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THE IMPLEMENTATION OF DIFFERENTIAL SUBORDINATION AND SUPERORDINATION THEOREMS FOR ACHIEVING POSITIVE ANALYTIC FUNCTIONS

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Abstract

Suppose assume that complex numbers such that $\alpha \neq 0, \alpha, \beta, \gamma$ *as well as* δ *define* φ on $D = C\{0\}$ *as well as* $\varphi(w, zw; a) = w^{\delta}(\beta w + \alpha \frac{zw'}{w} + \gamma), z \in E,$ *Where* $E = \{z; |z| \prec 1\}$. *The conditions are satisfactory for analytic function* $p, p(z) \neq 0$ *Cauchy's Riemann equations are satisfied for the functions* $q_1, q_2(z) \neq 0$ *in in E such that* $\phi(q_1(z), zq_1(z), zq_1(z); z) \prec \phi(p(z), zp'(z); z \prec \phi(q_2)z, zq'_2(z) \to q_1(z) \prec p(z) \prec (q_2)z.$ *Here observe that* q_1 and q_2 *the most excellent subordinate and best leading. The applications are applied of those results are equivalent;* $\varphi \approx$ like as well as $p \approx$ va ln et *ask as well as results to generalize and number of known results. By using a method based upon the Briot-Bouquet differential subordination, we prove several subordination results involving starlike and convex functions of complex order. Some special cases and consequences of the main subordination results are also indicated [I]. The main object of the present sequel to the aforementioned works is to apply a method based upon the Briot-Bouquet differential subordination in order to derive several subordination results involving starlike and convex functions of complex order[II],[III]. We also indicate some interesting special cases and consequences of our main subordination results.*

Keywords: convex function; Star like function $\varphi \mapsto$ *like* function, Differential subordination, Differential super ordination.

I. Introduction

A denotes a new class of the functions of the notation is $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ to satisfies the analytic and univalent of the disk $U = \{z \in C : |z| < 1\}$.. If f and g function of analytic at U[II],[III], we defined as f is subordinate to g, i.e., $f(z) \prec g(z)$. We use Schwarz function $w(z)$, which is an analytic function of U with $w(z) = 0$ *and* $|w(z)| < 1, \forall z \in U$. *Such that* $f(z) = g(w(z))z \in U$, *suchthat* $f(z) = f(w(z))z \in U$.

Additionally, the meaning g is univalent in U or Equivalence. $f(z) \prec g(z) \Leftrightarrow f(0) = g(0)$ *as well as* $f(U) \subset g(U)$. We defined as the functions $f(z)$ as well as [IV],[V] $g(z)$ *is* $g(z) = z + \sum_{k=2}^{\infty} b_k z^k$. We apply convolution theorem

$(f * g)(z) = z + \sum_{k=2}^{\infty} a_k z^k = (g * f)(z)$.

The parameters of complex numbers are $\alpha_1, \alpha_2, \ldots, \alpha_q$ *as well as* $\beta_1, \beta_2, \beta_3, \ldots, \beta_s$. Here ($\beta_j$ *does not belongs* $\bar{z}_0 = \{0, -1, -2, \ldots\}, i = 1, 2, 3, \ldots$ We developed the hyper geometric function generalization of F if [III],[VI]

$F_S\,(\alpha_1, \alpha_2, \ldots, \alpha_q$ *as well as* $\beta_1, \beta_2, \beta_3, \ldots, \beta_s; z)$

$= \sum_{k=0}^{z} \frac{(\alpha_1)k-1)\ldots(\alpha_q)k}{(\beta_1,)k-1\ldots(\beta_j)k-1} \frac{z^k}{k!}$ $(q \le s+1, q, s \in N_0 = N\,U\{0\}, N = \{1, 2, \ldots\} z \in U$.

