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TOPOLOGICAL AND SPECTRAL ASPECTS OF MONOMIAL IDEALS OFSEMIRINGS

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Abstract

In this article, we introduce the monomial ideals of semirings and study some of its properties. Main objective of this articleis to investigate prime spectrum of monomial ideals of semirings and discuss its topology.

Keywords: Monomial Ideals, Prime Spectrum, Topological Semirings, Zariski Topology

I. Introduction

Semirings are generalization of rings and bounded distributive lattices. H.S Vandiver[IV] presented the idea of semirings in 1935. Semirings arise naturally in such diverse areas of Mathematics as Combinatorics, Functional Analysis, Topology, Graph Theory, Automata Theory, Formal Languages Theory, Mathematical Modelling of Quantum Physics and Computational Systems (see [II, VI, XIV]). Over the years, semirings have been studied by various researchers in an attempt to broaden techniques coming from ring theory or in connection with their applications. Here, we generalize the concept of monomial ideals in polynomial semirings. Our focus on monomial ideals is that these ideals are simplest, in the sense, since the generators have only one term each. Also the monomial ideals have incredible connection to the other areas of Mathematics. For instance, one can use monomial ideals to study certain objects in Combinatorics, Geometry, Graph theory and Topology [I, V, XV]. The prime spectrum of ideals of polynomial rings and its associated topology have been studied by different Mathematicians in rings, modules and lattices (see [III, XI, XII]). In this article, we study some properties of monomial

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ideals in polynomial semirings and prove that the monomial ideals of semirings are subtractive (k-ideals) and extraordinary. We study about the topology of prime monomial ideals of semirings and prove that $Spec(S)$ is $T_o - space$, compact space and it is Monomial Noetherian space. The notion of irreducible topology is due to R. Aren and J. Dugundgi [VIII]. In this connection, we show that primeness of Nil radicals is connected with irreducibility of the topology associated to prime monomial ideals.

II. Preliminaries

For completion, we recall some definitions that will be useful for the sequel. By a semiring $(S,+,.)$ we mean a nonempty set S equipped with two binary operations '$+$'and '$.$' such that $(S,+)$and $(S,.)$are semi groups with absorbing zero$'0'$, i.e. $a+0=0+a=a$ and $a.0=0.a=0$ for all $a \in S$ and the multiplication is both left andright distributive over addition. A semiring S is commutative if it is commutative with respect to multiplication. All semirings in this paper are commutative with identity. A semiring S is said to be a semidomain if $ab=0$ for some $a,b \in S$, then either $a=0$ or $b=0$. Asemifield is a semiring in which non-zero elements form a group under multiplication. An ideal I of S is $k-ideal$ if $x+y \in I$ with $x \in I$ implies $y \in I$. A proper ideal P of S is prime if and only if whenever $IJ \subseteq P$ for some ideals I,J of S implies that $I \subseteq P$ or $J \subseteq P$. Allen [VII] presented the notion of $Q-ideal$ I in the semiring S and constructed the quotient semiring S/I (also see [IX, X, XIII]). An ideal I of S is called extraordinary if whenever A and B are semiprime $k-ideal$ of S with $A \cap B \subseteq I$, then $A \subseteq I$ or $B \subseteq I$. If a semiring which satisfies the ascending chain condition on ideals $I_1 \subseteq I_2 \subseteq .... \subseteq I_n \subseteq .....,$ then there exists an $n \in N$ such that $I_n = I_{n+1}......$ then it is called Noetheriansemiring. A topological space X is said to be irreducible if $X \neq \phi$ and if every pair of non-voidopen sets in X intersects (see [VIII]). Let X be topological space and $A \subset X$ is said tobe dense in X if and only if $A \cap G \neq \phi$, for every non-void open subset $G \subset X$. Thus X is irreducible if and only if every non-void open subset of X is dense.

III. Monomial Ideals of Semirings

Let $S = R[x_1, x_2, ..., x_n]$ be the polynomial semiring over a commutative semi domain R with unity. Any product of indeterminate in the form $m_i = x_1^{\alpha_1} x_2^{\alpha_2} .... x_n^{\alpha_n}$ is a monomial with $\alpha_1, \alpha_2, ..., \alpha_n$ are positive integers. An ideal I of S is called a monomial ideal if it is generated by monomials. For convenience, we denote polynomial semiring with S, monomials with m_i (where $i \in \mathrm{N}$) and I for monomial ideals.

