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OPTIMIZED FORCE DISTRIBUTION ON A COUPLED, SELF-ADAPTIVE, THREE PHALANXES PROSTHETIC FINGER

Mahmood Hamid Yasen[1], Nabil Hassan Hadi[2]

[1]University of Baghdad/College of Engineering/Mechanical Engineering Department, Baghdad, Iraq, 2019

[2]University of Baghdad/College of Engineering/Aeronautical Engineering Department, Baghdad, Iraq, 2019

m.al-nuaimi1303@coeng.uobaghdad.edu.iq

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Abstract

The significance of prosthesis and amputation have been presented, then the concept of under-actuation mechanism has been demonstrated, followed by an optimization procedure to get equal force distribution on a Coupling and Self-Adaptive three phalanxes prosthetic finger (iso-forced finger)Developing kinematic-mathematical model to get the required relations, to derive the objective function, then using multi-variable optimization with constraints, to determine the state of iso-forced finger. Discussing the results of the optimization and finding the average of the lengths of each link, finally explaining the stability of the new configuration, and the advantages of the new methodology.

Keywords: Prosthesis, Amputation, iso-forced finger, multi-variable optimization

I. Introduction

The name prosthesis refers to a device that has been manufactured and fabricated to replace a part of the human body [XI], the adjective of prosthesis is prosthetic.

Amputation is the surgical procedure used to cut off a limp or a part of a limp, the individual who undergo such kind of operation is called amputee, the main cause of amputation is a severe trauma which stands behind 80 percent of upper limp amputees, the second main cause is serious illness like cancer or tumors and the diseases that cause vascular circulation problems [IV].

Due to war operations in the last several years, the number of amputations has increased dramatically, most of them are in the developing countries, the majority of amputations caused by war are lower limp amputees.

The amputation or limp-loss has a great effect on the human in many aspects, functional, social and psychological [III], the functional ability of the body will decrease in a manner depends on the amputation kind and characteristics, if it was a lower limp amputation the mobility of the body will be effected, if it was an upper limp amputation the Daily-activities will be effected also depending on the situation of the amputation.

The main problem that this article is trying to solve is the cost of a prosthetic hand, where the high-quality commercial hand ranges between 35000 $ and 70000$ [VI].

I.i. Theoretical Background

The major task in manufacturing a prosthetic hand is how to provide functionality. And designing a human hand is a very complex and tedious task, the main reason is the small size constraint compared to the number of DOFs (degrees of freedom) required to be available. The size of a human hand is another variable in this field, which depends mainly on the age and gender, another main reason for the complexity of designing a human hand is sensation and feedback control. The control of a prosthetic hand could be divided into two main branches, object-adaptation, and holding force.

- Object adaptation means that the actuated part (in this case the finger) can take the shape of the outer perimeter of the object. This purpose has been achieved well in the BHG-1 hand [II]. This purpose or goal can be done in more than one way, in BHG-1 hand only one motor has been used to actuate the finger along with a four bar mechanism were enough to self-adapting the wanted object. another way is to use three motors for a single finger (each phalanx-to-phalanx joint has a motor) and then each phalanx has a touch sensor to tell when the phalanx is contact with the object, but the last method is coasty because it requires a very small motors and will drag a lot of current to hold the object.
- Holding force…for example if the object was an egg or an open bottle of water the holding force provided by fingers should be as small as possible to prevent breaking the egg or spoiling the water, on the other hand if the object was a door handle or a filled grocery sack the holding force should be high enough to not lose the handle or slipping the sack.

This process requires a pre-determined or even a pre-approximated value of the force that should be exerted on the object.

To reduce the cost and weight of motors for the prosthetic hand another solution can be followed which is under-actuation, which means that the number of actuators is less than the number of degrees of freedom that a finger can cover; this concept has been clarified in [VII].

One of the main methods to provide under-actuation is coupling, coupling (in the field of prosthetic fingers) means that the movement of and position of each phalanx is determined by one input only which is in this case the angle of the motor.

I.ii. Coupling and Self-Adaptive Mechanism

The word COSA stands for Coupling and self-adaptation in which each phalanx move in respect to the other phalanx with a pre-determined angle, and the finger can take (as much as possible)the shape of the outer perimeter of the object [X].

COSA mechanism can reduce the cost and weight of the finger because each finger will be actuated using one motor only, also it will reduce the possibility of hardware failure due to low number of electrical actuators.

II. Finger Design

In this study, the goal of the optimization is to obtain the lengths of the links that provide Iso-forced finger; the term Iso-forced finger means that each phalanx has the same force of the other two phalanxes.