Let, $h_{q,s}(\propto_1, \beta_1) f(z) = f(z) * h_{q,s}(\propto_1, \beta_1; z); z) = z + \sum_{k=0}^{\infty} \frac{(\propto_1)k-1 \ldots (\propto_q)k}{(\beta_1)k-1 \ldots (\beta_j)k-1} z^k$, *as well as by means of the Hadmard product, the operator of the definition* [VII],[VIII].

$$I_{q,s\lambda}^{m,l}((\propto_1, \beta_1) f : U \to U \; by \; I_{q,s,\lambda}^{0,l}((\propto_1, \beta_1) f(z) = f(z) * h_{q,s}(\propto_1, \beta_1, z);$$

$$I_{q,s,\lambda}^{0,l}\left((\propto_1, \beta_1) f(z) = (1-\lambda) f(z) * h_{q,s}(\propto_1, \beta_1, z)\right) + \frac{\lambda}{(1+l) z^{l-1}} (z^l f(z) * h_{q,s}(\propto_1, \beta_1, z)'$$

$I_{q,s\lambda}^{m,l}((\propto_1, \beta_1) f(z) = I_{q,s,\lambda}^{m,l}(I_{q,s,\lambda}^{m-1,l}(\propto_1, \beta_1) f(z)$ The mechanist of $I_{q,s\lambda}^{m,l}((\propto_1, \beta_1)$ was introduced by El-Ashwah and Aouf.

$I_{q,s\lambda}^{m0}((\propto_1, \beta_1) f(z)$
$= D_\lambda^m((\propto_1, \beta_1) f(z)$ *was studied by Selvaraj and karthikeyan.*

II. Preliminares

The Proofs of main results and lemmas are used follows.[X],[XII],[XII].

Definition 1 The set of functions *p as well as on injective , analytic then* $\overline{\mathbb{E} \setminus \mathbb{B}(p)}$,where$\mathbb{B}(p) = \{\zeta \in \partial\mathbb{E} : \log_{z\to\zeta} p(z) = \infty\}$, *as well as* $\exists\, p'(\zeta) \neq 0$ *for* $\zeta \in \partial\mathbb{E} \setminus \mathbb{B}(p)$.

Lemma 1

If $\mathbb{E}$and θ *are Equivalent and* φ *be and abalytic ina domain* D *containing* $q(\mathbb{E})$, *with* $\varphi(w) \neq 0$, *when* $w \in q(\mathbb{E})$. *Here* q *is an univalent.* $Q_1(z) = zq'(z), h(z) = \theta\{a(z)\} + Q_1(z)$and expect that either

$(i) h\ is\ conver, or\ (ii) Q_1 is\ starlike. In\ addition, assume\ that\ (iii)\Re\frac{zh'(z)}{Q_1(z)} > 0, z \in \mathbb{E}. If\ p\ is\ ananlytic\ in \mathbb{E}, with\ p(0) = q(0), p(\mathbb{E}) \subset D and\ \theta\{p(z)\} + zp'(z)\varphi(p(z) \prec \theta\{q(z)\}, then\ p \prec q\ as\ well\ as\ q\ is\ the\ best\ dominant.$

Lemma 2 Suppose $\mathbb{E}$ and $\theta\ are\ Equivalent\ and\ \varphi be\ and\ abalytic\ ina\ domain\ D$ contains $q(\mathbb{E})$. $Q_1(z) = zq'(z), h(z) = \theta\{a(z)\} + Q_1(z)$as well as expect that (i) $Q_1 is\ starlike\ in$ $\mathbb{E}$ as well as (ii) $\mathfrak{R}\frac{\theta'(q(z))}{\varphi(q(z))} > 0, z \in \mathbb{E}. If\ p \in TH[(0), 1] \cap Q, with\ p(\mathbb{E}) \subset Dand\ \theta[p(z)] + zp'(z)\varphi[p(z)] is\ univalent\ in\ \mathbb{E}\ and\ \theta[q(z)] + zq'(z)\varphi[q(z)] \prec \theta[p(z)] + zp'(z)\varphi[p(z)]$ then $q \prec p\ as\ well\ as\ q\ is\ the\ best\ subordinan$ [XIII], [XIV].

III. Main Theorems

Theorem 1 Let $q, q(z) \neq 0, The\ function\ of\ \mathbb{E}\ is\ univalent\ \exists\ (i)[1 + \frac{zq''(z)}{q(z)} + \frac{(\delta-1)zq'(z)}{q(z)}] > 0\ as\ well\ as\ (ii)\ [1 + \frac{zq''(z)}{q(z)} + \frac{(\delta-1)zq'(z)}{q(z)} + \frac{\beta(\delta+1)q(z)}{\alpha} + \gamma\delta/\alpha] > 0$. If p is the analytic function, then p(z) $\neq 0, z \in \mathbb{E}$ is satisfies the degree of difference subordination $\varphi[\ p(z), zp'(z); zp'(z)\ \prec\ \varphi[q(z), zq'(z); z]$. Where $\alpha, \beta, \gamma\ as\ well\ as\ \delta$ is complex information, Here $p(z) \prec q(z) as\ wellas\ q\ is\ the\ best\ dominant.$ [X],[XV].