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Lemma 3.1. Let S be a polynomial semiring and $I =< m_i, i \in Z^+$ a monomial ideal in S. Then any polynomial $P \in I$ if and only if each term in P is divisible by some m_i.

Proof. It is certain that if each term in P is divisible by some monomial m_i then $P \in I$. On the other hand, suppose that $P \in I$, then $P = \sum_i a_i m_i$, where $a_i = \sum_{i,j} n_{ij}$, and each n_{ij} is monomial.Therefore $P = \sum_i \sum_j n_{ij} m_i$ .Hence each term is divisible by some m_i.

Proposition 3.2. Every Monomial ideal I in S is a $k-ideal$.

Proof. Let I be a monomial ideal in S and P_1, P_2 are two polynomials of S satisfying $P_1 + P_2 \in I$ and $P_2 \in I$. Then by using the fact that $P_1 + P_2$ is also polynomial and by previous lemma 3.1, each term of $P_1 + P_2$ is divided by some monomial generator m_i of I. This implies that each term of P_1 is also divisible by m_i. Hence $P_1 \in I$ thus I is $k-ideal$. k-ideal.

Lemma 3.3. Let $S = R[x_1, x_2, ..., x_n]$ be the polynomial semiring with identity. Then $S = R[x_1, x_2, ..., x_n]$ has atleast one Maximal Monomial ideal.

Proof. We have $< x_i >, i \in \{1, 2, ...n\}$ is proper monomial ideal of $S = R[x_1, x_2, ..., x_n]$, therefore the set X of all proper monomial ideals of S is non-empty. By using the inclusion relation which is partial order on X and by using Zorn's Lemma to this poset, a maximal member of $(X, \subset)$ is maximal monomial ideal of $S = R[x_1, x_2, ..., x_n]$.

Proposition 3.4. Let I be a proper monomial ideal of semiring S. Then I is prime monomial ideal if and only if SI is semidomain.

Proof. Let I is prime monomial ideal and $(m_1 + I).(m_2 + I) = I$ i.e. $m_1 m_2 + I = I$ for some polynomials $m_1, m_2 \in S$. Then by Lemma 3.2 we get that $m_1 m_2 \in I$.This implies that $m_1 \in I$ or $m_2 \in I$, since I is prime. Therefore, $m_1 + I = I$ or $m_2 + I = I$, hence SI is a semidomain. For converse, we suppose that SI is semidomain and $m_1, m_2 \in S$ such that $m_1 \notin I$ and $m_1 m_2 \in I$. Then $m_1 m_2 + I = (m_1 + I)(m_2 + I) = I$ and SI is semidomain therefore $m_2 + I = I$.By using Lemma 3.2, we get $m_2 \in I$, hence I is prime.

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Theorem 3.5 Let $S[x_1,x_2,......,x_n]$ be polynomial semidomain with S is a semidomain. Then $<x_1>\subset<x_1>+<x_2>\subset.........\subset<x_1>+<x_2>+....+<x_n>$ are strict and all these are prime.

Proof. As canonical to ring theory, this can be shown that $S[x_1,x_2,......,x_n]<\ x_1>+<x_2>+....+<x_j>\cong S\left[x_{j+1},\ x_{j+2},\ ......,\ x_n\right]$ which is a semidomain, since S is semidomain and a polynomial semiring over semidomain issemidomain. Hence the ideal $<x_1>+<x_2>+....+<x_n>$ is necessarily prime.

The collection of all prime monomial ideals of $S=R[x_1,x_2,......,x_n]$, where R is a semidomain, is called the spectrum of S and denoted by $Spec(S)$. It is obvious from previous theorem that $Spec(S)\neq\phi$. For any monomial ideal I of S, we collect the set of all proper prime monomial ideal P of S containing I, denoted by $C(I)$. i.e. $C(I)=\{P\in Spec(S): I\subset P\}$.

