



SOFT $g^*\beta$ CLOSED SETS IN SOFT TOPOLOGICAL SPACES

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Abstract

We introduce a new class of soft generalized star β -closed sets (briefly soft g^β -closed set), soft $g^*\beta$ - open set in soft topological spaces (from now on STS). We have studied the relationship between this type of closed sets and other existing closed sets in STS and some of their basic properties.*

Keyword : Soft closed, Soft generalized closed, Soft $g^*\beta$ -closed set, Soft $g^*\beta$ -open set, Soft topological spaces.

I. Introduction

The soft set theory is a rapidly processing field of Mathematics. This new set theory has found its applications in Game Theory, Operations Research, Theory of Probability, Riemann Integration, Perron Integration, Smoothness of functions, etc. In 1999, the concept of soft set theory was initiated by Molodtsov [V] as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects which contains sufficient parameters such that it is free from the corresponding difficulties. In 2010 Muhammad Shabir and Munazza Naz [VI] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighbourhood of a point and soft separation axioms and also defined the theory of soft topological space over an initial universe with a fixed set of parameters. The concept of generalized closed sets was introduced by N. Levine [IV]. Punitha Tharani. A and Sujitha. H [VII] introduced the concept of $g^*\beta$ -closed sets in

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Topological spaces. Kannan [III] introduced soft g-closed sets in soft topological spaces.

In this paper we define soft $g^*\beta$ -closed, soft $g^*\beta$ -open in a soft topological space. We also investigate related properties of these sets and compared their properties with other existing soft closed sets in STS.

Let (\mathcal{U}, τ, E) represents a nonempty STS on which no separation axioms are assumed unless stated otherwise. For a subset (S, E) of \mathcal{U} , the closure, the interior and the complement of (S, E) are denoted by $cl(S, E)$, $int(S, E)$ and $(S, E)^c$ respectively.

II. Preliminaries

Definition: 2.1 Let \mathcal{U} be an initial universal set and E be the set of parameters. Let $P(\mathcal{U})$ denote the power set of \mathcal{F} and $S \subseteq E$. The pair (\mathcal{F}, S) is called a soft set over \mathcal{U} , where \mathcal{F} is a mapping given by $\mathcal{F}: S \rightarrow P(\mathcal{U})$.

Definition: 2.2 A soft (\mathcal{F}, E) over \mathcal{U} is said to be

- (1) A null soft set, denoted by Φ , if $\forall e \in E, \mathcal{F}(e) = \Phi$.
- (2) An absolute soft set, denoted by X , if $\forall e \in E, \mathcal{F}(e) = \mathcal{U}$.

The soft sets (\mathcal{F}, E) over an universe \mathcal{U} in which all the parameters of the set E are same is a family of soft sets, denoted by $SS(\mathcal{U})$.

Definition: 2.3 Let τ be the collection of soft sets over \mathcal{U} , then τ is said to be a soft topology on \mathcal{U} if

- (1) \mathcal{U} are belongs to τ .
- (2) The union of any number of soft sets in τ belongs τ .
- (3) The intersection of any two soft sets in τ belongs τ .

The triplet (\mathcal{U}, τ, E) is called a STS over \mathcal{U} and any member of τ is called soft open set in \mathcal{U} . The complement of a soft open set is called soft closed set over \mathcal{U} .

Definition: 2.4 Let (\mathcal{U}, τ, E) be a STS over \mathcal{U} and (\mathcal{F}, E) be a soft set over \mathcal{U} . Then

- (1) Soft interior of a soft set (\mathcal{F}, E) is defined as the union of all soft open sets contained in (\mathcal{F}, E) . Thus $int(\mathcal{F}, E)$ is the largest soft open set contained in (\mathcal{F}, E) .
- (2) Soft closure of a soft set (\mathcal{F}, E) is the intersection of all soft closed super sets (\mathcal{F}, E) . Thus $cl(\mathcal{F}, E)$ is the smallest soft closed set over \mathcal{U} which contains (\mathcal{F}, E) .

