JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-15, No.-8, August (2020) pp 93-108 ISSN (Print) 0973-8975

ON TRANSVERSAL VIBRATIONS OF AN AXIALLY MOVING STRING UNDER STRUCTURAL DAMPING

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https://doi.org/10.26782/jmcms.2020.08.00010

(Received: June 20, 2020; Accepted: July 26, 2020)

Abstract

In this paper, the transversal vibrations of axially moving strings under structural damping are studied. The focus on the possibility of the truncation method has been discussed. Governing equations of motion are modelled as second order linear homogeneous partial differential equations with constant coefficients. The string is taken to be fixed at both ends. To construct the asymptotic approximations, the Fourier expansion method in conjunction with the two timescales perturbation method is employed. Amplitude-response of individual mode is computed under the effect of various structural damping parameter values. It is obtained that the response of individual mode decays as time increases. Furthermore, to investigate the applicability of the truncation method, the method of characteristic coordinates and two timescales perturbation methods are used in conjunction with each other. The amplitude-response subject to the specific initial conditions under the effect of various structural damping parameter values is computed. It turned out that amplitude-response decay as time increases. The energy of the system is also computed and found to be decaying as time progresses. From the amplitude-response of the system and individual mode amplitude-response, it is found out that the mode-truncation is allowed in the structural damping case.

Keywords : Transverse vibrations, asymptotic approximations, structural damping, perturbation method.

I. Introduction

Many real life necessities like weighing cables, elevator cables, vertical beams, power transmission wires, tram-ways, and oil pipelines fall in the category of axially moving strings. These systems have been studied for last seven decades, and still gain attention of the researchers, because of their huge applications. Despite their wide range of applications in engineering, the vibrations have limited their applications. It is therefore essential to analyze the dynamic of the systems in terms of vibrations, specifically, the transverse vibrations and stability analysis of the systems. Wind, earthquake and other external excitations induce vibrations in these systems. The transversal vibrations [VIII], [XII], [XIII], [XIV], longitudinal vibrations [V] as well as coupled (transversal together with longitudinal) motion [VI], [XI] helps to study the behavior of axially moving system.

To control the vibrations in structural and mechanical systems, various kinds of damping such as external (viscous) damping [VII], [XIII], [XX] and internal (structural) damping [X] are used. In addition, most of the studies on axially moving string are also concerned with the vibrations at the boundaries [II], [III], [IX], [XVII]. In most physical systems, the damping is incorporated by means of dissipation of the energy and isolation of the energy [XVIII]. The damping is of great importance in the design of engineering structures, as it brings sustainability into systems. The response behavior and undesired oscillations are controlled by means of dampers. Few examples are viscous drag, resistance in electronic frequencies, and absorption in scattering of light waves. Damping is essential in oscillating systems occurring in bikes and biological systems, which is not related with energy loss. Damping can be classified into two classes: External (viscous) damping and internal (structural) damping. In this work, we consider the structural damping, with damping parameter, denoted by parameter δ, ranging between 0 and1. The effect of damping parameter is discussed in details and respective results are presented in this work.

Mathematically, the transversal and/or longitudinal vibrations are modeled as (non) homogeneous (non) linear second order partial differential equations (string-like models) [XIV], [XIX], [XXI] and fourth order partial differential equations (beam-like models) [XXII] with or without variable coefficients. Such type of PDEs are developed by means of Newton’s second law of motion (vector approach) and extended Hamilton’s principle (scalar approach).

In literature, the approximate-analytical technique together with the truncation method is employed to compute the transverse vibrations for string-like models [IV], [VII], [XX]. However, the application of truncation method on string-like models is questionable; see for instance [I], [IX], [XII], [XVII]. Maitlo et. al. [I] found out that the truncation method for problems governing vertical vibrations in string-like models under external (viscous) damping is not possible.

However, the applicability of truncation method for string-like model under the effect of structural damping in terms of all-mode response is not studied. This study will focus on the investigation of application of truncation method for axially moving string with structural damping via method of characteristic coordinates.

In this paper, the work is carried out as follows.

In section II, we construct the asymptotic approximations for the equations of motion via Fourier expansion method in conjunction with two timescales perturbation method.

In section III, computed the amplitude-response of the system via two timescales perturbation method together with method of characteristic coordinates (close-form solution).

In section IV, computed the energy of the system. In section V, some conclusions has been given, based on obtained results. Finally, the verification of results is given in section VI of appendix.

II. Application of Fourier-Expansion Method

In this section, the following equations of motion for string-like model are studied under structural damping which in dimensionless form [X] are given as:

$$u_{tt}+2Vu_{xt}+\left(V^2-1\right)u_{xx}-\delta\left(u_{txx}+Vu_{xxx}\right)=0,\quad t\geq 0,\ 0<x<1, \tag{1}$$

$$u\left(0,t\right)=u\left(1,t\right)=0,\quad t\geq 0, \tag{2}$$

$$u\left(x,0\right)=u_0(x),\ u_t\left(x,0\right)=u_1\left(x\right),\qquad 0<x<1, \tag{3}$$