Proof: The parameters $\theta\ as\ wellas\ \emptyset\ are\ define\ \theta(w) = (\beta w + \gamma)w^{\delta} as\ well\ as\ \varphi(w) = \alpha\ w^{\delta-1}$ apparently, the satisfies of analytic function of the parameters of $\theta\ as\ wellas\ \emptyset\ in\ the\ domain$

$D = \mathbb{C}\ \{0\} as\ well\ as\ \emptyset(w) \neq 0, w \in D.$ We use the differential subordination function $Q_1 = za'(z)\varphi(q(z)) = \alpha zq'(z)(q(z)^{\delta-1}$, as well as $h(z) = \theta(q(z)) + Q_1(z) = \emptyset(q(z), zq'(z); z)$.

We carry out the calculation of $\frac{zQ'_1(z)}{Q_1(z)} = 1 + \frac{zq''(z)}{q'(z)} + \frac{(\delta-1)zq'(z)}{q(z)}$ as well as [XVI],[XVII],[XVIII].

$\frac{zh'(z)}{Q_1(z)} = 1 + \frac{zq''(z)}{q'(z)} + \frac{(\delta-1)zq'(z)}{q(z)} + \frac{\beta(\delta+1)q(z)}{\alpha} + \frac{\gamma^{\delta}}{\alpha}$the conditions are applicable for (i) as well as (ii) we search out of $Q_1\ is\ star\ like\ in\ \mathbb{E}\ as\ \ well\ as$

$\mathfrak{R}\frac{zh\prime z)}{Q_1(z)} > 0, z \in\ \mathbb{E}. The\ conditions\ are\ applicable\ (i), (ii)$Outlookof $\varphi[\ p(z) + zp'(z)\ \varphi(p(z)) \prec\ \theta(q(z)) + zq'(z)\ \varphi[\ q(z).$

Theorem 2 Let the analytic function is univalent at $\mathbb{E}, \exists\ the\ conditions\ are\ (i)\mathfrak{R}[1 + \frac{zq''(z)}{q(z)} + \frac{(\delta-1)zq'(z)}{q(z)}] > 0\ as\ well\ as\ (ii)\mathfrak{R}[\frac{\beta(\delta+1)q(z)}{\alpha} + \frac{\gamma^{\delta}}{\alpha}] > 0. If\ p \in H[(q(0), 1] \cap Q, with\ p(z) \neq 0, z \in \mathbb{E}$, to fulfill the differential super ordination$\varphi[\ q(z) + zq'(z); z \prec\ \varphi(p(z)), zp'(z); z$, here $\alpha, \beta, \gamma\ as\ well\ as\ \delta$ are complex information with $\propto \neq 0, \varphi[p(z), zp'(z); z]\ is\ univalent\ in\ \mathbb{E}\ as\ well\ as\ \varphi$ is $q(z) \prec p(z)\ as\ well\ as\ q\ is\ good\ subordinant.$ [XIX], [XX],[XXI].

Proof : The parameters of the function θ *as wellas* $\emptyset$ to satisfies $\theta(w) = (\beta w + \gamma)w^{\delta}$ *as well as*

$\varphi(w) = \alpha\, w^{\delta-1}$, *since the parameters of the function* θ *as wellas* $\emptyset$are analytic in the domain

$D = \mathbb{C}\{0\}$ *as well as* $\varphi(w) \neq 0, w \in D$. *We assuming that the function* Q_1 *as well as* h go after$Q_1 = zq'(z)\varphi(q(z)) = \alpha zq'(z)(q(z)^{\delta-1}$ *it means that* [XIX],[XXII].

$h(z) = \theta(q(z)) + Q_1(z) = \emptyset(q(z), zq'(z); z)$. The fulfill calculation of $\frac{zQ_1{'}(z)}{Q_1(z)} = 1 + \frac{zq''(z)}{q'(z)} + \frac{(\delta-1)zq'(z)}{q(z)}$ as well as$\frac{\theta'(q(z))}{\varphi(q(z))} = \frac{\beta(\delta+1)q(z)}{\alpha} + \gamma\delta/\alpha$. The conditions are applicable for [XXIII],[XXIV].