Definition: 2.5 A subset (S, E) of STS (\mathcal{U}, τ, E) is said to be

- (1) a soft semi open set if $(S, E) \subseteq cl(int(S, E))$ and soft semi-closed if $int(cl(S, E)) \subseteq (S, E)$
- (2) a soft pre-open set if $(S, E) \subseteq int(cl(S, E))$ and soft pre-closed if $cl(int(S, E)) \subseteq (S, E)$

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- (3) a soft α open set if $(S, E) \subseteq \text{int}(\text{cl}(\text{int}(S, E)))$ and a soft α closed set if $\text{cl}(\text{int}(\text{cl}(S, E))) \subseteq (S, E)$
- (4) a soft regular set if $\text{int}(\text{cl}(S, E)) = (S, E)$ and a soft regular closed set if $\text{cl}(\text{int}(S, E)) = (S, E)$.
- (5) a soft β -open set if $(S, E) \subseteq \text{cl}(\text{int}(\text{cl}(S, E)))$.
- (6) a soft πg - closed if $\text{cl}(S, E) \subseteq (G, E)$ whenever $(S, E) \subseteq (G, E)$ and (G, E) is soft π - open in \mathcal{U} .
- (7) a soft πsg - closed if $\text{scl}(S, E) \subseteq (G, E)$ whenever $(S, E) \subseteq (G, E)$ and (G, E) is soft π - open in \mathcal{U} .
- (8) soft generalized-semi closed (soft gs - closed) if $\text{scl}(S, E) \subseteq (G, E)$ whenever $(S, E) \subseteq (G, E)$ and (G, E) is soft open in \mathcal{U} .

The intersection of all soft semi-closed (resp. pre-closed, α -closed) sets containing a subset (S, E) of (\mathcal{U}, τ, E) is called soft semi-closure (resp. soft pre closure, soft α -closure) of (S, E) and is denoted by $\text{scl}(S, E)$ (resp. $l(S, E)$, $\text{cl}_\alpha(S, E)$). The soft semi-interior of (S, E) is the largest soft semi open set contained in (S, E) and denoted by $\text{sint}(S, E)$.

Definition: 2.6 [1] A subset (S, E) of a topological space \mathcal{U} is called a soft generalized β closed (soft $g\beta$ -closed) in a STS (\mathcal{U}, τ, E) , if $\beta\text{cl}(S, E) \subseteq (G, E)$ whenever $(S, E) \subseteq (G, E)$ and (G, E) is soft open in \mathcal{U} .

III. Soft $g^*\beta$ -Closed Sets

Definition: 3.1 A subset (\mathcal{F}, E) of a STS (\mathcal{U}, τ, E) be an and (\mathcal{F}, E) is called a soft $g^*\beta$ -closed set, if $\beta\text{cl}(\mathcal{F}, E) \subseteq (G, E)$ whenever $(\mathcal{F}, E) \subseteq (G, E)$ and (G, E) is soft g^* -open set in \mathcal{U} .

Definition: 3.2 Let (S, E) be a soft set in STS. Then soft $g^*\beta$ -closure and soft $g^*\beta$ -interior of (S, E) are defined as follows:

- (1) $\tilde{\text{s}}g^*\beta\text{cl}(S, E) = \tilde{\cap} \{(T, E) : (T, E) \text{ is soft } g^*\beta\text{-closed set and } (S, E) \subseteq (T, E)\}$
- (1) $\tilde{\text{s}}g^*\beta\text{int}(S, E) = \tilde{\cup} \{(\mathcal{R}, E) : (\mathcal{R}, E) \text{ is soft } g^*\beta\text{- open set and } (\mathcal{R}, E) \subseteq (S, E)\}$

