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ON DIVERSITY OF GENERALIZED REVERSE DERIVATIONS IN RINGS

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Abstract

In this article, we study the diversity in generalized reverse derivation by defining L, R* and (α,β)-*- Generalized reverse derivation in rings. We introduce some conditions which make these generalized reverse derivations and their associated *-reverse derivations to be commuting. Moreover, we discuss the conditions on these mappings that enforce the rings to be commutative.*

Keywords: Reverse derivations, Prime rings, Semiprime rings, Involution.

I. Introduction

The derivation is defined as additive mappings d: R → R (i.e., endomorphism of the additive group of R) such that d(xy) = d(x)y + xd(y) for all x,y ∈ R. Herstein, introduced reverse derivation as additive mappings d: R → R if d(xy) = d(y)x+yd(x) for all x,y ∈ S and study the commutatively of rings with reverse derivations (see [VI, VII]).Furthermore, Brˇesar[XV]generalize the concept of derivation by introducing an additive mapping D: R → R associated to a derivation d: R → R such that D(xy) = D(x)y + xd (y) for all x,y ∈ R, and D is termed as generalized derivation on R. Such mappings were studied by several authors and obtained commutatively of prime and semiprime rings in which derivations or generalized derivations satisfy certain functional identities or have some additional properties (see [XII, XIII, XVI, XVIII]). Gölbasi and Kaya [V] represent the definition of Breˇsargeneralized derivation as left-generalized derivation associated to a derivation d and discriminate it from right-generalized derivations associated to d, termed as the additive mappings Δ : R → R satisfying Δ(xy) = d(x)y + xΔ(y) for all x,y ∈ R.In [XIV], Breˇsar and Vukman defined *-derivation to be an additive map d of R satisfying d(xy)=d(x)y* +xd(y) for all x,y∈R, where * is an anti-automorphism of period 2 on a ring R and is said to be an involution. A ring R equipped with * is called a ring with involution or *- ring.S. Ali [XVII] defined the generalized *-derivation as an additive mapping F:R→R associated with a *-derivationd of RifF (xy)=F(x)y* +xd(y) for all x,y∈R.

Recently, Aboubakr and Gonzalez [I] generalized the notion of reverse derivation to generalized reverse derivation defined as an additive mapping Δ : R → R, termed as left generalized reverse derivation (right generalized reverse derivation) associated with reverse derivation δ, if it holds Δ(ab) = Δ(b)a + bδ(a) (Δ(ab) = δ(b)a + bΔ(a)) for all a, b$\in$ R. The reverse derivations in the case of prime Lie and prime Malcev algebras were studied by Hopkins and Filippov. Those papers provided some examples of nonzero reverse derivations for the simple 3-dimensional Lie algebra sl-2 (see VIII) and characterized the prime Lie algebras admitting a nonzero reverse derivation (see [II, III]). In particular, Filippov proved that each prime Lie algebra, admitting nonzero reverse derivation is a PI-algebra. Filippov also described all reverse derivations of prime Malcev algebras [IV]. The supercase of reverse derivations (anti super derivations) of simple Lie super algebras was studied by Kaygorodov in [IX, X]. He proved that every reverse superderivation of a simple finite-dimensional Lie superalgebra over an algebraically closed field of characteristic zero is the zero mapping. After that, Kaygorodov proved that every r-generalized reverse (or l-generalized) derivation of a simple (non-Lie) Malcev algebra is the zero mapping (see [XI]).

We extend the idea of Gölbasi and Kaya [V] to reverse derivations in ring with involution and define the additive mapping dfrom a ring R to R which is termed as *-reverse derivation satisfying $d(xy)=d(y)x^*+yd(x)$ for all $x,y\in R$. The notion of L*-generalized reverse derivation (R*- generalized reverse derivation) are introduced and denotes for additive mappings Δ: R $\rightarrow$ R associated with *-reverse derivation δsatisfies$\Delta(xy) = \Delta(y)x^* + y\delta(x)$ $(\Delta(xy) = \delta(y)x^* + y\Delta(x))$for all $x,y \in R$. Furthermore, the additive mapping Δ: R $\rightarrow$ R is termed to be (α,β)-*- generalized reverse derivation ((α,β) generalized reverse derivation) if it satisfies $\Delta(xy) = \Delta(y)\alpha(x^*) + \beta(y)\delta(x)$ $(\Delta(xy) = \Delta(y)\alpha(x) + \beta(y)\delta(x))$for all $x, y \in R$.