(i) as well as (ii) that Q_1 *is star like in* $\mathbb{E}$ *as well as* $\Re\frac{\theta'(q(z))}{\varphi(q(z))} > 0, z \in \mathbb{E}$.

$\theta[\, q(z) + zq'(z)\, \varphi(q(z)) \prec \theta(p(z)) + zp'(z)\, \varphi[\, p(z)$.

IV. Applications to Univalent Functions

Newly we define $p(z) = \frac{(f*\varphi)(z)}{(f*\Psi)(z)}$, the follows the Theorem 1 as well as the result [XX],[XXI],[XXV].

Theorem 4 We assume that $q, q(z) \neq 0$,tosatisfy$\mathbb{E}$ the conditions (i) as well as (ii) of the Theorem 1. Suppose $f \in A$ *as well as the functions of analytic parameters* φ , Ψ with $\frac{(f*\varphi)(z)}{(f*\Psi)(z)} \neq 0, z \in \mathbb{E}$ satisfies the differential superordination[XXVI],[XXVII].$\varphi(q(z)), zq'(z); z) \prec \varphi\left[\frac{(f*\varphi)(z)}{(f*\Psi)(z)}, z\left(\frac{(f*\varphi)(z)}{(f*\Psi)(z)}\right)'; z\right] = h(z)$,

Where α, β, γ *as well as* δ are complex information with $\alpha \neq 0$, *here* h *is in*$\mathbb{E}$ as well as φ *is* given by then $q(z) \prec \frac{(f*\varphi)(z)}{(f*\Psi)(z)}$, *as well as q is the best subordinant.*

Remark 1 The exact values of α, β, γ *as well as* δ are complex information in Theorem 1 as well as Theorem 3[XXVIII],[XXIX] as well as by assuming that the cases of the functions φ *as well as*Ψ follows the Theorem 3, knows that results them are as follows:

(i) To take $\gamma = 1 - \beta, \delta = 1$ to get hold of, Lemma 1 of [XII].
(ii) On replacing $\gamma = 1$ *as well as* $\beta = 0$ *in Theorem*1, to get hold of, Corollary 3.2 of [XXI].
(iii) To take $\alpha = \delta = 1$ *as well as*$\gamma = 0$ *in Theorem*1, to get hold of, 3.4 of [22]
(iv) To take $\alpha = \delta = 2, \beta = 0$ *as well as* $\gamma = 1$ *in Theoremk* 1 , to get hold of, Corollary 3.3 of [XXI] .

(v) To take $\beta = 0 \; as \; well \; as \; \gamma = \delta = 1 \; in \; Theorem \; 1$, to get hold of , Corollary 3.4 of [XXI]

(vi) To take $\alpha = 1, \beta = \delta = 0 \; as \; well \; as \; \delta = -1$ in Theorem 1, Based on the Result of the Authors Ravichandran as well as Darus [XV].

(vii) To take $\alpha = \gamma = 1, \beta = 0 \; as \; well \; as \delta = 1/\lambda$in Theorem 1, to get hold of Lemma 1 of [XIII].

(viii) To take $\varphi(z) = \sum_{n=1}^{\infty} nz^n$, $\Psi(z) = \sum_{n=1}^{\infty} z^n, \beta = \alpha, \gamma = 1 - \alpha \; as \; well \; as \; \delta = 1$ in Theorem 3, to get hold of the Theorem 3of [XII].

(ix) To take $\varphi(z) = \sum_{n=1}^{\infty} nz^n$, $\Psi(z) = \sum_{n=1}^{\infty} z^n, \beta = 1, as \; well \; as \; \gamma = \delta = 0$ in Theorem 3, to get hold of, Theorem 4.3 of [XXII].

(x) To take $\varphi(z) = \sum_{n=1}^{\infty} nz^n$, $\Psi(z) = \sum_{n=1}^{\infty} z^n$, $\gamma = \beta = 1, \gamma = 0$ as well as $\delta = -1$ in Theorem3, to get hold of , Theorem 4.5 of [XXII].