Theorem: 3.3

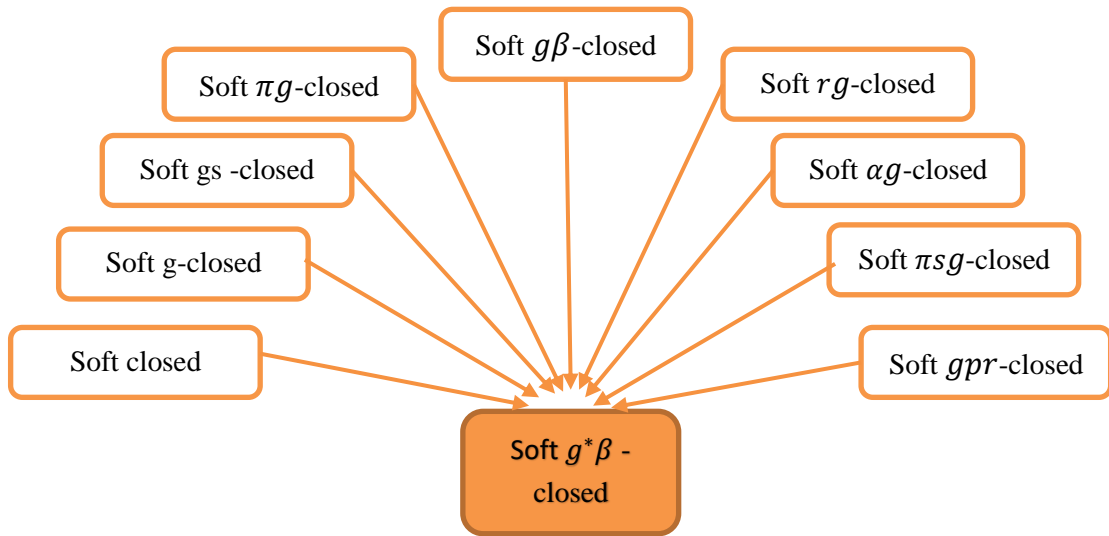
- (1) Every soft closed set is soft $g^*\beta$ -closed.
- (2) Every soft $g^*\beta$ - closed set is soft g -closed.
- (3) Every soft $g^*\beta$ - closed set is soft gs -closed.
- (4) Every soft $g^*\beta$ - closed set is soft πg -closed.
- (5) Every soft $g^*\beta$ - closed set is soft $g\beta$ -closed.
- (6) Every soft $g^*\beta$ - closed set is soft rg -closed.
- (7) Every soft $g^*\beta$ - closed set is soft αg -closed.
- (8) Every soft $g^*\beta$ - closed set is soft πsg -closed.
- (9) Every soft $g^*\beta$ - closed set is soft gpr -closed.

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Proof:

- (1) Let (\mathcal{F}, E) be soft closed set in (\mathcal{U}, τ, E) and (G, E) be soft g^* -open in \mathcal{U} , such that $(\mathcal{F}, E) \subseteq (G, E)$.
Then $\beta cl(\mathcal{F}, E) = (\mathcal{F}, E) \subseteq (G, E)$. Therefore (\mathcal{F}, E) is $g^*\beta$ -closed.
- (2) Let (\mathcal{F}, E) be $g^*\beta$ -closed in (\mathcal{U}, τ, E) . Let (G, E) be soft open in \mathcal{U} such that $(\mathcal{F}, E) \subseteq (G, E)$.
Since every soft open set is soft g^* -open, we have $\beta cl(\mathcal{F}, E) \subseteq (G, E)$.
Therefore (\mathcal{F}, E) is g -closed.
- (3) Let (\mathcal{F}, E) be $g^*\beta$ -closed in (\mathcal{U}, τ, E) . Let (G, E) be soft open in \mathcal{U} such that $(\mathcal{F}, E) \subseteq (G, E)$.
Since every soft open set is soft g^* -open, we have $\beta cl(\mathcal{F}, E) \subseteq (G, E)$.
But $scl(\mathcal{F}, E) \subseteq \beta cl(\mathcal{F}, E) \subseteq (G, E)$. Therefore (\mathcal{F}, E) is gs -closed.
- (4) Obvious.
- (5) Proof follows from the definition.
- (6) Let (\mathcal{F}, E) be $g^*\beta$ -closed in (\mathcal{U}, τ, E) . Let (G, E) be soft regular open in \mathcal{U} such that $(\mathcal{F}, E) \subseteq (G, E)$. Since every soft regular open set is soft g^* -open, we have $\beta cl(\mathcal{F}, E) \subseteq (G, E)$.
Hence (\mathcal{F}, E) is gr -closed.
- (7) Let (\mathcal{F}, E) be $g^*\beta$ -closed in (\mathcal{U}, τ, E) . Let (G, E) be soft open in \mathcal{U} such that $(\mathcal{F}, E) \subseteq (G, E)$.
Since every soft open set is soft g^* -open, we have $\beta cl(\mathcal{F}, E) \subseteq (G, E)$.
But $\alpha cl(\mathcal{F}, E) \subseteq \beta cl(\mathcal{F}, E) \subseteq (G, E)$. Hence (\mathcal{F}, E) is αg -closed.
- (8) Let (\mathcal{F}, E) be $g^*\beta$ -closed in (\mathcal{U}, τ, E) . Let (G, E) be soft π -open in \mathcal{U} such that $(\mathcal{F}, E) \subseteq (G, E)$.
Since every soft π -open set is soft g^* -open, we have $\beta cl(\mathcal{F}, E) \subseteq (G, E)$.
But $scl(\mathcal{F}, E) \subseteq \beta cl(\mathcal{F}, E) \subseteq (G, E)$. Hence (\mathcal{F}, E) is πsg -closed.
- (9) Proof follows from definition.

