



COVID-19 IN INDIA AND SIR MODEL

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Abstract

In the present numerical investigation, the epidemic patterns of Covid-19 in India is studied from a mathematical modeling perspective. The study is based on the simple SIR (Susceptible-Infectious-Recovered) deterministic compartmental model. It is analyzed fully and then calibrated against publicly available epidemiological data from late January until 10 July 2020 for interpreting the transmission dynamics of the novel coronavirus disease (COVID-19) in India. The purpose of this study is to give a tentative prediction of the epidemic peak and sizes in our country.

Keywords: COVID-19, India, Parameter Estimation, SIR Model, Simulation.

I. Introduction

The Government of India confirmed India's first case of Coronavirus disease 2019 on 30 January 2020 in the state of Kerala, when a university student from Wuhan travelled back to the state [IX]. The World Health Organization on 11 March 2020, declared COVID-19 as a pandemic. The only solution to control the spread of coronavirus is to break the cycle of transmission through social distancing. Our country has been under lockdown since 25 March 2020 following 'Janata Curfew' (people's curfew) on Sunday, 22 March. In the present numerical study, SIR model is applied to the real data to investigate the dynamics of Covid 19 in India.

II. Mathematical Model & Materials

Any model can be typically thought of as a conceptual tool that explains how an object (or a system of objects) will behave. A mathematical model uses the language of mathematics to produce a more refined and precise description of the system. Models for infectious diseases predict things such as how a disease spreads, the total number infected, the duration of an

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epidemic, and to estimate various epidemiological parameters such as the reproductive number. Such models can show how different public health interventions may affect the outcome of the epidemic, e.g., what the most efficient technique is for issuing a limited number of vaccines in a given population.

In the present study, the simplest compartmental model, i.e., SIR model Kermack and McKendrick [V] is proposed. In this model, the population is divided into susceptible, infective and recovered individuals, with the functions $S(t)$, $I(t)$ and $R(t)$ at time t (measured, for example, in days). The evolution of these quantities is described by the differential equations “frequency dependent” formulation:

$$\frac{dS}{dt} = -\beta SI \tag{1}$$

$$\frac{dI}{dt} = \beta SI - \gamma I \tag{2}$$

$$\frac{dR}{dt} = \gamma I \tag{3}$$

To recap:

$S(t)$ is the number of ‘susceptibles’, i.e. the number in the community of N ($N=S+I+R$) ready and waiting to be infected with the disease at time t .

$I(t)$ is the number who actually have the disease and are capable of infecting others (the susceptibles) and are no longer susceptible themselves.

$R(t)$ is the number ‘removed’ from the system because they are neither infected nor susceptible since they have either recovered, and so acquired immunity, or, in severe cases, are dead.

The origin of the simple model represented by the right-hand side of these equations is very simple to understand in the light of the above examples. Equation (3) says that the rate at which people are removed (die or recover) is proportional to the number of cases I . Here

$$\gamma = \text{removal rate of infectives } (\gamma > 0) \tag{4}$$

The second term in equation (2) reflects the corollary to this statement since these same individuals are being lost from class I . The first term in this equation is the statement that the number of new cases per unit time is proportional both to the number of potential victims (susceptibles) S and to the

number already having the disease I and so capable of infecting others. The constant of proportionality is

$$\beta = \text{infection rate } (\beta > 0) \tag{5}$$

Finally, equation (1) represents the corollary to this statement since these same individuals are being lost to class S.

The particular model is now fully specified by the infection and removal rates β and γ and by a set of initial conditions. Usually, one takes $R(0) = 0$ since no one has yet had the chance to recover or die. Thus, a choice of $I(0) = I_0$ is enough to define the system at $t = 0$ since then $S_0 = N - I_0$. Typically, one takes $t = 0$ to be the moment at which $I = I_0 = 1$ if the epidemic is triggered by a single infected individual.