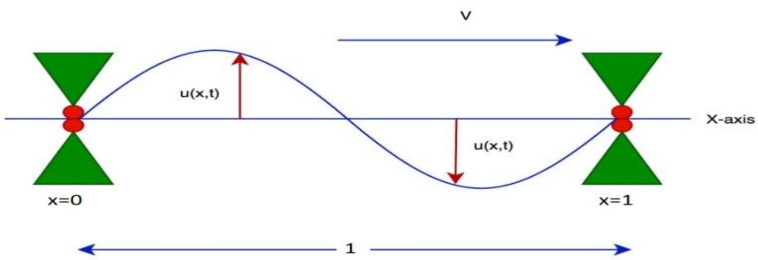


Fig. 1: A schematic model for axially moving string between two fixed ends

where, $u\left(x,t\right)$ is the transversal displacement, t is the time, x is the axial displacement, V is axial speed in belt and assumed to be constant, δ is damping coefficient and assumed to be constant, $u_0\left(x\right)$ is the initial displacement and $u_1\left(x\right)$ is the initial velocity of the system. Both V and δ are taken to be of order ε .

In the following, we consider the mathematical Eq. (1)-(3), with the following assumptions;

$V=O\left(\varepsilon\right),\ \delta=O\left(\varepsilon\right).$

The solution of Eq. (1)-(3) is assumed as series expansion as:

$$u\left(x,t\right)=\sum_{n=1}^{\infty}u_n\left(t;\varepsilon\right)\sin\left(n\pi x\right), \tag{4}$$

Where, u_n is the amplitude.

Putting Eq. (4) and its derivatives in Eq. (1), we obtain;

$$\sum_{n=1}^{\infty}\left[\ddot{u}_n+(n\pi)^2 u_n\right]\sin(n\pi x)=-\varepsilon\sum_{n=1}^{\infty}\begin{bmatrix}2V\dot{u}_n(n\pi)\cos(n\pi x)\\ +\delta\dot{u}_n(n\pi)^2\sin(n\pi x)\end{bmatrix}+O(\varepsilon^2) \tag{5}$$

By multiplying the function $\sin(k\pi x)$, for fixed k , and by using the orthogonally relation for the *sine* and *cosine* functions, we obtain;

$$\sum_{n=1}^{\infty}\left[\ddot{u}_n+(n\pi)^2 u_n\right]\int_0^1\sin(n\pi x)\sin(k\pi x)\,dx=$$

$$-\varepsilon\sum_{n=1}^{\infty}\begin{bmatrix}2V\dot{u}_n(n\pi)\int_0^1\sin(k\pi x)\cos(n\pi x)\,dx\\ +\delta\dot{u}_n(n\pi)^2\int_0^1\sin(n\pi x)\sin(k\pi x)\,dx\end{bmatrix} \tag{6}$$

$$\ddot{u}_k+(k\pi)^2 u_k=\varepsilon\left[-\delta(k\pi)^2\dot{u}_k+\sum_{\substack{n=1\\ n\pm k\\ \text{is odd}}}^{\infty}\frac{8nkV}{n^2-k^2}\right] \tag{7}$$

Eq. (7) represents the system of coupled infinitely many ordinary differential equations. The system given in Eq. (7) is not easy to solve analytically. Therefore, to compute the solution of Eq. (7), two timescales perturbation method is used.

The solution of Eq. (7) is assumed to be in the form;

$$u(t)=w_k(t_0,t_1;\varepsilon)=w_{k_0}(t_0,t_1;\varepsilon)+\varepsilon w_{k_1}(t_0,t_1;\varepsilon)+O(\varepsilon^2) \tag{8}$$

Where, $t_0=t$ (Fast scale) $t_1=\varepsilon t$ (Slow scale), $0<\varepsilon<<1$.

For the derivatives, the following transformation is considered:

$$\frac{du_k}{dt}=\frac{\partial w_k}{\partial t_0}+\varepsilon\frac{\partial w_k}{\partial t_1} \tag{9}$$

$$\frac{d^2u_k}{dt^2}=\frac{\partial^2 w_k}{\partial t_0^2}+2\varepsilon\frac{\partial^2 w_k}{\partial t_0\partial t_1}+O(\varepsilon^2) \tag{10}$$

Putting Eq. (9)-Eq. (10), into Eq. (7) and equating the coefficients of ε^0 and ε^1 we obtain;

$$O(1): \frac{\partial^2 w_{k_0}}{\partial t_0^2} + (k\pi)^2 w_{k_0} = 0 \tag{11}$$

$$O(\varepsilon): \frac{\partial^2 w_{k_1}}{\partial t_0^2} + (k\pi)^2 w_{k_1} = -2\frac{\partial^2 w_{k_0}}{\partial t_0 \partial t_1} + \sum_{\substack{n=1 \\ n\pm k \\ \text{is odd}}}^{\infty} \frac{8(nk)V}{n^2 - k^2} \dot{w}_{k_0} - (k\pi)^2 \delta \frac{\partial w_{k_0}}{\partial t_0} \tag{12}$$

We shall solve the Eq. (11) by direct integration as:

$$w_{k_0}(t_0, t_1) = A_{k_0}(t_1)\cos(k\pi t_0) + B_{k_0}(t_1)\sin(k\pi t_0) \tag{13}$$