Here, we study the discrimination in characteristics of L* and R*-generalized reverse derivations in semiprime rings. We prove that if there exist non-zero L*-generalized reverse derivations with associated *-reverse derivation δ in semiprime ring then $\delta(R)$ contain in Z(R). On the other hand, the existence of R*-generalized reverse in semiprime rings ensure that $\Delta(R)$ contain in centre Z(R). We proved that if Δ is a L*-generalized reverse (or R*-generalized reverse)derivation on a prime ring R, then it is L*- generalized (or R*-generalized) derivation. Also, we discuss the functional condition $\Delta([x,x^*]) = 0$for commutatively in prime rings byR*-reverse generalized derivation. Furthermore, we study that(α, β)-generalized reverse derivation and (α, β)-*-generalized reverse derivation enforces the prime ring to be commutative.

Throughout this paper R denotes an associative ring with involution and its centre is denoted by Z(R). Recall that R is prime if $aRb = \{0\}$ implies that $a = 0$ or $b = 0$. The ring R is semiprime if $aRa = 0$ implies $a = 0$.As usual, $[x,y]$ denotes the commutator$xy-yx$. We will make extensive use of the basic Jacobi identities $[xy,z] = x[y,z] + [x,z]y$ and $[x,yz] = y[x,z] + [x,y]z$.

Theorem I

Suppose that R is a semiprime rings with involution and Δ is a L*-generalized reverse derivation associated with *- reverse derivation δ, then δ maps R into the centre Z (R).

Proof

By assumption, we have

$$\Delta(xy) = \Delta(y)x^* + y\delta(x), \quad \text{for all } x,y \in R \tag{1}$$

On substituting xz in place of x in (1), we have

$\Delta(xzy) = \Delta((xz)y) = \Delta(y)(xz)^* + y\delta(xz)$

$$\Delta(xzy) = \Delta(y)z^*x^* + y\delta(z)x^* + yz\delta(x) \tag{2}$$

Also, $\Delta(xyz) = \Delta((x(zy)) = \Delta(zy)x^* + zy\delta(x)$, this implies that

$$\Delta(xyz) = \Delta(y)z^*x^* + y\delta(z)x^* + zy\delta(x) \tag{3}$$

From (2) and (3), we get

$$[y,z]\delta(x) = 0 \tag{4}$$

On replacing y by $\delta(x)y$ in (4) to obtain $[\delta(x)y,z]\delta(x) = 0$ or

$$\delta(x)[y,z]\delta(x) + [\delta(x),z]y\delta(x) = 0 \tag{5}$$

By using (4), we get

$$[\delta(x),z]y\delta(x) = 0 \tag{6}$$

Now replacing y by yz in (6), we have

$$[\delta(x),z]yz\delta(x) = 0 \tag{7}$$

On multiplying (6) by z, implies

$$[\delta(x),z]y\delta(x)z = 0 \tag{8}$$

From (7) and (8) we get $[\delta(x),z]y[\delta(x),z] = 0$. By using semi primeness of R, we have $[\delta(x),z] = 0$ for all $z \in R$. Hence $\delta(R)$ contain in $Z(R)$.

Theorem II

Suppose that R is a prime ring with involution. If there exists a L*-generalized reverse derivation Δ associated with *- reverse derivation, then Δ is L*-generalized derivation.

Proof

From theorem I, we have

$$[y,z]\delta(x) = 0, \ \forall\, x,y,z \in R \tag{9}$$

On replacing xy instead of y in (9), we have $x[y,z]\delta(x) + [x,z]y\delta(x) = 0$. By using (9) again we have $[x,z]y\delta(x) = 0$. This implies that

$$[x,z]\, R\delta(x) = 0 \tag{10}$$

By primeness, we have $[x,z] = 0$ or $\delta(x) = 0$. Therefore, we have two subsets $A = \{x \in R | [x,z]=0,\ z \in R\}$ and $B = \{x \in R \,|\delta(x) = 0\}$ which are subgroups such that $A \cup B = R$, this leads to contradiction therefore we get either A = R or B = R. If A =R, i.e $\delta(x)=0$, $x \in R$ which is contrary to assumption, therefore A = R implies that

$[x,z]=0$, for all $x,z \in R$. Thus R is commutative. Hence $\Delta(xy) = \Delta(yx) = \Delta(x)y^* + x\delta(y)$ which is L*- generalized derivation.

Theorem III

Suppose that R is a semiprime ring with involution. If Δ is R*- generalized reverse derivation associated with *-reverse derivation δ, then Δ maps R into centre of R.