Remark: 3.4 We elaborate the above theorem 3.3 in the following schematic and the converse is not true.



Theorem: 3.5 Union of two soft $g^*\beta$ -closed set in a STS is soft $g^*\beta$ - closed.

Proof: Let (\mathcal{F}, E) and (\mathcal{H}, E) be two soft $g^*\beta$ - closed sets in (\mathcal{U}, τ, E) . Let (G, E) be soft g^* -open sets in (\mathcal{U}, τ, E) such that $(\mathcal{F}, E) \cup (\mathcal{H}, E) \subseteq (G, E) \Rightarrow (\mathcal{F}, E) \subseteq (G, E)$ and $(\mathcal{H}, E) \subseteq (G, E)$. Again $\beta cl(\mathcal{F}, E) \subseteq (\mathcal{U}, E)$ and $\beta cl(\mathcal{H}, E) \subseteq (G, E) \Rightarrow \beta cl(\mathcal{F}, E) \cup (\mathcal{H}, E) \subseteq (G, E) \Rightarrow (\mathcal{F}, E) \cup (\mathcal{H}, E)$ is soft $g^*\beta$ -closed.

Theorem: 3.6 Intersection of two soft $g^*\beta$ -open sets in a STS is soft $g^*\beta$ -open.

Proof: Let (S, E) and (T, E) be two soft $g^*\beta$ open sets $\Rightarrow (S, E)^c$ and $(T, E)^c$ are soft $g^*\beta$ closed sets $\Rightarrow (S, E)^c \cup (T, E)^c$ is soft $g^*\beta$ - closed set $\Rightarrow ((S, E) \cap (T, E))^c$ is soft $g^*\beta$ closed set. Therefore, $(S, E) \cap (T, E)$ is soft $g^*\beta$ -open set.

Theorem: 3.7 Every soft semi closed set in a STS is soft $g^*\beta$ -closed but not conversely.

Proof: Let (S, E) be a soft semi closed set in an STS and (G, E) be a soft g^* -open set such that $(S, E) \subseteq (G, E)$. since (S, E) is soft semi closed, $\beta cl(S, E) = (S, E)$ and so (S, E) is soft $g^*\beta$ -closed set.

Theorem: 3.8 Let (T, E) be a soft $g^*\beta$ -closed set in (\mathcal{U}, τ, E) iff $\beta cl(T, E) \setminus (T, E)$ does not contain any non-empty soft g^* -closed set.