The importance of having an equal force on each phalanx lays in dividing the working load and the stresses between the phalanxes rather than having most of the working load and stress on one phalanx only.And that will extend the life of the mechanism and lower the possibility of failure.

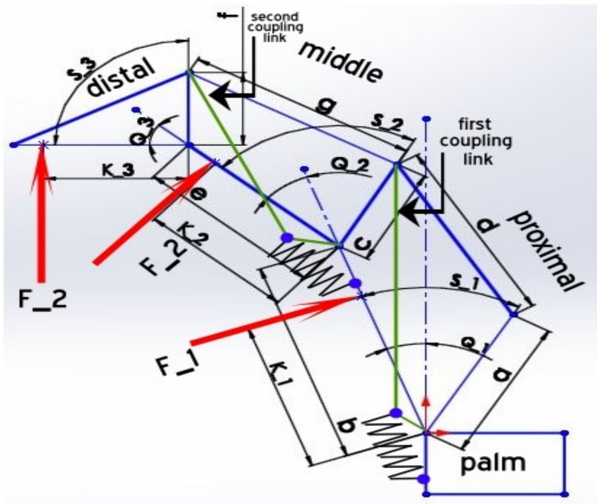

Fig.1: Coupling and Self- mechanism

II.i. Kinematic System *Adaptive*

As shown (figure 1), the kinematic system of the finger consists of three phalanxes, proximal, middle, and distal phalanx, in this finger there are two Coupling and Self-Adaptive mechanisms, which is enough to make the finger movement coupled and self-adaptive. The coupling link of the proximal phalanx will ensure the coupling of the middle and proximal phalanxes so that the angle Q_2 is related to the angles Q_1 and $S_1$when the finger is moving without contact with the object, also the angle Q_3 is related to the two angles Q_2 and S_2 because of the coupling link in the middle phalanx, it should be noted that the coupling feature is only available when the phalanxes are free from load (i.e. the finger is not in contact with the object). However, when the contact between the finger and the object occur, the self-adaptive mechanism starts to work, to adapt the shape of the object that is in contact with.

II.ii. Modal Description

a: is the length of the actuating bar in the proximal phalanx four bar mechanism, in other words (a) is the link of the motor.b: is the length of the second link in the proximal phalanx four bar mechanism, and (b) is the link in contact with the object. c:is the third link in the proximal phalanx four bar mechanism, (c) represents the actuation link for the middle phalanx four bar mechanism.d: is the fourth bar in the proximal phalanx four bar mechanism, (d) represents the link which is in the outer perimeter of the finger. e: is the length of the second link in the middle phalanx four bar mechanism, (e) is the link in contact with the object.f: is the third link in the middle phalanx four bar mechanism, (f) represents the actuation link for the distal phalanx four bar mechanism. g: is the fourth bar in the middle phalanx four bar mechanism, (g) represents the link which is in the outer perimeter of the finger.h: is the length of the link in the distal phalanx which is in contact with the object. The distal phalanx is composed by three links only. The three links form a right triangle, whichis specified by (h) and (f) only, since they are enough to fully describe the distal phalanx. All the angles are measured in degrees, and as followsS_1: is the angle between (a) and (b) in the proximal phalanx, S_2:is the angle between (c) and (e) in the middle phalanx. S_3: is the angle between (f) and (h) in the distal phalanx. Q_0: is the angle of the link (a) and the horizon, or the angle provide by the motor. Q_1:is the angle between (b) and the vertical, measured counter clockwise. Q_2: is the angle between (e) in the middle phalanx and the propagation of (b) in the proximal phalanx also measured counter clockwise. Q_3: is the angle between (h) in the distal phalanx and the propagation of (e) from the middle phalanx measured counter clockwise. All the toques are measured in N.mm and the counter clock wise as the positive direction. T_m: is the actuation torque exerted by the motor on the (a) in the proximal phalanx. T_1: Is the torque exerted by the first coupling mechanism on the link (c).T_2:is the torque exerted by the second coupling mechanism on the link (f). All the forces are measured in (N)F_1:is the force exerted by the object on the proximal phalanx specifically on link (b), with a point of action located on link (b) by the distance (k_1) which is measured in (mm).$\mathrm{F}_2$is the force exerted by the object on the middle phalanx specifically on link (e), with a point of action located on link (e) by the distance (k_2) which is measured in (mm). F_3:is the force exerted by the object on the distal phalanx specifically on link (h), with a point of action located on link (h) by the distance (k_3) which is measured in (mm).

all the three forces should be positive to say that the mechanism is stable, stability means the sense of the three forces, if the three forces were positive the mechanism is stable if at least one of the forces is negative then the mechanism is un-stable as slipping will occurs in the phalanx with the negative force.