Proof

By hypothesis, we have

$$\Delta(xy) = \delta(x)y^* + y\Delta(x) \tag{11}$$

Replacing y by yz in (11), we have $\Delta(xyz) = \delta(z)(xy)^* + z\Delta(xy)$ or

$$\Delta(xyz) = \delta(z)y^*x^* + z\delta(y)x + zy\Delta(x) \tag{12}$$

On the other hand, by associativity of R, we get $\Delta(xyz) = \delta(yz)x^* + yz\Delta(x)$ or

$$\Delta(xyz) = \delta(z)y^*x^* + z\delta(y)x^* + yz\Delta(x) \tag{13}$$

From (12) and (13), we have

$$[y,z]\Delta(x) = 0 \tag{14}$$

By replacing y with $\Delta(x)y$, we get

$$\Delta(x)[y,z]\Delta(x) + [\Delta(x),z]y\Delta(x) = 0$$

By using (14), we obtain

$$[\Delta(x),z]y\Delta(x) = 0 \tag{15}$$

On substituting yz in place of y yields

$$[\Delta(x),z]yz\Delta(x) = 0 \tag{16}$$

Multiplying (15) by z from right to get

$$[\Delta(x),z]y\Delta(x)z = 0 \tag{17}$$

From (16) and (17), we get $[\Delta(x),z]R[\Delta(x),z] = 0$. Since R is semiprime, we get that $[\Delta(x),z] = 0$, $\forall\, x,z \in R$. Hence $\Delta(R)$ contain in $Z(R)$.

Theorem IV

Consider R is prime ring with involution. If R admits R*-generalized reverse derivation Δ, associated with *-reverse derivation δ, then Δ is R*-generalized derivation.

Proof

From (14) of Theorem III, we have

$$[y,z]\Delta(x) = 0 \tag{18}$$

Replace y by xy in (18) and using it again, we get $[x,z]y(x) = 0$ for all $y \in R$ or $[x,z]R\Delta(x) = 0$.

By primeness of R, we get, $[x,z] = 0$ or $\Delta(x) = 0$, therefore we have two subsets $A = \{x \in R | [x,z] = 0, \forall z \in R\}$ and $B = \{x \in R | \Delta(x) = 0\}$. Then by Brauer's trick, either A = R or B = R. Here B = R gives contradiction. Therefore, we have A = R then $[x,z] = 0, \forall x,z \in R$, i.e R is commutative. Hence $\Delta(xy) = \Delta(yx) = \delta(x)y^* + x\Delta(y)$, which is R*-generalized derivation.

Theorem V

Let R be prime ring with involution with $char(R) \neq 2$ and $S \cap Z \neq \{0\}$.and Δ is a R*- generalized reverse derivation associated with any *-reverse derivation δ, if it satisfies $\Delta([x,x^*]) = 0$, then R is commutative.

Proof

Replace x with k+h, with $h \in H$, $k \in S$,

$$\Delta([h,k]) = 0 \tag{19}$$

Taking $h = k_0k_1$, where $k_0 \in S$, $k_1 \in S \cap Z$, we get, $\Delta([k_0k_1,k] = 0$. This implies that $\Delta([k_0,k]k_1) = 0$, therefore $\alpha(k_1)[k_0,k] + k_1\Delta([k_0,k]) = 0$ or

$$\Delta[k_0,k]k_1 + \delta(k_1)[k_0,k] = 0 \tag{20}$$

Substitute h_0k_1 fork in (20), where $h_0 \in H$, $K_1 \in S \cap Z$, we get

$$\Delta([k_0,h_0k_1])k_1 + [k_0,h_0k_1]\,\delta(k_1) = 0$$

$$\Delta([k_0,h_0]k_1)k_1 + [k_0,h_0]\,\delta(k_1)k_1 = 0$$

$$k_1\,\delta(k_1)[k_0,h_0]^* + k_1^2\Delta([k_0,h_0]) + [k_0,h_0]\,\delta(k_1)k_1 = 0$$

By using (19), we have $k_1\delta(k_1)[k_0,h_0] + k_1\,\delta(k_1)[k_0,h_0] = 0$. Since $char(R) \neq 2$, this implies that

$[k_0,h_0]\,\delta(k_1)k_1 = 0$, $k_0 \in S$, $h_0 \in H$, $k_1 \in S \cap Z$. As centre of prime ring is free of zero divisor, therefore, $[k_0,h_0] = 0$ or $\delta(k_1)k_1 = 0$.