Putting Eq.(13) into Eq. (12) we obtain;

$$\begin{aligned} \frac{\partial^2 w_{k_1}}{\partial t_0^2} + (k\pi)^2 w_{k_1} &= -2\left[-A'_{k_0}(t_1)(k\pi)\sin(k\pi t_0) + B'_{k_0}(t_1)(k\pi)\cos(k\pi t_0)\right] \\ &\quad - \delta(k\pi)^2\left[-A_{k_0}(t_1)(k\pi)\sin(k\pi t_0) + B_{k_0}(t_1)(k\pi)\cos(k\pi t_0)\right] \\ &\quad + \sum_{\substack{n=1 \\ n\pm k \\ \text{is odd}}}^{\infty} \frac{8(nk)V}{n^2 - k^2}\left[-A_{n_0}(t_1)(n\pi)\sin(n\pi t_0) + B_{n_0}(t_1)(n\pi)\cos(n\pi t_0)\right] \end{aligned} \tag{14}$$

Eliminating the secular terms from Eq. (12), we obtain;

$$\begin{cases} A'_{k_0}(t_1) + \frac{\delta}{2}(k\pi)^2 A_{k_0}(t_1) = 0, \\ B'_{k_0}(t_1) + \frac{\delta}{2}(k\pi)^2 B_{k_0}(t_1) = 0. \end{cases} \tag{15}$$

The solution of Eq. (15) is;

$$\begin{cases} A_{k_0}(t_1) = e^{-\frac{\delta}{2}(k\pi)^2 t_1} \\ B_{k_0}(t_1) = e^{-\frac{\delta}{2}(k\pi)^2 t_1} \end{cases} \tag{16}$$

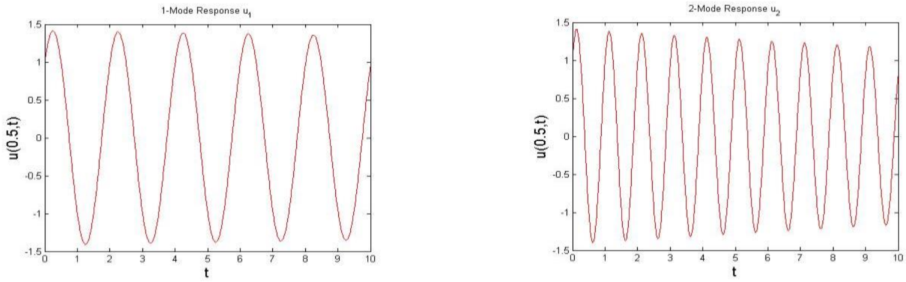
Putting Eq. (16) into Eq. (13), we obtain;

$$w_{k_0}(t_0, t_1) = e^{-\frac{\delta}{2}(k\pi)^2 t_1}\left[\cos(k\pi t_0) + \sin(k\pi t_0)\right] \tag{17}$$

Thus the solution of Eq. (1)-(3) is;

$$u(x,t) = e^{-\frac{\delta}{2}(k\pi)^2 t_1}\left[\cos(k\pi t_0) + \sin(k\pi t_0)\right] + O(\varepsilon) \tag{18}$$

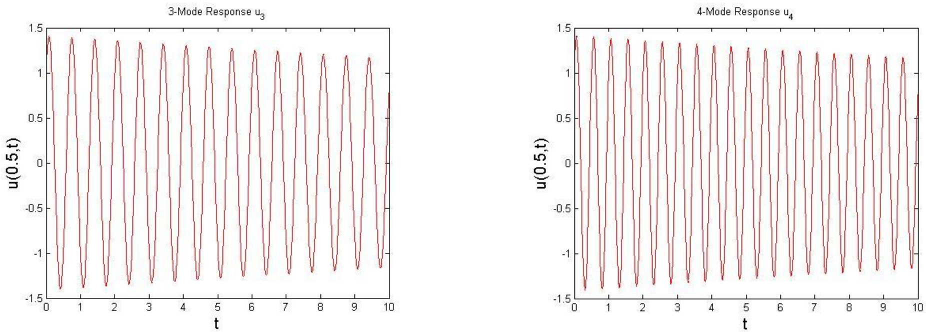
To plot the mode-response Eq. (18), the computer software MATLAB is used. In this work, the response of the system up to ten-modes is computed, by taking $k=1, k=2$ up to $k=10$. Figure 2- Figure 6 depict the amplitude of individual modes of the axially moving system under the effect of structural damping. It is evident that the amplitude of each mode decays with the increasing of time.



(a) (b)

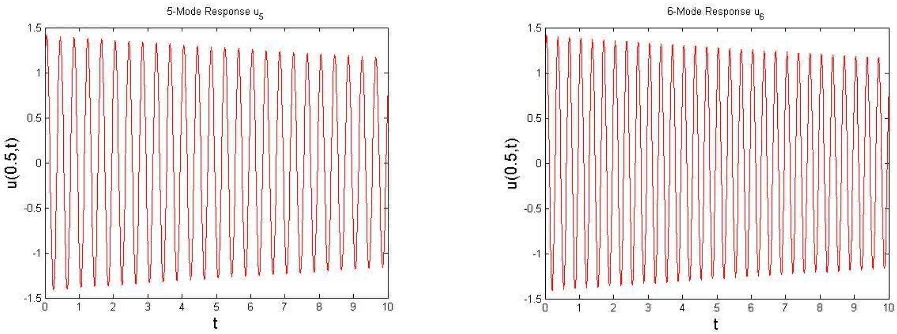
Fig. 2: Mode response for $\delta = 0.1, \varepsilon = 0.01$ (a) $k=1$, (b) $k=2$