II.iii. Mathematical Model

The input power to the system has the form of torque and angular velocity or:

$$W_{in} = T \times w_a \tag{1}$$

Where:

W_{in} : is the power provided to the system.

T: is the torque (in matrix form) provided to the links.

w_a: is the angular velocity (in matrix form) of the driving crank.

The output power from the system has the form of force and linear velocity, the force is the grasping force which is interpreted by the three contact points k_1,k_2, andk_3.and the linear velocity is for the three contact points (in matrix form).

$$W_{out} = F \times V\# \tag{2}$$

Where:

W_{out}:is the power provided by the system.

F:is the force (in matrix form) exerted on the body that being grasped.

V:is the linear velocity (in matrix form) of the contact points.

$$\mathrm{T} = [\mathrm{T_m} \quad \mathrm{T_1} \quad \mathrm{T_2}]\# \tag{3}$$

$$\mathrm{F} = [\mathrm{F_1} \quad \mathrm{F_2} \quad \mathrm{F_3}]\# \tag{4}$$

$$V = \begin{bmatrix} v1 \\ v2 \\ v3 \end{bmatrix}\# \tag{5}$$

$$w_a = \begin{bmatrix} Q_0^{\cdot} \\ Q_2^{\cdot} \\ Q_3^{\cdot} \end{bmatrix}\# \tag{6}$$

$\mathrm{T_m}$: is the torque provided by the motor.

$\mathrm{T_1}$: is the torque provided by the first coupling link.

$\mathrm{T_2}$: is the torque provided by the second coupling link.

$\mathrm{F_1}$,$\mathrm{F_2}$,$\mathrm{F_3}$are the force exerted on the first, second, and third contact points respectively.

v_1,v_2,$v_3$are the linear velocities of the first, second, and third contact points respectively.

$$v_1 = k_1 \times Q_1^{\cdot}\# \tag{7}$$

$$v_2 = (k_2 \times Q_2^{\cdot}) + \left(Q_1^{\cdot} \times \left((b \cos Q_2) + k_2\right)\right)\# \tag{8}$$

$$v_3 = (Q_3^{\cdot} \times k_3) + \left(Q_2^{\cdot} \times (e \cos Q_3 + k_3)\right) + \left(Q_1^{\cdot} \times (b \cos(Q_2 + Q_3) + e \cos Q_3 + k_3)\right)\# \tag{9}$$

Let

$$L_1 = \left((b \cos Q_2) + k_2\right)\# \tag{10}$$

$$L_2 = (e \cos Q_3 + k_3)\# \quad (11)$$

$$L_3 = (b \cos(Q_2 + Q_3) + e \cos Q_3 + k_3) \quad (12)$$

k_1,k_2,k_3 are the points of intersections of the three forces respectively .

$Q_1^{\cdot}$,$Q_2^{\cdot}$,$Q_3^{\cdot}$Are the angular velocities .

or in matrix form:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} k_1 & 0 & 0 \\ \big((b \cos Q_2) + k_2\big) & k_2 & 0 \\ (b \cos(Q_2 + Q_3) + e \cos Q_3 + k_3) & (e \cos Q_3 + k_3) & k_3 \end{bmatrix} \times \begin{bmatrix} Q_1^{\cdot} \\ Q_2^{\cdot} \\ Q_3^{\cdot} \end{bmatrix} \#(13)$$

Let

$$J_v = \begin{bmatrix} k_1 & 0 & 0 \\ \big((b \cos Q_2) + k_2\big) & k_2 & 0 \\ (b \cos(Q_2 + Q_3) + e \cos Q_3 + k_3) & (e \cos Q_3 + k_3) & k_3 \end{bmatrix} \# \quad (14)$$

$$Q^{\cdot} = \begin{bmatrix} Q_1^{\cdot} \\ Q_2^{\cdot} \\ Q_3^{\cdot} \end{bmatrix} \# \quad (15)$$

So

$$V = J_v \times Q^{\cdot} \# \quad (16)$$

Using Freudenstein equation between the two angles, it should be noted that Freudenstein equation (generally) can be used in two ways, the first one is to provide the equation with the lengths of the four links and the input angle to get the output angle, and the second way is to provide the equation with combinations of the input and output angles, to get the lengths of the links that satisfies these combinations [IX]. In this mathematical approach the second way will be used.