This model has the following deficiencies in representing Covid-19 infections:

- Constant (closed) population size, N.
- No demography (i.e., births and deaths).
- Constant rates (e.g., transmission, removal rates)
- Latent period for the disease is ignored.
- All those who recover from the disease are assumed to be immune (at least within the time period considered).
- Well-mixed population.

Properties of Solutions [I, II]

Property 1. Model (1-3) is well posed. By well-posed-ness we mean that nonnegative initial conditions lead to nonnegative solutions, namely, $S(0) \geq 0$, $I(0) \geq 0$, and $R(0) \geq 0$ imply $S(t) \geq 0$, $I(t) \geq 0$, and $R(t) \geq 0$ for $t \geq 0$.

Property 2. Total population is constant.

Property 3. Solutions to (1-3) exist for $t \in [0, +\infty]$.

Property 4. $\lim_{t \rightarrow \infty} (S(t), I(t), R(t)) = (S(\infty), I(\infty), R(\infty))$ exists.

Property 5. $S(0) > 0$ and $I(0) > 0$ imply $0 < S(\infty) < S(0)$ and $I(\infty) = 0$.

Database

The analysis is based on the data collected and made available in the website [IV]. It gives the number of confirmed cases day wise in India from 30 Jan 2020 to 10 July 2020 as shown in Table 1.

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Table 1: Daily confirmed cases

Date	Confirmed case
30 Jan 2020	1
31 Jan 2020	1
.....
.....
09 July 2020	793802
10 July 2020	820916

III. Simulation & Results

The mathematical model is fully specified by the infection and removal rates β and γ , and by a set of initial conditions. A set of codes in Matlab is developed for the differential equations of the model (1-3) with the reported values [VI] of the parameters β and γ , and then solved with built-in Matlab solver *ode45*. Total population of India is $N = 1380004385$ [VIII]. The outbreak is triggered by a single infected individual at $t = 0$ (on 30 Jan 2020), so the set of initial conditions is then:

$$I(0) = 1, S(0) = N - I(0) = 1380004384, R(0) = 0 \quad (6)$$

Table 2 describes a set of Matlab routines for the solution of Eqs. (15) along with the initial conditions (6).

Table 2: A set of Matlab routines used sequentially to solve Equations (1-3).

Matlab code	Brief Description
<i>sir.m</i>	Defines the differential Eqs. (1-3).
<i>incond.m</i>	Describes initial values for integration Eq (6).
<i>ode45</i>	Integrates as initial value problem.
<i>nlinfit.m</i>	Gives the best-fit values of the parameters, β and γ .
<i>nlparci.m</i>	Gives 95% confidence intervals of the parameters.
<i>results.m</i>	Gives the results (Figs 1-3).

Parameter Estimation & Calibration the model with the data.

Built-in function *nlinfit* [VII] based on Gauss-Newton's algorithm is used to find the least square fitting. It gives the best fit values of the parameters β and γ . 95% confidence interval estimation is achieved with the function *nlparci* [VII]. Best-fit values of the parameters and their confidence intervals (CIs) are shown in Table 3 and Fig 1.

Table 3: Best-fit values of β and γ , and their confidence intervals.

Parameters	Best-fit values	95% CIs
Infection rate, β	3.6462	3.6362 - 3.6569
Removal rate, γ	3.5860	3.5727 - 3.5989