(a)



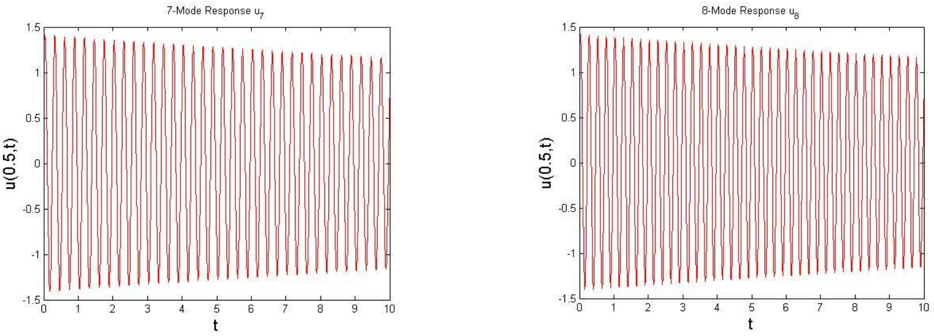
(b)

Fig. 3: Mode response for $\delta = 0.1, \varepsilon = 0.01$ (a) $k=3$, (b) $k=4$



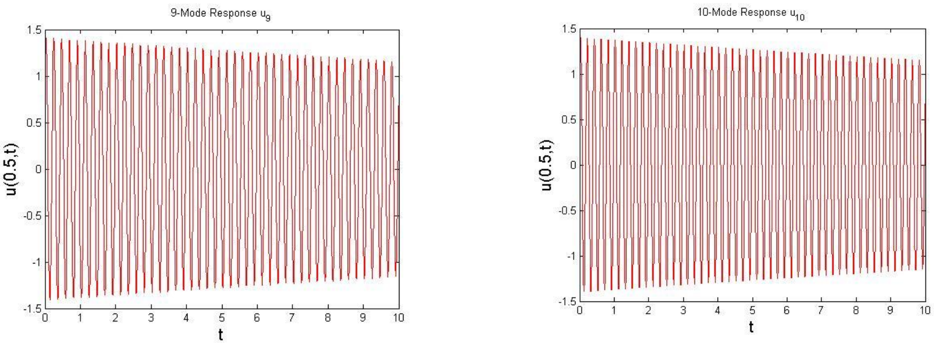
(a) (b)

Fig. 4: Mode response for $\delta = 0.1, \varepsilon = 0.01$ (a) $k=5$, (b) $k=6$



(a) (b)

Fig. 5: Mode response for $\delta = 0.1, \varepsilon = 0.01$ (a) $k = 7$, (b) $k = 8$



(a) (b)

Fig. 6: Mode response for $\delta = 0.1, \varepsilon = 0.01$ (a) $k = 9$, (b) $k = 10$

III. Application of Characteristic Coordinates Method:

In this section, the characteristic coordinate method together with the two time scales perturbation method is employed. In order to apply the method of characteristic coordinates, we need to convert the initial-boundary value problem to an initial-value problem [XIV], [XVII]. For doing so, we will extend the domain and each term in the Eq. (1) to 2-periodic and odd. Since the term u_{xt} and u_{xxx} are not odd, these terms can be made odd by multiplying the term:

$$R(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases} \tag{19}$$

The Fourier series of $R(x)$ is given as:

$$H(x)=\sum_{n=1}^{\infty}\frac{4}{(2n-1)\pi}\sin((2n-1)\pi x), \tag{20}$$

Multiplying the term $H(x)$ with the terms u_{xt} and u_{xxx}, we obtain;

$$u_{tt}+2Vu_{xt}H(x)+(V^2-1)u_{xx}-\delta(u_{txx}+Vu_{xxx}H(x))=0,\ t\geq 0,\ \ 0<x<1 \tag{21}$$

$$u(x,0)=u_0(x),\ \ u_t(x,0)=u_1(x) \tag{22}$$

In this study, we choose the specific initial conditions:

$$u_0(x)=\sin(\pi x),\ u_1(x)=0 \tag{23}$$

Further, by applying perturbation method, we assume the solution of Eq. (21) in the form;

$$u(x,t)=w(\sigma,\xi,\tau), \tag{24}$$

Where $\sigma=x-t,\ \xi=x+t$ and $\tau=\varepsilon t$.