Theorem 3.6 Let $S=R[x_1,x_2,......,x_n]$ be polynomial semiring over semidomainR.Then

(i) $C(0)=Spec(S)$ And $C(S)=\phi$.

(ii) For any monomial ideal I generated by set of monomials m and $R(I)$ is radicalof I, then $C(m)=C(I)=C(R(I))$

(iii) $C(I)\cup C(J)=C(IJ)=C(I\cap J)$

(iv) If $\{E_i\}_{i\in\Delta}$ be any family of subsets of monomials of S, then $C\left(\bigcup_{i\in\Delta}E_i\right)=\bigcap_{i\in\Delta}C(E_i)$

(v) If $\{I_i\}_{i\in\Delta}$ is a family of monomial ideals of S, then $C\left(\sum_{i\in\Delta}I_i\right)=\bigcap_{i\in\Delta}C(I_i)$

Proof. (i) Is straight forward.

(ii). Suppose that I is the ideal generated by monomial m. Then $m\subset I\subset R(I)$ and we clearly have $C(R(I))\subset C(I)\subset C(m)$. However, I is the smallest idealcontaining m so that $P\in C(m)$ implies that $P\in C(I)$. Hence, $C(m)=C(I)$. Also, $R(I)$ is the intersection of all prime ideals containing I therefore if $P\in C(I)$, then $I\subset P$, whereas $I\subset R(I)\subset P$ this implies that

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$P \in C(R(I))$, hence $C(R(I)) = C(I)$. Therefore, for any monomial m we get that $C(m) = C(I) = C(R(I))$.

(iii). we have I and J are monomial ideals, therefore $IJ \subset I \cap J \subset I$ or $IJ \subset I \cap J \subset J$. This implies that $C(I) \cup C(J) \subset C(I \cap J) \subset C(IJ)$.

For converse, it suffices to show that $C(IJ) \subset C(I) \cup C(J)$. Let $P \in C(IJ)$, then $IJ \subset P$. If $I \subset P$, then $P \in C(I)$ and it is done. On the other hand if $I \not\subset P$ andthere exist some monomial $m_i \in I - P$. Then by taking a monomial $n_i \in J$, we get $m_i n_i \in IJ \subset P$. Here P is prime monomial ideal therefore $n_i \in P$ and hence $P \in C(J)$. Therefore $C(IJ) \subset C(I) \cup C(J)$ and we get the desired result.

(iv). Let $P \in C\left(\bigcup_{i\in\Delta} E_i\right)$. Then $\bigcup_{i\in\Delta} E_i \subset P$, this implies that $E_i \subset P$ for each $i \in \Delta$. Therefore $P \in C(E_i)$ for each $i \in \Delta$ i.e. $P \in \bigcap_{i\in\Delta} C(Ei)$. Hence $C\left(\bigcup_{i\in\Delta} E_i\right) \subset \bigcap_{i\in\Delta} C(E_i)$. Now let $P \in \bigcap_{i\in\Delta} C(E_i)$. This implies that $P \in C(E_i)$ for each $i \in \Delta$. Therefore $\bigcup_{i\in\Delta} E_i \subset P$ which conclude that $P \in C\left(\bigcup_{i\in\Delta} E_i\right)$ and we achieve the result.

(v). Let $P \in C\left(\bigcap_{i\in\Delta} E_i\right)$. Then $I_i \subseteq P$ for all $i \in \Delta$, therefore $\sum_{i\in\Delta} I_i \subseteq P$. This impliesthat $\bigcap_{i\in\Delta} C(I_i) \subseteq C\left(\sum_{i\in\Delta} I_i\right)$.

Conversely, suppose that $P \in C\left(\sum_{i\in\Delta} I_i\right)$. then $I_i \subset C\left(\sum_{i\in\Delta} I_i\right) \subset P$ for all $i \in \Delta$. therefore, we get $P \in C(I_i)$ for all $i \in \Delta$, hence $P \in \sum_{i\in\Delta} C(I_i)$. this gives thedesired result.

The following Corollary can be followed from Theorem 3.6(ii).

Corollary 3.7. Let I and J be two monomial ideals of polynomial semiring S. Then $C(I) \subset C(J)$ if and only if $R(J) \subset R(I)$.