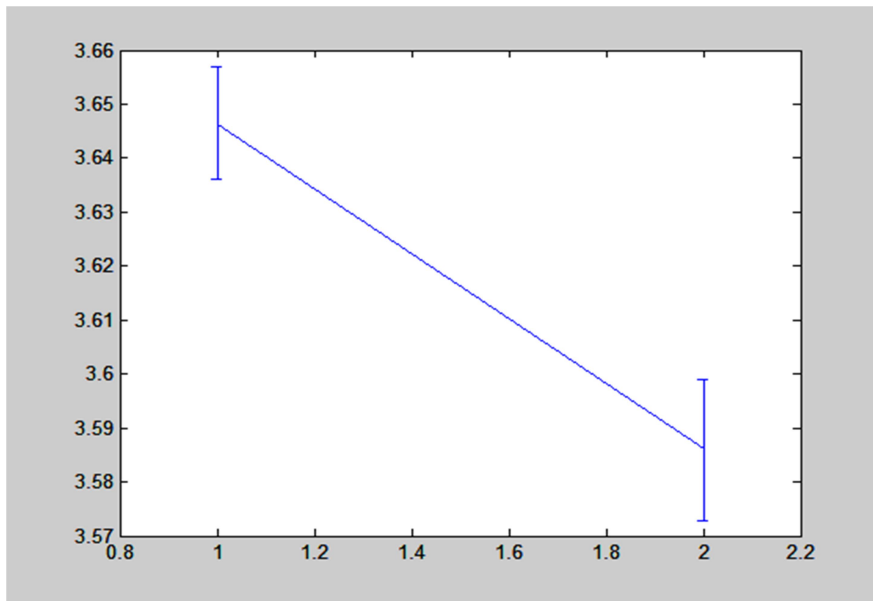


Fig 1. Best-fit parameters (β and γ) and confidence intervals

Results & Discussion

Finally, with the best-fit values of the parameters of β and γ , Matlab solver ode45 is again run, and the results are shown in Fig 2 and Fig 3.

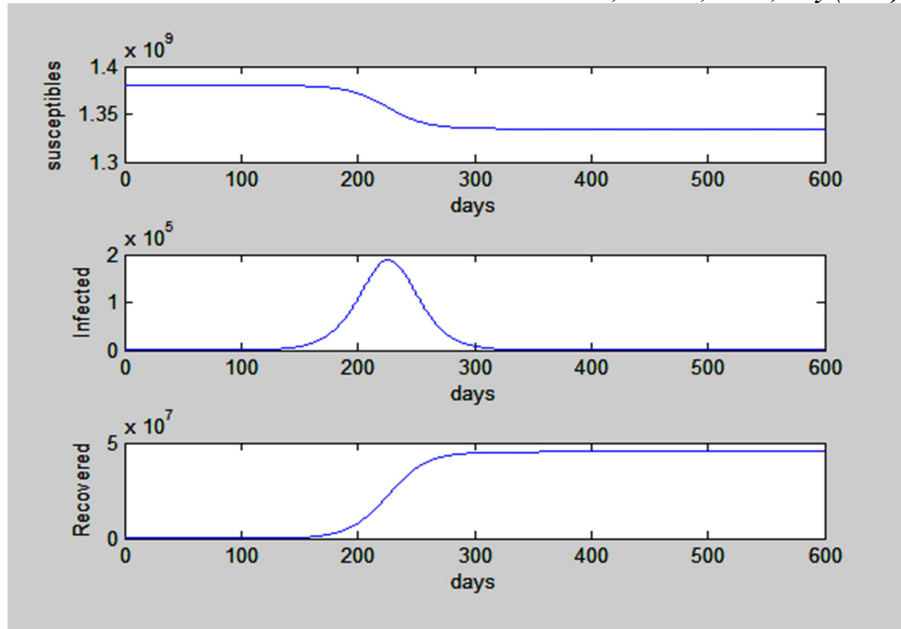


Fig 2. Solution of the model using best-fit parameter values

Note that the graphs fit well with our general intuition of how the graphs should look: the graph for S only decreases, the graph for R only increases, and the graph for I increases to a peak and then decreases.

The solution shows very interesting behaviour. The number $I(t)$ of infecteds grows initially (middle plot of Fig 2). It peaks near $t = 225$ with $I(225) \approx 1.89$ lacs, which is approximately 0.014% of total population. It occurs on 11 September 2020, and then quickly decreases to close to zero after 320 days. So, epidemic proceeds to its end 320 days after 30 Jan 2020, i.e., around middle of December 2020.