$(S_2 - Q_2)$as the first angle or the middle phalanx and S_1 as the second angle or the proximal phalanx.

$$K_{11} \cos(S_2 - Q_2) - K_{21} \cos S_1 + K_{31} = \cos(S_2 - Q_2 - S_1) \# \quad (17)$$

Where :$K_{11} = {}^{b}/_{a}$, $K_{21} = {}^{b}/_{c}$,$K_{13} = (c^2 - d^2 + a^2 + b^2)/(2ca)$

Differentiates(17) with respect to time:

$$\big(-K_{11}(S_2^{\cdot} - Q_2^{\cdot})(\sin(S_2 - Q_2))\big) + (K_{21}\, S_1^{\cdot} \sin(S_1)) = (-(S_2^{\cdot} - Q_2^{\cdot} - S_1^{\cdot}) \sin(S_2 - Q_2 - S_1))\#(18) \quad (18)$$

Extracting the common terms from eq (18):

$$S_1^{\cdot}[K_{21}\sin(S_1) - \sin(S_2 - Q_2 - S_1)] = S_2^{\cdot}[K_{11}\sin(S_2 - Q_2) - \sin(S_2 - Q_2 - S_1)] + Q_2^{\cdot}[-K_{11}\sin(S_2 - Q_2) + \sin(S_2 - Q_2 - S_1)] \quad (19)$$

Or

$$S_1^{\cdot} = \frac{[K_{11}\sin(S_2-Q_2)-\text{si}\ \ (S_2-Q_2-S_1)]}{[K_{21}\sin(S_1)-\sin(S_2-Q_2-S_1)]}S_2^{\cdot} + \frac{[-K_{11}\sin(S_2-Q_2)+\sin(S_2-Q_2-S_1)]}{[K_{21}\sin(S_1)-\sin(S_2-Q_2-S_1)]}Q_2^{\cdot} \quad (20)$$

After substitution the values of K_{11}, and K_{21} in the above equation:

$$S_1^{\cdot} = -\frac{c[b\sin(Q_2-S_2)-a\ \sin(S_1-S_2+Q_2)]}{a[b\sin(S_1)+c\sin(S_1-S_2+Q_2)]}S_2^{\cdot} + \frac{c[b\sin(Q_2-S_2)-a\ \sin(S_1-S_2+Q_2)]}{a[b\sin(S_1)+c\sin(S_1-S_2+Q_2)]}Q_2^{\cdot} \dots \quad (21)$$

Or

$$S_1^{\cdot} = -AS_2^{\cdot} + AQ_2^{\cdot}\# \quad (22)$$

Where

$$A = \frac{c[b\sin(Q_2-S_2)-a\ \sin(S_1-S_2+Q_2)]}{a[e\sin(S_1)+c\sin(S_1-S_2+Q_2)]}\# \quad (23)$$

In the other hand, applying Freudenstein equation between the angle $(S_3 - Q_3)$as the first angle or the distal phalanx and (S_2)as the second angle or the middle phalanx:

$$K_{12}\cos(S_3 - Q_3) - K_{22}\cos S_2 + K_{32} = \cos(S_3 - Q_3 - S_2)\# \quad (24)$$

Where:$K_{12} = {}^{e}/_{c}$, $K_{22} = {}^{e}/_{f}$, $K_{32} = (f^2 - h^2 + c^2 + e^2)/(2fc)$

Differentiate eq(24) with respect to time:

$$-K_{12}(S_3^{\cdot} - Q_3^{\cdot})\sin(S_3 - Q_3) + K_{22}S_2^{\cdot}\sin S_2$$
$$= -(S_3^{\cdot} - Q_3^{\cdot} - S_2^{\cdot})\sin(S_2 - Q_3 - S_1)\# \quad (25)$$

Extracting the common terms from eq(25).

$$S_2^{\cdot}[K_{22}\sin S_2 - \sin(S_2 - Q_3 - S_1)] = S_3^{\cdot}[K_{12}\sin(S_3 - Q_3) - \sin(S_2 - Q_3 - S_1)] + Q_3^{\cdot}[-K_{12}\sin(S_3 - Q_3) + \sin(S_2 - Q_3 - S_1)] \quad (26)$$

In here, we have that $S_3^{\cdot} = 0$ because the distal phalanx has a solid geometry that is invariant with time.