Remark 2 From the selections of the same as in Remark 1 , as well as Theorem 2 ,4 to get hold of the matching results meant for superordination follows that :

(i) To take $\delta = 1 \; in \; Theorem 2$,to get hold of, Lemma 2.1 of [XVII].
(ii) To take $\gamma = \delta = 0 \; The \; theorem \; 2$, to get hold of, Lemma 2.4 of [XVII].
(iii) To,take $\varphi(z) = \sum_{n=1}^{\infty} nz^n$, $\Psi(z) = \sum_{n=1}^{\infty} z^n, \alpha = \beta, \gamma = 1 - \beta \; as \; well \; as \; \delta = 1$ the Theorem 4, to get hold of, Theorem 2.2 of [XVII].
(iv) To take $\varphi(z) = \sum_{n=1}^{\infty} nz^n$, $\Psi(z) = \sum_{n=1}^{\infty} z^n, \beta = 1 \; as \; well \; as \; \gamma = \delta = 0$ in the Theorem 4, to get hold of, Theorem 2.5 of [XVII].

V. Applications to Multivalent Functions

We assuming that $A(P)$symbolize the form of the functions are $f(z) = z^p + \sum_{n=1}^{\infty} ap + z^{P+k} \; (P \in \{1,2,3, \ldots, \}), to \; satisfies \; the \; analytic \; as \; well \; as \; p - Valent \; in \; \mathbb{E}$. We characterize the function in Theorem 1, is $P(z) = \frac{1}{P}\frac{zf'(z)}{f(z)}$.

Theorem 5 We assume that $q, q(z) \neq 0$, to satisfy $\mathbb{E}$ the conditions (i) as well as (ii) of the Theorem 2.

Suppose

$f \in A \; as \; well \; as \; the \; functions \; of \; analytic \; parameters \; \varphi$, Ψ with $\frac{1}{P}\frac{zf'(z)}{f(z)} \in H[q(0), 1] \cap Q$, by means of $\frac{(f*\varphi)(z)}{(f*\Psi)(z)} \neq 0, z \in \mathbb{E}$, to satisfies the differential superordination as the function exist i.e.,

$\varphi\left[\frac{1}{P}\frac{zf'(z)}{f(z)}, z\left(\frac{1}{P}\frac{zf'(z)}{f(z)}\right)'; z\right] \prec \varphi\big(q(z)\big), zq'(z); z)$ Where $\alpha, \beta, \gamma \; as \; well \; as \; \delta$ are complex information with $\alpha \neq 0, here \; h \; is \; in \mathbb{E}$ as well as $\varphi \; is$ given by then $\frac{1}{P}\frac{zf'(z)}{f(z)} \prec q(z) \; , as \; well \; as \; q \; is \; the \; best \; subordinant.$ we define and writing

$P(z) = \frac{1}{P}\frac{zf'(z)}{f(z)}$

Theorem 6 We assume that $q, q(z) \neq 0$, to satisfy $\mathbb{E}$ the conditions (i) as well as (ii) of the Theorem 2.

Suppose

$f \in A$ *as well as the functions of analytic parameters* φ, Ψ with $\frac{1}{P}\frac{zf'(z)}{f(z)} \in H[q(0),1] \cap Q$, by means of $\frac{(f*\varphi)(z)}{(f*\Psi)(z)} \neq 0, z \in \mathbb{E}$, to satisfies the differential superordination as the function exist i.e.,

$\left(q(z)\right), zq'(z); z) \prec \varphi\left[\frac{1}{P}\frac{zf'(z)}{f(z)}, z\left(\frac{1}{P}\frac{zf'(z)}{f(z)}\right)'; z\right] = h(z)$,Where
α, β, γ *as well as* δ are complex information with
$\alpha \neq 0$, *here h is in* $\mathbb{E}$ as well as φ *is* given by then
$\frac{1}{P}\frac{zf'(z)}{f(z)} \prec q(z)$, *as well as q is the best subordinant.*

Remark 3 We define and exciting consequences for $\mathrm{P}-$ valent functions by selecting parameters of the functions values α, β, γ *as well as* δ are complex interesting valued in Theorem 5.Example for $\beta = P, \gamma = 0$ *as well as* $\delta = 0$ *in Theorem* 5, to get hold of the Theorem 1 of [XXV]. Also to selection of the Theorem 6, to get hold of corresponding result for super ordination.