The number $S(t)$ of susceptibles (top plot of Fig 2) initially decreases and then almost levels off as $t \rightarrow \infty$. However, note that $S(t)$ does not tend to zero as $t \rightarrow \infty$. Rather, it tends toward $S \approx 1.33 \times 10^9$ (96.72% of total population). This 96.72% of total population never gets the disease, i.e., they do not fall prey to the disease even though they lack any special immunity.

Finally, the total in the R category at the end of the epidemic can be treated as total infected during the pandemic, because in order for someone to be removed, they must have first gotten sick. Accordingly, 4.53 crores (3.28% of total population) of people contract the disease during the course of the epidemic (bottom plot of Fig 2).

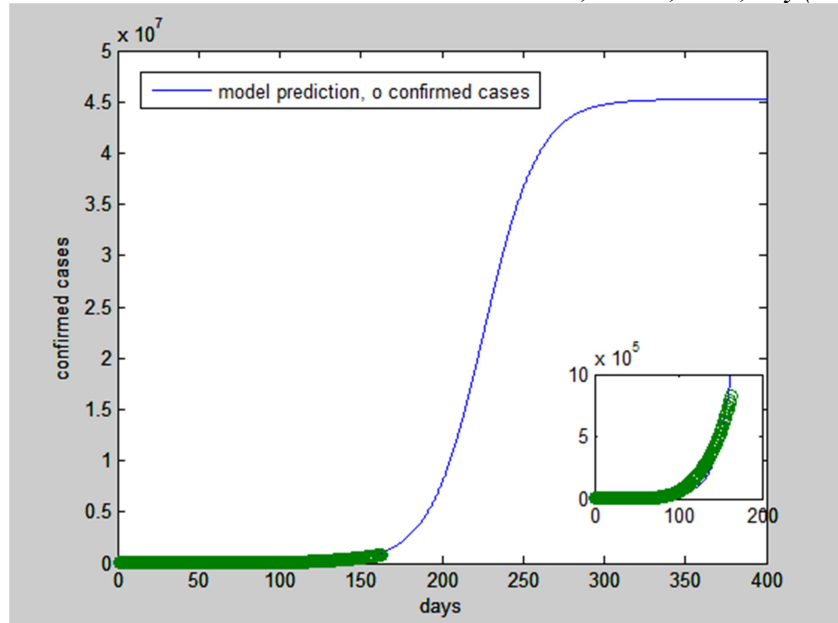


Fig 3. Model prediction with confirmed cases

Fig 3 shows an excellent agreement between confirmed cases with the present model.

IV. Limitations

Models have their limitations. It is impossible to build a fully accurate model: there always be some element of the host behaviour or quirk of the disease that is unknown or even unknowable. As such, the present study has several limitations (besides those mentioned in the Mathematical Model section). Population density variation across the entire country, weather variation from 30 Jan to 10 July 2020 (data collection period) [III], difference between the transmission dynamics of coronavirus disease before and after lockdown, levels of different preventive measures taken by state governments etc. are not considered in this study. Further, the epidemiological characteristic of COVID-19 is not clear. The present work is based on mathematical modeling perspective.

V. Conclusion

In the present numerical study, observed data of Covid-19 in India during the period 30 Jan to 10 July 2020 is analyzed using SIR model with an aim to predict the precise description of the disease, such as, the spread of the disease, the probable peak date, the total number infected, the duration of an epidemic and various epidemiological parameters. Values of the parameters of the differential equations of the model, as reported in literature, is initially used to solve them, and judged with observed data using Gauss-Newton's algorithm for least square fitting to find their

best fit values. Based on the projection as of 10 July 2020, the outcome of the present numerical study can be sum up as:

- The model predicts that the population that is infected will reach a maximum of approximately 0.014% (of total population) around the second week of September 2020.
- The population that contracts the disease during the epidemic is approximately 4.58 crores (3.28% of total population). Accordingly, 96.72% of total population escape the disease.
- Pandemic is expected to come to an end around middle of December 2020.

Transparency Declaration: The author declares no conflicts of interest.

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