Proof: Necessity: Given (\mathcal{C}, E) is soft g^* -closed subset of $\beta cl(T, E) \setminus (T, E) \Rightarrow (\mathcal{C}, E) \subseteq \beta cl(T, E)$ and $(T, E) \subseteq \mathcal{U} \setminus (\mathcal{C}, E)$. Since $\mathcal{U} \setminus (\mathcal{C}, E)$ is soft g^* -open set, (T, E) is soft $g^*\beta$ -closed and $\beta cl(T, E) \subseteq \mathcal{U} \setminus (\mathcal{C}, E)$. Therefore, $(\mathcal{C}, E) \subseteq \beta cl(T, E) \cap (\mathcal{U} \setminus \beta cl(\mathcal{C}, E)) = \Phi$. Hence $\beta cl(T, E) \setminus (T, E)$ does not contain any non-empty soft g^* -closed set.

Sufficiency: Given $\beta cl(T, E) \setminus (T, E)$ does not contain any non-empty soft g^* -closed set.

Let $(T, E) \subseteq (G, E)$ and (G, E) is soft g^* -open. Suppose that $\beta cl(T, E)$ is not contained in (G, E) , $\beta cl(T, E) \subseteq (G, E)^c$ is a nonempty soft g^* -closed set of $\beta cl(T, E) \setminus (T, E)$. Which is a contradiction. Hence $\beta cl(T, E) \subseteq (G, E)$ and hence (T, E) is soft $g^*\beta$ -closed set.

Theorem: 3.9 If (S, E) be a soft $g^*\beta$ closed set in (\mathcal{U}, τ, E) such that $(S, E) \subseteq (T, E) \subseteq \beta cl(S, E)$ then (G, E) be a soft $g^*\beta$ -closed set in (\mathcal{U}, τ, E) .

Proof: Given (S, E) is soft $g^*\beta$ -closed set in (\mathcal{U}, τ, E) such that $(S, E) \subseteq (T, E) \subseteq \beta cl(S, E)$.

Let (G, E) be a soft g^* -open set in (\mathcal{U}, τ, E) such that $(T, E) \subseteq (G, E)$.

By hypothesis, we have $\beta cl(S, E) \subseteq (G, E)$.

Now, $\beta cl(T, E) \subseteq \beta cl(\beta cl(S, E)) = \beta cl(S, E) \subseteq (G, E)$.

Therefore (T, E) is $g^*\beta$ -closed in (\mathcal{U}, τ, E) .

Theorem: 3.10 If $(S, E) \subseteq \mathcal{V} \subseteq \mathcal{U}$ and (S, E) is soft $g^*\beta$ -closed in (\mathcal{U}, τ, E) , then (S, E) is soft $g^*\beta$ -closed relative to \mathcal{V} .

Proof: Given $(S, E) \subseteq \mathcal{V} \subseteq \mathcal{U}$ and (S, E) is soft $g^*\beta$ -closed in (\mathcal{U}, τ, E) .

Let $(S, E) \subseteq \mathcal{V} \cap (G, E)$, where (G, E) is soft g^* -open in \mathcal{U} . Since (S, E) is soft $g^*\beta$ -closed, $(S, E) \subseteq (G, E) \Rightarrow \beta cl(S, E) \subseteq (G, E)$.

In continuation to that $\mathcal{V} \cap \beta cl(S, E) \subseteq \mathcal{V} \cap (G, E)$.

Therefore (S, E) is soft $g^*\beta$ -closed set relative to \mathcal{V} .

Theorem: 3.11 Let (T, E) be a subset of a STS namely \mathcal{U} , then the following are equivalent:

- (1) (T, E) is soft regular open.
- (2) (T, E) is soft open and soft $g^*\beta$ -closed.

Proof: (1) \Rightarrow (2). Let (\mathcal{B}, E) be a soft g^* -open in \mathcal{U} containing (T, E) .

Since every soft regular open set is open,

$(T, E) \cup \text{int}(\text{cl}((T, E))) \subseteq (T, E) \subseteq (\mathcal{B}, E)$. Hence $\beta cl(T, E) \subseteq (\mathcal{B}, E)$, (T, E) is soft $g^*\beta$ -closed.