Theorem 3.8. Every monomial ideal in $S = R[x_1, x_2, \ldots, x_n]$ is extraordinary.

Proof. Let P be any prime monomial ideal of $S = R[x_1, x_2, \ldots, x_n]$ and let I and J be semiprime monomial ideals of $S = R[x_1, x_2, \ldots, x_n]$ such that $I \cap J \subseteq P$. Then by theorem 3.6, $C(I) \cup C(J) = C(U)$ for some monomial ideal

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U of S .Here I is semiprime therefore $I=\bigcap_{i\in\Delta} P_i$, where P_i are prime monomial idealsof $S[x_1,x_2,.....,x_n]$. Therefore for each $i\in\Delta$, $P_i\in C(I)\subseteq C(U)$ so $U\subseteq P_i$. Thus $U\subseteq I$. In similar way, we get $U\subseteq J$. Thus $U\subseteq I\cap J$. Hence $C(I)\cup C(J)\subseteq C(I\cap J)\subseteq C(I)=C(I)\cup C(J)$. Therefore $C(I)\cup C(J)=C(I\cap J)$. This impliesthat $P\in C(I\cap J)$, this gives that P is prime monomial ideal so $I\subseteq P$ or $J\subseteq P$.

IV. Topology of Monomial Ideals

Consider τ, the collection of all sets U of $Spec(S)$ such that $U(I)=Spec(S)-C(I)$.hen from previous theorem it can be easily verified that τ satisfies all axioms oftopology and $U(I)$ are its open sets and $C(I)$ are closed sets of this topology. Forany monomial x of S, we have $B(x)=Spec(S)-C(x)$ is open set and next theoremwill show that it makes basis of this topology.

Theorem 4.1 Let x,y be non-nilpotent monomials of polynomial semirings S. then

(i). $B(m_1)\cap B(m_2)=B(m_1m_2)$

(ii). The collection $\beta=\{B(m): m \text{ is monomial of } S\}$ is basis of topology on $Spec(S)$

(iii). $B(m)=\phi$ if and only if monomial m is nilpotent.

Proof.(i).Let $A\in(B(m_1)\cap B(m_2))$.Then $A\in(Spec(S)-C(m_1))\cap(Spec(S)-C(m_2))$ $=Spec(S)-(C(m_1)\cup C(m_2))$.This implies that $m_1\notin A$ and $m_2\notin A$.If we consider that $m_1m_2\in A$, then primenessof A gives the contradiction. Hence $m_1m_2\notin A$, therefore $A\in B(m_1m_2)$, that is $B(m_1)\cap B(m_2)\subseteq B(m_1m_2)$. On the other hand, suppose that $A\in B(m_1m_2)$, thistells that $m_1m_2\notin A$, then $m_1\notin A$ otherwise $m_1m_2\in A$, since A is monomial ideal.Similarly $m_2\notin A$. It follows that $A\in B(m_1)\cap B(m_2)$, hence we get the requiredresult.

(ii). Let $A\in Spec(S)$. Since $A\neq S$, therefore there exist some monomial $m\in S-A$ that satisfies $m\notin A$. This implies that $A\notin C(m)$. Hence $A\in B(m)$ where $B(m)\in\beta$.Moreover, if $A\in B(m_1)\cap B(m_2)$ for some monomials m_1,m_2,

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then by using (i) weget $A \in B(m_1 m_2) = B(m_1) \cap B(m_2)$. Therefore β is basis of this topology.

(iii). Let m is nilpotent and $P \in Spec(S)$. Then for some $t \in \mathbb{N}$, we have $mt = 0 \in P$. Byprimeness of P we have $m \in P$. Therefore, $P \notin B(m)$ for all $P \in Spec(S)$ and itgives $B(m) = \phi$.

Conversely, let $B(m) = \phi$. Then for each $P \in Spec(S)$ we have $m \in P$, this impliesthat $m \in \bigcap_{P \in Spec(S)} P = Rad\{0\}$. Thus m is nilpotent monomial.

Theorem 4.2. $Spec(S)$ is a T_o space.