The introduction of σ,ξ and τ transformation as above, the solution and its derivative, as needed in Eq. (1) will take form as follows:

$$u_t=-w_\sigma+w_\xi+\varepsilon w_\tau, \tag{25}$$

$$u_{tt}=w_{\sigma\sigma}+w_{\xi\xi}-2w_{\sigma\xi}+2\varepsilon(w_{\xi\tau}-w_{\sigma\tau})+\varepsilon^2 w_{\tau\tau}, \tag{26}$$

$$u_{xx}=w_{\sigma\sigma}+w_{\xi\xi}+2w_{\sigma\xi}, \tag{27}$$

$$u_{xxx}=w_{\sigma\sigma\sigma}+w_{\xi\xi\xi}+3w_{\sigma\sigma\xi}+3w_{\sigma\xi\xi}, \tag{28}$$

$$u_{xt}=-w_{\sigma\sigma}+w_{\xi\xi}+\varepsilon(w_{\sigma\tau}+w_{\xi\tau}), \tag{29}$$

$$u_{xxt}=-w_{\sigma\sigma\sigma}+w_{\xi\xi\xi}-w_{\sigma\sigma\xi}+w_{\sigma\xi\xi}+\varepsilon(w_{\sigma\sigma\tau}+w_{\xi\xi\tau}+2w_{\sigma\xi\tau}). \tag{30}$$

Now substitute equations (24)-(30) in Eq. (21), we get,

$$4w_{\sigma\xi}=\varepsilon\begin{bmatrix}2w_{\xi\tau}-2w_{\sigma\tau}+2V(w_{\xi\xi}-w_{\sigma\sigma})H\left(\dfrac{\sigma+\xi}{2}\right)\\ +\delta(w_{\sigma\sigma\sigma}-w_{\xi\xi\xi}+w_{\sigma\sigma\xi}-w_{\sigma\xi\xi})\end{bmatrix}+O(\varepsilon^2). \tag{31}$$

Moreover, we assume that the function $w(\sigma,\xi,\tau)$ can be expanded as;

$$w(\sigma,\xi,\tau)=w_0(\sigma,\xi,\tau)+\varepsilon w_1(\sigma,\xi,\tau)+O(\varepsilon^2). \tag{32}$$

Plugging Eq. (32) into Eq. (31), and equating the terms of order ε^0 and ε^1, obtained the problems of $O(1)$ and $O(\varepsilon)$;

$$O(1):\begin{cases} 4w_{0\sigma\xi}=0, & -\infty<\sigma<\xi<\infty,\ \tau>0 \\ w_0(\sigma,\sigma,0)=u_0(\sigma), & -\infty<\sigma=\xi<\infty,\quad \tau=0 \\ -w_{0\sigma}(\sigma,\sigma,0)+w_{0\xi}(\sigma,\sigma,0)=u_1(\sigma), & -\infty<\sigma=\xi<\infty,\quad \tau=0 \end{cases} \tag{33}$$

And

$$O(\varepsilon):\begin{cases} 4w_{1\sigma\xi}=2w_{0\xi\tau}-2w_{0\sigma\tau}+2V\left(w_{0\xi\xi}-w_{0\sigma\sigma}\right)H\left(\dfrac{\sigma+\xi}{2}\right) & \\ \qquad +\delta\left(w_{0\sigma\sigma\sigma}-w_{0\xi\xi\xi}+w_{0\sigma\sigma\xi}-w_{0\sigma\xi\xi}\right), & -\infty<\sigma<\xi<\infty,\ \tau>0 \\ w_1(\sigma,\sigma,0)=0, & -\infty<\sigma=\xi<\infty,\quad \tau=0 \\ -w_{1\sigma}(\sigma,\sigma,0)+w_{1\xi}(\sigma,\sigma,0)=w_{0\tau}(\sigma,\sigma,0), & -\infty<\sigma=\xi<\infty,\quad \tau=0 \end{cases} \tag{34}$$

By direct integration, the solution of $O(1)$ -problem is obtained as;

$$w_0(\sigma,\xi,\tau)=f_0(\sigma,\tau)+g_0(\xi,\tau). \tag{35}$$

Substitute Eq. (35) into Eq. (34), we obtain:

$$4w_{1\sigma\xi}=2g_{0\xi\tau}-2f_{0\sigma\tau}+2V\left(g_{0\xi\xi}-f_{0\sigma\sigma}\right)H\left(\frac{\sigma+\xi}{2}\right)+\delta\left(f_{0\sigma\sigma\sigma}-g_{0\xi\xi\xi}\right). \tag{36}$$

Integrate Eq. (36) w.r.t $'\xi'$, we obtain;

$$\begin{aligned} 4w_{1\sigma}(\sigma,\xi,\tau)&=4w_{1\sigma}(\sigma,\sigma,\tau)+(\xi-\sigma)\left(\delta f_{0\sigma\sigma\sigma}-2f_{0\sigma\tau}\right) \\ &+\int_{\sigma}^{\xi}\left[g_{0\bar{\xi}\tau}+2V\left(g_{0\bar{\xi}\bar{\xi}}-f_{0\sigma\sigma}\right)H\left(\frac{\sigma+\bar{\xi}}{2}\right)-\delta g_{0\bar{\xi}\bar{\xi}\bar{\xi}}\right]d\bar{\xi}+F(\sigma,\tau), \end{aligned} \tag{37}$$

Where, $F(\sigma,\tau)$ is an arbitrary function. Again, by integrating Eq. (36) w.r.t $'\sigma'$, we obtain:

$$\begin{aligned} 4w_{1\xi}(\sigma,\xi,\tau)&=4w_{1\xi}(\sigma,\sigma,\tau)+(\xi-\sigma)\left(-\delta g_{0\xi\xi\xi}+2g_{0\xi\tau}\right) \\ &+\int_{\sigma}^{\xi}\left[f_{0\bar{\sigma}\tau}+2V\left(f_{0\bar{\sigma}\bar{\sigma}}-g_{0\xi\xi}\right)H\left(\frac{\sigma+\xi}{2}\right)-\delta f_{0\bar{\sigma}\bar{\sigma}\bar{\sigma}}\right]d\bar{\sigma}+G(\xi,\tau), \end{aligned} \tag{38}$$

Where, $G(\xi,\tau)$ is an arbitrary function. Since $\xi-\sigma=2t$ is the term which grows without bound. By eliminating the secular (unbounded) terms from Eq. (37), we have;

$$f_{0\sigma\tau}-\frac{\delta}{2}f_{0\sigma\sigma\sigma}=0. \tag{39}$$

From (38), we have;

$$g_{0\xi\tau} - \frac{\delta}{2} g_{0\xi\xi\xi} \quad (40)$$

Eq. (39) and Eq. (40) are equivalent.

