itself is a left centralizer with the identity element of $S$ if and only if $G$ represented in the form $G(x) = cx$, this being true for all $c \in S$ where $c \in S$ is a fixed element. It is easy to define the right centralizer. If $G$ is both left and right centralizer of $S$ then $G$ is called centralizer of $S$. The concepts of commutator's are useful in developing the theory of centralizers. The study of various sorts of centralizers and derivations in inverse semirings are not well developed to compare to rings. In the last few years the study of centralizers has been done in prime and semiprime rings, $C^*$algebras and $H^*$algebras. In this connection we study centralizers of inverse semirings and using simple conditions under which additive mappings are centralizers. In [IX, X] shows the results concerning centralizers in semiprime rings. The first result of this paper is motivated by the classical results inspired by Irena [III] and based on some observations we prove the following: Let $S$ be an inverse semiring with suitable torsion restrictions, and let $G$ be an additive mapping of $S$ in to itself such that $G(x^{p+q+1}) + x^p G'(x) x^q = 0$ fulfilled for all $x \in S$, $p$ and $q$ are distinct integers. Then $G$ is a centralizer. The second aim of this paper is to prove the conjecture of Koshi-ulbal and J. Vukman [VI] in the settings of inverse semiring.

## II. Preliminaries

**Lemma 2.1** If $S$ represent an inverse semiring if for $b, c \in S$, $b + c = 0$ implies $b = c'$

Lemma 2.1 is proved in [VII].

**Lemma 2.2** $S$ will represent a semiprime inverse semiring with 2-torsion free and $G$ be an additive mapping of $S$ into itself, satisfying for all $x \in S, G(x^2) + G(x)x' = 0$, forces $G$ to be left centralizer.

The proof of above lemma is shown in [VII].

**Lemma 2.3** In any MA-Semiring the following identities $[uv, w] = u[v, w] + [u, w]v$ and $[u, vw] = v[u, w] + [u, v]w$ are valid.

The above identities are proved in [IV].

## III. Our Results

**Theorem 3.1**Let $S$ will represent a semiprime inverse semiring with $k$-torsion free and has identity element, where $k \in \{pq(p + q + 2), p + q\}$. An additive mapping $G$ of $S$ in to itself satisfies $G(x^{p+q+1}) + x^p G'(x) x^q = 0$ fulfilled for all $x \in S$ and distinct integers $p, q \geq 1$ is a centralizer.

**Proof**

We have

$$G(x^{p+q+1}) + x^p G'(x) x^q = 0 \qquad (1)$$

Putting $x$ by $x + te$ in (1), $t$ be fixed in $S$ and $e \in S$ be the identity in $S$, we get

$$G\left(\sum_{i=0}^{p+q+1}\binom{p+q+1}{i}x^{p+q+1+i'}(te)^i\right)$$
$$+\sum_{i=0}^{p}\binom{p}{i}x^{p+i'}(te)^i\big(G'(x)+tG'(e)\big)\sum_{i=0}^{q}\binom{q}{i}x^{q+i'}(te)^i=0$$

We have therefore

$$\sum_{i=0}^{p+q+1}\binom{p+q+1}{i}G\big(x^{p+q+1+i'}\big)(te)^i$$
$$+\sum_{i=0}^{p}\binom{p}{i}x^{p+i'}(te)^i(G'(x)+t'c)\sum_{i=0}^{q}\binom{q}{i}x^{q+i'}(te)^i=0 \tag{2}$$

Where $G(e)$ is denoted by $c$. Collecting the coefficients of $t^{p+q}$ in the above expression, we get

$$\binom{p+q+1}{p+q}G(x)+\binom{p}{p+1'}xc'+c'\binom{q}{q+1'}x+\binom{p}{p}\binom{q}{q}G'(x)=0$$

This implies, $(p+q+1)G(x)+pxc'+qc'x+G'(x)=0$

$$(p+q)G(x)+pxc'+qc'x=0 \tag{3}$$

Therefore we get, $(p+q)G(x^2)+px^2c'+qc'x^2=0$ (4)

Now collecting the coefficient of $t^{p+q+1'}$ from (2), we get

$$\frac{(p+q)(p+q+1)}{2}G(x^2)+G'(x)qx+pxG'(x)+pxc'qx+\frac{(p^2+p')}{2}x^2c'+$$
$$c'\frac{(q^2+q')}{2}x^2=0$$

Applying lemma 2.1 in (4) and using it in the above, we get

$(px^2c+qcx^2)(p+q+1)+2G'(x)qx+2pxG'(x)+2pxc'qx+(p^2+p')x^2c'+(q^2+q')c'x^2=0, x\in S$

$2qcx^2+2px^2c+px^2cq+qpx^2c+2G'(x)qx+2pxG'(x)+2qxc'qx=0$

$2q(cx^2+G'(x)x)+2p\big(x^2c+xG'(x)\big)+pq(cx^2+x^2c+2xc'x)=0$

Pre-multiplying the above by $(p+q)$, we get

$$2q\big((p+q)cx^2+(p+q)G'(x)x\big)+2p\big((p+q)x^2c+(p+q)xG'(x)\big)+$$
$$pq\big((p+q)cx^2+(p+q)x^2c+2(p+q)xc'x\big)=0 \tag{5}$$