Proof. Let $P, P_1 \in Spec(S)$ with $P \neq P_1$. Considering $D(a) = \{P \in Spec(S) : a \notin P\}$. It is clearly viewed that $D(a)$ is a neighbourhood of P if and only if $a \notin P$. Supposethat $P_1 \in D(a)$, for all $a \notin P$. Then $a \in P_1$ implies that $a \in P$ that is $P_1 \subseteq P$ thisleads to contradiction. Now consider that $b \in P - P_1$. Then $b \notin P_1$ which gives $D(b)$ is a neighbourhood of P_1. Also $b \in P$ therefore $P \notin D(b)$. Hence $Spec(S)$ is T_o.

Theorem 4.3. $Spec(S)$ is compact.

Proof. By theorem 4.1(ii), we can assume that an open covering of $Spec(S)$ is ofthe form $\lambda = \{B(m_\alpha),\ m_\alpha \text{ is monomial}\}$ and $\alpha \in J$ with J is index set. We have $Spec(S) = \bigcup_{\alpha \in J} B(m_\alpha) = \bigcup_{\alpha \in J} (Spec(S) - C(m_\alpha)) = Spec(S) - \bigcap_{\alpha \in J} C(m_\alpha)$. This implies that

(1) $\bigcap_{\alpha \in J} C(m_\alpha) = \phi$

Also,

(2) $\bigcap_{\alpha \in J} C(m_\alpha) = \bigcap_{\alpha \in J} Spec(S) - B(m_\alpha) = Spec(S) - B(I) = C(I)$

Where I is monomial ideal generated by monomials $\{m_\alpha\}_{\alpha \in J}$. From (1) and (2), weget $C(I) = \phi$. Thus there doesn't exist any prime monomial ideal which contains $I = <m_\alpha, \alpha \in J>$ if and only if $C(I) = \phi \Leftrightarrow Spec(S) - C(I) = Spec(S)$. While $C(1) = \phi$ therefore there exist some monomial ideal $I = <m\alpha,\ \alpha \in J> = <1> = S$ therefore there exist a finite subset of monomials $m_1, m_2, \ldots\ldots, m_k$ of m_α and $\{x_1, x_2, \ldots\ldots, x_k\} \in S$ such that $m_1 x_1 + m_2 x_2 + \ldots.. + m_k x_k = 1$, since S is finitely generated. This implies that $I = <m_1, m_2, \ldots\ldots, m_k> = S$. Hence

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$C(I)=Spec(S)-B(I)=Spec(S)-\bigcup_{i=1}^{k}B(m_i)=\bigcap_{i=1}^{k}(Spec(S)-B(m_i)) =\bigcap_{i=1}^{k}C(m_i)=\phi$ and $Spec(S)=Spec(S)-\bigcap_{i=1}^{k}C(mi) =\bigcup_{i=1}^{k}(Spec(S)-C(m_i))=\bigcup_{i=1}^{k}B(m_i)$. Therefore $B(m_i)_{i=1}^{k}$ is a finite subcollection of $B(m_\alpha)$ that covers $Spec(S)$.Hence $Spec(S)$ is compact.

Recall that every topological space is distributive lattice of open sets thereforeit forms inverse semirings.

Proposition 4.4. Let $S=R[x_1,x_2,.....,x_n]$ bea polynomial semiring. If the semiring R isNoetherian, then $Spec(S)$ is Monomial Noetherian space.

Proof. Let $C(I_1)\supseteq C(I_2)\supseteq........\supseteq C(I_m)\supseteq.....$ be a decreasing sequence of closedMonomial sets. Then we may assume that $I_i=R(I_i)$ for $i\in\mathbb{N}$. By using Corollary3.7, there exist an ascending chain condition of monomial ideals $I_1\subseteq I_2\subseteq............\subseteq I_n\subseteq......$ By Hilbert Basis theorem in semiring [XV], $S=R[x_1,x_2,.....,x_n]$ is alsoNoetherian, so that there exist some positive $r\geq 0$ such that $I_r = I_{r+l}$ if $l\geq 1$. Thus $C(I_r)=C(I_{r+l})$. Hence $Spec(S)$ is Monomial Noetherian space.

Proposition 4.5. For the topological space $Spec(S)$ we have

(i). If Y is an irreducible subspace of $Spec(S)$, then the closure of Y is irreducible.