(2) \Rightarrow (1). Since (T, E) is soft open and soft $g^*\beta$ -closed, then $\beta cl(T, E) \subseteq (T, E)$ and so $(T, E) \cup \text{int}(\text{cl}((T, E))) \subseteq (T, E)$, but (T, E) is soft open,

$\text{int}(\text{cl}((T, E))) \subseteq (T, E)$. Since every soft open set is soft pre-open, we have

$(T, E) \subseteq \text{int}(\text{cl}((T, E)))$. Therefore $(T, E) = \text{int}(\text{cl}((T, E))) \Rightarrow (T, E)$ is soft regular open.

IV. SOFT $g^*\beta$ -OPEN SET

Definition: 4.1 A subset (S, E) of a topological space \mathcal{U} is called soft $g^*\beta$ - open in a soft topological space (\mathcal{U}, τ, E) , if $(\mathcal{F}, E) \subseteq \beta int(S, E)$ whenever $(\mathcal{F}, E) \subseteq (S, E)$ and (\mathcal{F}, E) is soft g^* -closed in \mathcal{U} .

Theorem: 4.2 If (S, E) is a soft $g^*\beta$ -open set of \mathcal{U} and $\beta int(S, E) \subseteq (T, E) \subseteq (S, E)$, then (T, E) is also soft $g^*\beta$ - open set of \mathcal{U} .

Proof: Let (S, E) be soft $g^*\beta$ -open set of \mathcal{U} . Suppose (G, E) is soft $g^*\beta$ -closed set such that $(\mathcal{H}, E) \subseteq (T, E)$. By assumption, $(T, E) \subseteq (S, E)$ we have $(\mathcal{H}, E) \subseteq (B, E)$. Since (S, E) be $g^*\beta$ - open set $(\mathcal{H}, E) \subseteq \beta int(S, E)$. Then $int(\beta int(S, E)) \subseteq (B, E)$. $\Rightarrow \beta int(S, E) \subseteq \beta int(B, E)$. Hence $(\mathcal{H}, E) \subseteq \beta int(S, E) \subseteq \beta int(B, E)$. Thus (B, E) is soft $g^*\beta$ - open set of \mathcal{U} .

Theorem: 4.3 If (\mathcal{F}, S) and (\mathcal{H}, T) are soft $g^*\beta$ - open sets, then $((\mathcal{F}, S) \tilde{\cap} (\mathcal{H}, T))$ is also soft $g^*\beta$ - open set.

Proof: Let (\mathcal{F}, S) and (\mathcal{H}, T) be soft $g^*\beta$ - open set. Suppose (\mathcal{B}, E) is soft g^* -closed set such that $(\mathcal{B}, E) \subseteq ((\mathcal{F}, S) \tilde{\cap} (\mathcal{H}, T))$. Then $(\mathcal{B}, E) \subseteq (\mathcal{F}, S)$ and $(\mathcal{B}, E) \subseteq (\mathcal{H}, T)$. Since (\mathcal{F}, S) and (\mathcal{H}, T) are soft $g^*\beta$ - open set, $(\mathcal{B}, E) \subseteq \beta int(\mathcal{F}, S)$ and $(\mathcal{B}, E) \subseteq \beta int(\mathcal{H}, T)$. Therefore $(\mathcal{B}, E) \subseteq \beta int(\mathcal{F}, S) \cap \beta int(\mathcal{H}, T)$. Thus $(\mathcal{B}, E) \subseteq \beta int((\mathcal{F}, S) \tilde{\cap} (\mathcal{H}, T))$.

Hence $((\mathcal{F}, S) \tilde{\cap} (\mathcal{H}, T))$ is a soft $g^*\beta$ - open set.

V. Conclusion

In this paper we had researched and found out a new set in Soft- $g^*\beta$ -closed set and Soft- $g^*\beta$ -open set in soft topological space and we studied the relationship between this type of closed sets and other existing closed sets in STS and also studied some of their basic properties.

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