In order to solve (39), let $f_{0\sigma} = h$, we obtain;

$$\frac{\partial h}{\partial \tau} - \frac{\delta}{2}\frac{\partial^2 h}{\partial \sigma^2} = 0, \quad (41)$$

With initial condition (23), we have;

$$h(\sigma,0) = \pi\cos(\pi\sigma). \quad (42)$$

By using Fourier transform method, we get the solution (41);

$$f_{0\sigma}(\sigma,\tau) = e^{-\frac{\delta\pi^2\tau}{2}} \pi\cos(\pi\sigma). \quad (43)$$

Integrating (43) w.r.t 'σ', we get;

$$f_0(\sigma,\tau) = e^{-\frac{\delta\pi^2\tau}{2}} \sin(\pi\sigma). \quad (44)$$

Thus, the solution of the mathematical model (1)-(3) for transversal vibrations of damped string system given as under;

$$u(x,t) = w_0(\sigma,\xi,\tau) + O(\varepsilon), \quad (45)$$

From Eq. (35), we have;

$$u(x,t) = f_0(\sigma,\tau) + g_0(\xi,\tau) \quad (46)$$

Since, $g_0(\xi,\tau) = -f_0(-\sigma,\tau)$, thus, the Eq. (46) becomes;

$$u(x,t) = f_0(\sigma,\tau) - f_0(-\xi,\tau). \quad (47)$$

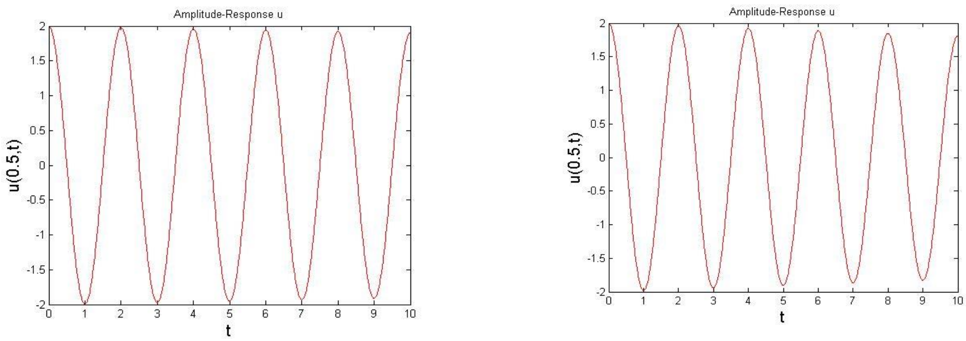
By plugging Eq. (44) into (47), we obtain;

$$u(x,t) = e^{-\frac{\delta\pi^2\tau}{2}} [\sin(\pi\sigma) + \sin(\pi\xi)] + O(\varepsilon) \quad (48)$$

$$u(x,t) = e^{-\frac{\delta\pi^2\varepsilon t}{2}} \left[\sin(\pi(x-t)) + \sin(\pi(x+t))\right] + O(\varepsilon) \quad (49)$$

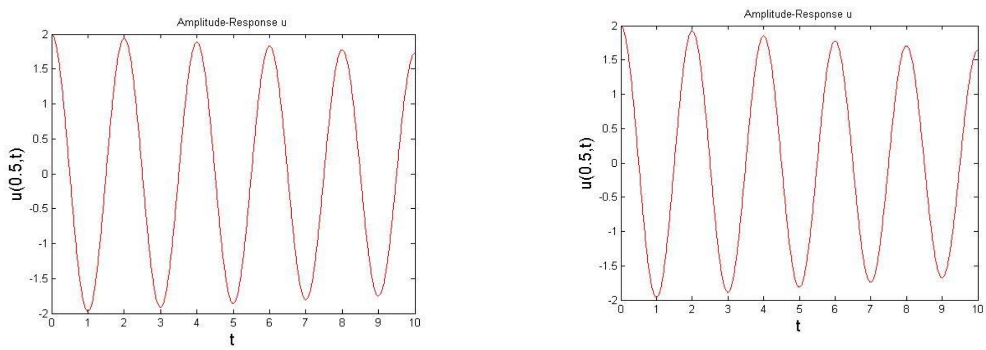
Eq. (49) is the solution of the IVP (1)-(3). From where, the amplitude response presented using MATLAB software in Figure 7 –Figure 11. One can clearly notice that amplitude response decays with the passage of time and further the amplitude response is inversely proportional to the structural damping parameter $0.1 \le \delta \le 1$.