(ii).Every irreducible subspace of $Spec(S)$ is contained in a maximal irreduciblesubspace.

Proof.(i). Let Y be irreducible subspace of $Spec(S)$ and let U and V be arbitraryopen subsets of $\overline{Y}$. Then $U\cap Y, V\cap Y$ are nonempty subsets of Y, since Y is densein $\overline{Y}$. As U is open in $\overline{Y}$ therefore there exists open set $G\subseteq Spec(S)$ such that $U=G\cap\overline{Y}$. This implies that $Y\cap U=Y\cap(\overline{Y}\cap G)=Y\cap G$. Hence $Y\cap U$ is open in Y. Similarly $Y\cap V$ is open in Y. Therefore $(Y\cap U)\cap(Y\cap V)\neq\phi$. In particular, $U\cap V\neq\phi$. Hence $\overline{Y}$ is irreducible.

(ii). Let $E\subset Spec(S)$ be an irreducible subspace. Consider $\omega=\{M_i\subset Spec(S):M_i \text{ is an irreducible subspace and } E\subseteq M_i\}$. Clearly $\omega\neq\phi$.Now let C be a chain in ω and $M =\bigcup_{M_i\in C}M_i$.we shall show that $M\in\omega$, then theexistence of a maximal element in ω is assured by Zorn's

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lemma.Let $U, V \subseteq M$ then there exist $M_1, M_2 \in C$ such that $M_1 \cap U \neq \phi$ and $M_1 \cap V \neq \phi$. Here C is a chain so we may assume that $M_1 \subseteq M_2$ therefore $M_2 \cap U$ and $M_2 \cap V$ are opensubsets of M_2 and since M_2 is irreducible. This implies that $\phi \neq (M_2 \cap U) \cap (M_1 \cap V) \subseteq U \cap V$. Hence M is an irreducible subspace of $Spec(S)$ and since U, V are arbitrary. Also, $E \subseteq M$ therefore $M \in \omega$. Hence we get the required result.

Theorem 4.6. Let $S = R[x_1, x_2, ..., x_n]$ be polynomial semirings. Then the nil radical N in $S = R[x_1, x_2, ..., x_n]$ is a prime monomial ideal if and only if $Spec(S)$ is irreducible.

Proof. Suppose the nil radical N is prime monomial ideal of $S[x_1, x_2, ..., x_n]$. Let $U, V \subseteq Spec(S)$ be open subsets. Take some prime monomial ideals $P_u \in U$ and $P_v \in V$. Here $U = Spec(S) - C(E)$ for some monomial $E \subset S[x_1, x_2, ..., x_n]$. Thus $P_u \in U$ implies that $E \not\subset P_u$ and $N \subseteq P_u$ implies that $E \not\subset N$. Hence $N \in U$. Similarly,we can get $N \in V$. Therefore $N \in U \cap V$ that is $U \cap V \neq \phi$. Hence $Spec(S)$ isirreducible.On the other hand, suppose that N is not a prime monomial ideal of $S = R[x_1, x_2, ..., x_n]$ then there exist some monomials $m_1, m_2 \in Spec(S) - N$ such that $m_1 m_2 \in N$. If $m_1 \notin N$, then $Spec(S) \neq C(m_1)$, this implies that $Spec(S) - C(m_1) \neq \phi$. Similarly, we can prove that $Spec(S) - C(m_2) \neq \phi$. Both $Spec(S) - C(m_1) \neq \phi$ and $Spec(S) - C(m_2) \neq \phi$ are open in $Spec(S)$. However, $B(m_1) \cap B(m_2) = B(m_1 m_2)$ $= Spec(S) - C(m_1 m_2) \subseteq Spec(S) - N = \phi$. Hence $Spec(S)$ is not irreducible and thisconcludes the proof.

V. Concluding Remarks

This article introduces the notion of monomial ideals of semirings and discussestheir spectral properties. Moreover it discusses the topology associated with themonomial ideals. The concepts presented in this article have a lot of potential for flourishing along with topological and algebraic entities. Therefore this article isvery useful as it invites the researchers to work on it to explore more on algebraicand topological grounds.

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