In (3) Pre and post multiplying by $x$, we get

$$(p+q)\ xG\ (x) + xpxc' + xqc'x = 0 \tag{6}$$

$$(p+q)\ G\ (x)\ x + pxc'x + qc'x^2 = 0\ ,\ x \in S \tag{7}$$

Applying lemma 2.1 in (7) and (6) and using it in (5), we get

$$2q\left((p+q)cx^2 + pxc'x + qc'x^2\right) + 2p\left((p+q)x^2c + xpxc' + xqc'x\right) + pq\left((p+q)cx^2 + (p+q)\ x^2c + 2\ (p+q)xc'x\right) = 0$$

Therefore we obtain

$$\left(2q(p+q) + 2qq' + pq(p+q)\right)cx^2 + \left(2p(p+q) + 2pp' + pq(p+q)\right)x^2c + (2qpxc'x + 2pqxc'x + 2pq(p+q)xc'x) = 0\ , x \in S$$

Thus, $pq(p+q+2)cx^2 + pq(p+q+2)x^2c + 2pq(p+q+2)xc'x = 0$

Since $S$ is $pq(p+q+2)$ torsion free semiring, gives

$$cx^2 + x^2c + 2xc'x = 0 \tag{8}$$

This is also expressed as, $\left[[c,x],x\right] = 0$ (9)

Replacing $x$ by $x+z$ in (9), gives $\left[[c,x],z\right] + \left[[c,z],x\right] = 0$ (10)

Substituting $z$ by $xz\ in\ (10),$ we get

$$\left[[c,x],xz\right] + \left[[c,xz],x\right] = 0$$

$$x\left[[c,x],z\right] + \left[[c,x],x\right]z + [c,x][z,x] + [x,x][c,z] + x\left[[c,z],x\right] = 0$$

Using (9) and (10) in the last relation, we get

$$[c,x][z,x] \tag{11}$$

Replacing $z$ by $zc$ in (11), gives

$$[c,x]z[c,x] + [c,x][z,x]c = 0$$

Using (11) in the above equation, we obtain $[c,x]z[c,x] = 0$

By the semiprimeness of $S$ gives $[c,x] = 0$. That is $cx + x'c = 0$

This implies $cx = xc$.

Applying the pseudo inverse of above in (3), we get $(p+q)G(x) + (p+q)xc' = 0$. By our assumption that $S$ is $(p+q)$- torsion free, we get $G(x) = xc$ and $G(x) = cx$, this is true for all $x \in S$. This completes the proof of the theorem.

We include similar proof for our second main result.

**Theorem 3.2**

Let $S$ be a $k$-torsion free semiprime inverse semiring with identity element, where $\in \{2,p\}$ . Suppose $G$ be an additive mapping of $S$ in to itself such that

$2G\,(x^{p+1}) + G'(x)x^p + x^p G'(x) = 0$ fulfilled for all $x \in S$ and integer $p \geq 1$. Then $G$ is a centralizer.

**Proof**

We have $2G(x^{p+1}) + G'(x)x^p + x^p G'(x) = 0$ (12)

Replacing $x$ by $x + te$ in (12), where $t$ be fixed in $S$ and $e \in S$ be the identity in $S$, we get

$$2G'\left(\sum_{i=1}^{p} \binom{p+i}{i} x^{p+1+i'} (te)^i\right) + \left(G'(x) + t'G(e)\right)\left(\sum_{i=1}^{p} \binom{p}{i} x^{p+i'} (te)^i\right) +$$
$$\left(\sum_{i=1}^{p} \binom{p}{i} x^{p+i'} (te)^i\right)\left(G'(x) + t'G(e)\right) = 0$$

We have therefore

$$2\left(\sum_{i=1}^{p} \binom{p+i}{i} G(x^{p+1+i'})(te)^i\right) + (G'(x) + t'c)\left(\sum_{i=1}^{p} \binom{p}{i} x^{p+i'} (te)^i\right) +$$
$$\left(\sum_{i=1}^{p} \binom{p}{i} x^{p+i'} (te)^i\right)(G'(x) + t'c) = 0, x \in S$$

(13)

Where $G(e)$ is denoted by $c$. Comparing the coefficients of $t^p$, we obtain

$$2\binom{p+1}{p} G(x) + G'(x) + c'px + G'(x) + pxc' = 0$$

This leads to $2(p+1)G(x) + 2G'(x) + p(c'x + xc') = 0$

That is, $2G(x) + c'x + xc' = 0, x \in S$ (14)

It follows by our assumption that $S$ is $p$- torsion free. Also from (14), we have

$2G(x^2) + c'x^2 + x^2c' = 0$ (15)

Now compare the coefficients of $t^{p+1'}$ from (13), we get

$$p(p+1)G(x^2) + G'(x)px + c'\left(\frac{p^2 + p'}{2}\right)x^2 + pxG'(x) + \left(\frac{p^2 + p'}{2}\right)x^2c' = 0$$

By our assumption that $S$ is $p$ -torsion free, we obtain

$2(p+1)G(x^2) + 2G(x)x' + 2x'G(x) + c'(p+1')x^2 + (p+1')x^2c' = 0$

Applying lemma 2.1 in (14) and using it in the last relation, we get

$2(p+1)G(x^2) + (cx + xc)x' + x'(cx + xc) + c'(p+1')x^2 + (p+1')x^2c' = 0$

In (15) using Lemma 2.1 then substituting it in the last relation, we reach

$(p+1)(cx^2+x^2c)+2xcx'+p(x^2c'+c'x^2)=0, x\in S$

Which implies that,

$$cx^2+x^2c+2xc'x=0 \quad (16)$$

The last equation is similar to equation (8). Thus we perform similar construction used after equation (8) in the proof of theorem 3.1. Hence we get the required result.

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