This means that the amplitude damps out at a certain time when larger value of damping δ is incorporated.



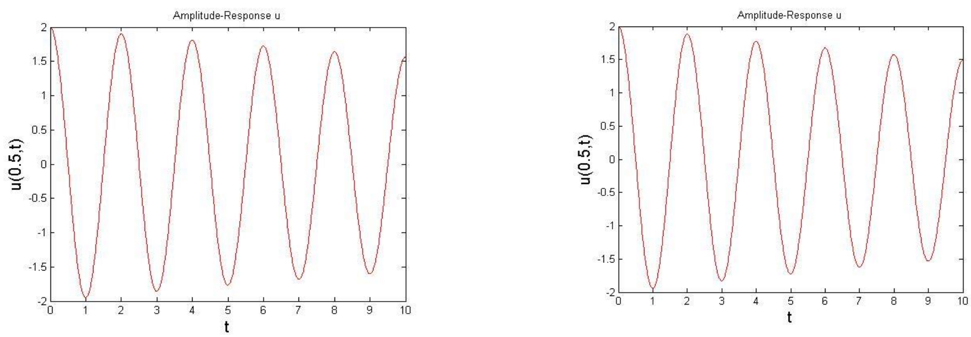
(a) (b)

Fig. 7: Amplitude-response u verses time t for $\varepsilon = 0.01$ (a) $\delta = 0.1$, (b) $\delta = 0.2$



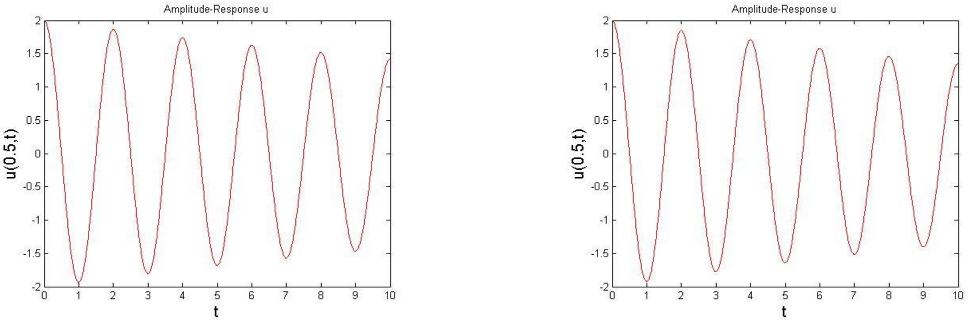
(a) (b)

Fig. 8: Amplitude-response u verses time t for $\varepsilon = 0.01$ (a) $\delta = 0.3$, (b) $\delta = 0.4$



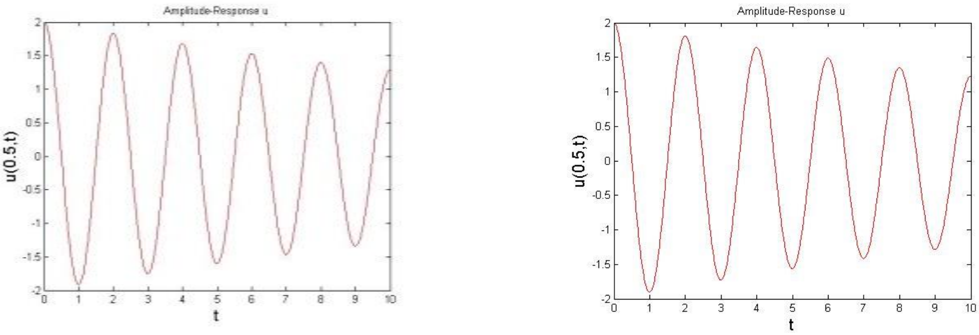
(a) (b)

Fig. 9: Amplitude-response u verses time t for $\varepsilon = 0.01$ (a) $\delta = 0.5$, (b) $\delta = 0.6$



(a) (b)

Fig. 10: Amplitude-response u verses time t for $\varepsilon = 0.01$ (a) $\delta = 0.7$, (b) $\delta = 0.8$



(a) (b)

Fig. 11: Amplitude-response u verses time t for $\varepsilon = 0.01$ (a) $\delta = 0.9$, (b) $\delta = 1.0$

IV. Energy of the System

Using the solution of problem governed by Eq. (1) – (3), as given in Eq.(49) and its derivatives;

$$u_x^2 = \pi^2 e^{-\delta\pi^2\varepsilon t}\left[\cos^2\left(\pi\left(x+t\right)\right)+\cos^2\left(\pi\left(x-t\right)\right)+2\cos\left(\pi\left(x+t\right)\right)\cos\left(\pi\left(x-t\right)\right)\right], \quad (50)$$

$$u_t^2 = \pi^2 e^{-\delta\pi^2\varepsilon t}\left[\cos^2\left(\pi\left(x+t\right)\right)+\cos^2\left(\pi\left(x-t\right)\right)-2\cos\left(\pi\left(x+t\right)\right)\cos\left(\pi\left(x-t\right)\right)\right], \quad (51)$$

in the equation of Energy for the string system , which is given by;