VI. Applications to $\varphi -$ *Like Functions*

On inscription $\frac{(f*\varphi)(z)}{(f*\Psi)(z)} = P(z)$, *in Theorem* 1 *to get hold of the result.*

Theorem 7 We assume that $q, q(z) \neq 0$, to satisfy $\mathbb{E}$ the conditions (i) as well as (ii) of the Theorem 2.

Suppose $g \in A \exists \frac{z(f*g)'(z)}{\varphi((f*g)(z))} \neq 0, z \in \mathbb{E}$, satisfies the differential subordination of the univalent function is $\left[\frac{z(f*g)'(z)}{\varphi((f*g)(z))}, z\left(\frac{z(f*g)'(z)}{\varphi((f*g)(z))}\right)'; z\right] \prec \varphi(q(z), zq'(z); z)$, we observe that parameters of the functions values α, β, γ *as well as* δ are complex numbers with$\alpha \neq 0$, φ *is an analytic* function

In domain containing $(f * g)(\,)$, $\varphi(0) = 0, \varphi'(0) = 1$ *as well as* $\varphi(w) \neq 0$ is $\frac{z(f*g)'(z)}{\varphi((f*g)(z))} \prec q(z)$, as well as q is the best dominant. Here $p(z) = \frac{z(f*g)'(z)}{\varphi((f*g)(z))} \in H[q(0),1] \cap$ *with* $\frac{z(f*g)'(z)}{\varphi((f*g)(z))} \neq 0$ satisfies the differential superordination.

$\varphi(q(z), zq'(z); z) \prec \varphi\left[\frac{z(f*g)'(z)}{\varphi((f*g)(z))}, z\left(\frac{z(f*g)'(z)}{\varphi((f*g)(z))}\right)'; z\right] = h(z)$, to the complex numbers $(f * g)(\mathbb{E}\,)$, $\varphi(0) = 0, \varphi'(0) = 1$ *as well as* $\varphi(w) \neq 0$ subsequently

$q(z) \prec \frac{z(f*g)'(z)}{\varphi((f*g)(z))}$as well as q is the best subordinant.

Remark 4 Assuming that $\gamma = 0$ *as well as* $\delta = 0$ *in Theorem* 7, get hold of that the Theorem 2.1 of [XIX] as well as by the equivalent selection in Theorem 8, to get hold of Theorem 2.5 of [XIX].

Remark 5 Suppose selection of $g(z) = \sum_{n=1}^{\infty} z^n \; in \; Theorem7 \; as \; well \; as \; Theorem8$, after that for $f \in A, having \; that \; \frac{z(f*g)'(z)}{\varphi((f*g)(z))} = \frac{zf'(z)}{\varphi f(z)}. To \; unsing \; of \; the \; applications$ Of Theorem7 as well as Theorem8, by gives the different complex parameters values of $\alpha, \beta, \gamma \; as \; well \; as \; \delta$. By doing so, we obtain the results of ([IV], [XIII],[XXIV]).

(i) $\sum_{n=1}^{\infty} z^n = g(z) \; rewriting \; of \; \alpha = \beta, \gamma = 1 - \beta \; as \; well \; as \; \delta = 1$ Theorem 7, to get hold of the Theorem 3 of[XIII].

(ii) $\sum_{n=1}^{\infty} z^n = g(z) \; rewriting \; \alpha = \gamma = 1, \beta = 0 \; as \; well \; as \; \delta = 1/\lambda$ The Theorem 7, to get hold of the Theorem 4 of [XIII].

VII. Results and Discussion

In the present paper we examined a few classes of scientific capacities; we acquire the quantity of adequate conditions and standardized systematic capacities in the unit plate. Likewise we give a few utilization of the First – Order Differential Subordination just as Super appointment for Generalized Positive Analytic capacities. Anyway utilization of Star-like capacities just as Univalent capacities is significant job of Differential subjection, super ordination of Analytic capacities.

Conclusion

We remark that several subclasses of analytic univalent functions can be derived using the operator $\alpha, \beta, \gamma \; as \; well \; as \; \delta$ to developed a new Results and Theorems. Mainly we define to work out Star like function as well as $\varphi - like$ functions. Also we introducing some applications as well as fractional derivatives operators.

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