If $[k_0,h_0] = 0$, $k_0 \in S$, $h_0 \in H$, then by lemma2.1 [7], we have R is commutative. On the other hand, let $\delta(k_1)k_1 = 0$, $k_1 \in S \cap Z$ therefore, $\delta(k_1) = 0$ or $k_1 = 0$. Since $k_1 = 0$ also implies that, $\delta(k_1) = 0$ for all $k_1 \in S \cap Z$. Thus (20), reduces to

$\Delta([k_0,k])k_1 = 0$, for all $k_0, k \in S$

$$\Delta([k_0,k]) = 0 \tag{21}$$

By taking $2x = h + k$, for all $h \in H$, $k \in S$, therefore, from (19) and (21), we get

$$\Delta([x,k]) = 0, \text{ for all } k \in S, x \in R \tag{22}$$

By substituting $k = hk_1$ in (22), we have $\Delta([x,hk_1]) = 0$ or $\Delta([x,h]k_1) = 0$, where $k_1 \in S \cap Z$. This implies that $\delta(k_1)[x,h]^* + k_1\Delta([x,h]) = 0$. By assumption $\delta(k_1) = 0$, therefore $k_1\Delta([x,h]) = 0$. By using primeness of R, we get

$$\Delta([x,h]) = 0 \tag{23}$$

From (22) and (23), we get that

$$\Delta([x,y]) = 0, \text{ for all } x,y \in R \tag{24}$$

this implies that $\Delta([x,yx]) = 0$ or $\Delta([x,y]x) = 0$ or $\delta(x)[x,y]^* + x\Delta([x,y]) = 0$. This yields that $\delta(x)[x,y]^* = 0$ or $[x,y](\delta(x))^* = 0$. By taking $x=xr$, we get $[xr,y](\delta(x))^* = 0$ or $[x,y]r(\delta(x))^* = 0$ or

$$[x,y]R(\delta(x))^* = 0$$

this implies that $[x,y] = 0$ or $(\delta(x))^* = 0$ i.e, $\delta(x) = 0$. If $\delta(x) = 0$, for all $x \in R$ then, $\Delta(xy) = \delta(y)x^* + y\Delta(x) = y\Delta(x)$. Hence $\Delta([x,yz]) = 0$ or $\Delta([x,y]z + y[x,z]) = 0$ or

$$\delta(z)[x,y] + z\Delta[x,y] + \delta[x,z]y + [x,z]\Delta(y) = 0$$

$$[x,z]\Delta(y) = 0$$

Hence, $\Delta(y)R[x,z] = 0$, for all $x,y\in R$. By primeness of R, we have, $\Delta = 0$ or R is commutative.

Theorem VI

Suppose that R is prime ring and Δ is (α,β)-generalized reverse derivation and if $\beta$is an automophism on R, then R is commutative.

Proof

By hypothesis, we have $\Delta(xy) = \Delta(y)\alpha(x) + \beta(y)\delta(x)$

Now consider$\Delta(x(yz)) = \Delta(yz)\alpha(x) + \beta(yz)\delta(x)$, this gives

$$\Delta(x(yz)) = \Delta(z)\alpha(y)\alpha(x) + \beta(z)\delta(y)\alpha(x) + \beta(y)\beta(z)\delta(x) \quad (25)$$

Also, by associativity of R, we get $\Delta((xy)z) = \Delta(z)\alpha(xy) + \beta(z)\delta(xy)$

$$\Delta((xy)z) = \Delta(z)\alpha(x)\alpha(y) + \beta(z)\delta(y)\alpha(x) + \beta(z)\beta(y)\delta(x) \quad (26)$$

From (25) and (26), we get

$$\Delta(z)\alpha([x,y]) + \beta([z,y])\delta(x) = 0 \quad (27)$$

By taking $y = x$, we have

$$\beta([z,x])\delta(x) = 0 \quad (28)$$

Replace z by zy in (28) and using it again, we get$\beta([z,x])\beta(y)\delta(x) = 0$. Here$\beta$ is an automophism, therefore we can write$[z,x]R\delta(x) = 0$. By using primeness, we get, $x \in Z(R)$ or $\delta(x) = 0$, $\forall\ x \in R$. This implies that we have two sets$A = \{h\in R|h\in Z\}$ and $B = \{h\in R|\delta(h) = 0\}$where either R=A or R=B.If R=B, then $\delta= 0$, which is a contradiction.Hence R=A, therefore R is commutative and$\Delta(xy) = \Delta(yx) = \Delta(x)\alpha(y) + \beta(x)\delta(y)$ is an (α,β) generalized derivation.

Corollary VII

If α,β are automorphisms in the (α,β)-generalized reverse derivation Δ, then in non-commutative prime ring the $\Delta= 0$.