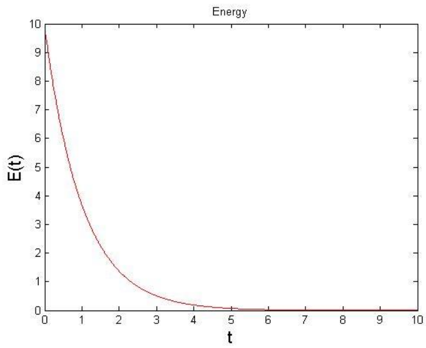
$$E\left(t\right)=\frac{1}{2}\int_0^1\left(u_t^2+u_x^2\right)dx+O\left(\varepsilon\right). \quad (52)$$

After lengthy calculation, we obtain the following expression for the energy of the string system:

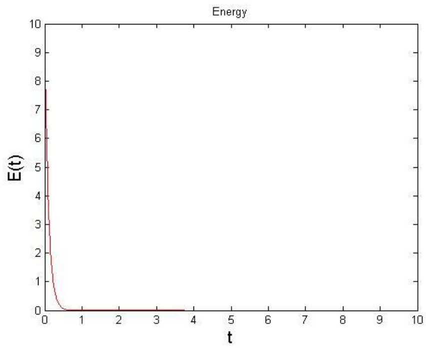
$$E(t) = \pi^2 e^{-\delta\pi^2 \varepsilon t}. \quad (53)$$

The energy in Eq. (53), so computed, is plotted using MATLAB software in Figures 12 (a) and 12 (b). The energy of the system also validates the claim that system is being damped as the time progresses. In addition to this, the energy is seen to be independent of mode-number(s).

Further, by comparing the energy of the system and individual mode-response, it turns out that the truncation is possible in string-like system for constant speed and structural damping.



(a)



(b)

Fig. 12: Energy of the system for $\varepsilon = 0.01$ (a) $\delta = 0.01$, (b) $\delta = 0.1$

V. Conclusion

This study examined the transverse vibrations of axially moving string (belt) system under the effect of structural damping. It was assumed that the belt moves with a constant speed. The string is taken to be fixed at the end points. Both the belt velocity and structural damping are considered to be small. Mathematically, the transverse vibrations of the system are modeled as a linear homogeneous partial differential equation with variable coefficients. The method of two timescales perturbation is used together with the Fourier expansion method and the method of characteristic coordinates. The individual mode response and the response of system are computed. The main purpose of computing the response of individual mode via Fourier expansion method and the response of the system via characteristic coordinate method was to examine the applicability of mode-truncation in string-like model with structural damping. It turned out that the individual mode response as well as the response of the system exhibit the similar behavior, that is, the amplitude response damped out as the time progresses. In addition to this, the energy of the system up to order ε is also seen to be damped out. In conclusion, the mode-truncation is not a problem in string-like model with the inclusion of structural damping

VI. Appendix

Verification of Eq. (39) with Eq. (15):

Consider the Eq. (39)

$$f_{0\sigma\tau} - \frac{\delta}{2} f_{0\sigma\sigma\sigma} = 0 \tag{A.1}$$

The Fourier series representation for the function f_0 is given as;

$$f_0 = \frac{1}{2}\sum_{n=1}^{\infty}\left[A_n(\tau)\sin(n\pi\sigma) + B_n(\tau)\cos(n\pi\sigma)\right] \tag{A.2}$$

By differentiating Eq. (A.2) w.r.t $'\sigma'$ and then w.r.t $'\tau'$, we obtain;

$$f_{0\sigma\tau} = \frac{1}{2}\sum_{n=1}^{\infty}\left[A_n'(\tau)(n\pi)\cos(n\pi\sigma) - B_n'(\tau)(n\pi)\sin(n\pi\sigma)\right] \tag{A.3}$$

Also differentiating Eq.(A.2) w.r.t $'\sigma'$ thrice, we obtain;

$$f_{0\sigma\sigma\sigma} = \frac{1}{2}\sum_{n=1}^{\infty}\left[-A_n(\tau)(n\pi)^3\cos(n\pi\sigma) + B_n(\tau)(n\pi)^3\sin(n\pi\sigma)\right] \tag{A.4}$$

Substitution of Eq. (A.3) and Eq. (A.4) into Eq. (A.1), we obtain

$$\begin{aligned}&\sum_{n=1}^{\infty}\left[A_n'(\tau)\cos(n\pi\sigma) - B_n'(\tau)\sin(n\pi\sigma)\right](n\pi)\\ &\qquad -\frac{\delta}{2}\sum_{n=1}^{\infty}\left[-A_n(\tau)\cos(n\pi\sigma) + B_n(\tau)\sin(n\pi\sigma)\right](n\pi)^3 = 0\end{aligned} \tag{A.5}$$

Multiplying Eq. (A.5) with $\sin(k\pi\sigma)$, and integrate the resulting equation w.r.t $'\sigma'$ as a period of 2, we get;

$$B_n'(\tau) + \frac{\delta}{2}(n\pi)^2 B_n = 0 \tag{A.6}$$

Multiplying Eq. (A.5) with $\cos(k\pi\sigma)$, and integrate the resulting equation w.r.t $'\sigma'$ as a period of 2, we get;

$$A_n'(\tau) + \frac{\delta}{2}(n\pi)^2 A_n = 0 \tag{A.7}$$

Eq. (A.6) and Eq. (A.7) are the same as Eq. (15).

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