Proof

By taking $y = z$ in (21), we get$\Delta(z)\alpha([x,z]) = 0$**.** After replacing z by yz, we obtain

$$\Delta(z)\alpha(y)\alpha([x,z]) = 0$$

Hereα is automorphism, therefore we have$\Delta(z)R[x,z] = 0$. By primeness of R, and using Brauer's trick either$\Delta(z)=0$ or $[x,z]=0$, here R is non-commutative this implies that $\Delta = 0$.

Theorem VIII

Suppose that R is prime ring with involution and Δ be (α,β)-*- generalized reverse derivation on R, then R is commutative.

Proof

Consider

$$\Delta(x(yz)) = \Delta(yz)\alpha(x^*) + \beta(yz)\delta(x)$$

$$\Delta(x(yz)) = \Delta(z)\alpha(y^*)\alpha(x^*) + \beta(z)\delta(y)\alpha(x^*) + \beta(y)\beta(z)\delta(x) \quad (29)$$

On the other hand

$$\Delta((xy)z) = \Delta(z)\alpha((xy)^*) + \beta(z)\delta(xy)$$

$$\Delta((xy)z) = \Delta(z)\alpha(y^*)\alpha(x^*) + \beta(z)\delta(y)\alpha(x^*) + \beta(z)\beta(y)\delta(x) \quad (30)$$

From relations (29) and (30), we have $\beta([y,z])\delta(x) = 0$. On substituting $y = xy$, we get

$$\beta([x,z])\beta(y)\delta(x) = 0$$

Here β is automorphism, then by primeness of R we have $[x,z] = 0$ or $\delta(x) = 0$.

Using Brauer's trick, we get that if $\delta \neq 0$, then R is commutative.

Theorem IX

Let R be semi prime, α is homomorphism and β is automophism in (α,β)-*-reverse generalized derivation Δ on R, then $\Delta(R)$ contain in Z(R).

Proof

Consider

$$\Delta(r(st)) = \Delta(st)\alpha(r^*) + \beta(s)\beta(t)\delta(r)$$

$$\Delta(r(st)) = \Delta(t)\alpha(s^*)\alpha(r^*) + \beta(t)\delta(s)\alpha(r^*) + \beta(s)\beta(t)\delta(r) \quad (31)$$

On the other hand,

$$\Delta((rs)t) = \Delta(t)\alpha((rs)^*) + \beta(t)\delta(rs)$$

$$\Delta((rs)t) = \Delta(t)\alpha(s^*)\alpha(r^*) + \beta(t)\delta(s)\alpha(r^*) + \beta(t)\beta(s)\delta(r) \quad (32)$$

From (31) and (32), we obtain

$$\beta([s,t])\delta(r) = 0 \quad (33)$$

Replacing s by $\beta^{-1}(\delta(r))\beta^{-1}(s)$, we have $\beta[\beta^{-1}(\delta(r))\beta^{-1}(s),t]\delta(r) = 0$.By Jacobi identity, we have

$$\beta(\beta^{-1}(\delta(r))[\beta^{-1}(s),t] + [\beta^{-1}(\delta(r)),t]\beta^{-1}(s))\delta(r) = 0$$

This implies that

$$(\delta(r)\beta([\beta^{-1}(s),t]) + \beta([\beta^{-1}(\delta(r),t])s)\delta(r) = 0$$

By using (33), we get$\beta([\beta^{-1}(\delta(r)),t])s\delta(r) = 0$.Now taking $t = \beta^{-1}(t)$, we have,

$$\beta([\beta^{-1}(\delta(r)), \beta^{-1}(t)])s\delta(r) = 0$$

$$\delta(r),t]s\delta(r) = 0 \tag{34}$$

Replace s by st, we have

$$[\delta(r),t]st\delta(r) = 0 \tag{35}$$

On multiplying (34) by t from right to get

$$[\delta(r),t]s\delta(r)t = 0 \tag{36}$$

From (35) and (36), we have$[\delta(r),t]s[\delta(r),t] = 0$**.** By primeness of R, we get $[\delta(r),t] = 0$, that is $\delta(R)$ contain in $Z(R)$.

The following result can be easily shown by taking $\Delta = \delta$.

Corollary X

If R is semi prime and δ is (α,β)-*- reverse derivation, then δ is (β,α)-*-derivation.

Conclusion

We have discussed the commutatively of rings and commuting conditions on reverse derivations from different types of Generalized reverse derivations defined here. These Generalized reverse derivations pave new paths of research on reverse derivations in prime rings. These derivations may also be helpful for prime Lie algebra to bring it to be PI-algebra. Furthermore the characterization of these mappings in Malcev algebra and BCK algebra would be interesting for researchers.

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