So

$$S_2^{\cdot}[K_{22}\sin S_2 - \sin(S_2 - Q_3 - S_1)] = Q_3^{\cdot}[-K_{12}\sin(S_3 - Q_3) + \sin(S_2 - Q_3 - S_1)]\#(27)$$

$$S_2^{\cdot} = \frac{[-K_{12}\sin(S_3-Q_3)+\sin(S_2-Q_3-S_1)]}{[K_{22}\,\text{si}\ \ {}_2-\sin(S_2-Q_3-S_1)]}Q_3^{\cdot}\# \quad (28)$$

Substituting the values of K_{12} , and K_{22} into eq(28).

$$S_2^{\cdot} = \frac{f[e\sin(Q_3-S_3)-c\sin(S_2-Q_3-S_1)]}{c[e\sin S_2+f\text{s}\ \ (S_2-Q_3-S_1)]}Q_3^{\cdot}\# \quad (29)$$

$$S_2^{\cdot} = BQ_3^{\cdot}\# \quad (30)$$

Where

$$B = \frac{f[e\sin(Q_3 - S_3) - c\, s \quad (S_2 - Q_3 - S_1)]}{c[e\, si \quad {}_2 + f\sin(S_2 - Q_3 - S_1)]} \# \tag{40}$$

Another relation could be noticed from the geometry:

$$Q_1 = S_1 + Q_0 - 90^o \# \tag{50}$$

Differentiate eq(50) with respect to time:

$$Q_1^{\cdot} = S_1^{\cdot} + Q_0^{\cdot} \# \tag{51}$$

Using eq(22) and eq(30) into eq(51):

$$Q_1^{\cdot} = Q_0^{\cdot} + AQ_2^{\cdot} - ABQ_3^{\cdot} \# \tag{52}$$

Or in matrix form

$$\begin{bmatrix} Q_1^{\cdot} \\ Q_2^{\cdot} \\ Q_3^{\cdot} \end{bmatrix} = \begin{bmatrix} 1 & A & -AB \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} Q_0^{\cdot} \\ Q_2^{\cdot} \\ Q_3^{\cdot} \end{bmatrix} \# \tag{52}$$

Let

$$J_w = \begin{bmatrix} 1 & A & -AB \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \# \tag{53}$$

$$w_a = \begin{bmatrix} Q_0^{\cdot} \\ Q_2^{\cdot} \\ Q_3^{\cdot} \end{bmatrix} \# \tag{54}$$

So

$$Q^{\cdot} = J_w \times w_a \# \tag{55}$$

Assuming there is not any loose of energy due to friction nor air resistance, we have

$$W_{in} = W_{out} \# \tag{56}$$

$$T \times w_a = F \times V \# \tag{57}$$

$$T \times w_a = F \times J_v \times J_w \times w_a \# \tag{58}$$

Or

$$F = T \times J_w^{-1} \times J_v^{-1} \# \tag{59}$$

We already have from eq(14),eq(53),and eq(3):

$$J_v^{-1} = \frac{adj(J_v)}{|J_v|} = \frac{1}{k_1 k_2 k_3} \begin{bmatrix} k_2 k_3 & 0 & 0 \\ -L_1 k_3 & k_1 k_3 & 0 \\ L_1 L_2 - L_3 k_2 & -L_2 k_1 & k_1 k_2 \end{bmatrix} \# \tag{60}$$

Starting from the left hand of the right side:

$$T \times J_w^{-1} = [T_m \quad -T_m A + T_1 \quad T_m AB + T_2] \# \tag{62}$$

Then:

$$T \times J_w^{-1} \times J_v^{-1} = [T_m \quad -T_m A + T_1 \quad +T_m AB + T_2] \times \frac{1}{k_{1k_{2k_3}}} \tag{63}$$

$$\begin{bmatrix} k_2 k_3 & 0 & 0 \\ -L_1 k_3 & k_1 k_3 & 0 \\ L_1 L_2 - L_3 k_2 & -L_2 k_1 & k_1 k_2 \end{bmatrix}$$

Or

$$F = \frac{1}{k_1 k_2 k_3} [T_m \quad -T_m A + T_1 \quad +T_m AB + T_2] \times \begin{bmatrix} k_2 k_3 & 0 & 0 \\ -L_1 k_3 & k_1 k_3 & 0 \\ L_1 L_2 - L_3 k_2 & -L_2 k_1 & k_1 k_2 \end{bmatrix} \#(64)$$

To make it in a column mode:

$$F^t = \frac{1}{k_1 k_2 k_3} \begin{bmatrix} (T_m)(k_2 k_3) + (-T_m A + T_1)(-L_1 k_3) + (T_m AB + T_2)(L_1 L_2 - k_2) \\ (-T_m A + T_1)(k_1 k_3) + (+T_m AB + T_2)(-L_2 k_1) \\ k_1 k_2 (T_m AB + T_2) \end{bmatrix} \#(65)$$

$$F^t = \frac{1}{k_1 k_2 k_3} \begin{bmatrix} k_2 k_3 T_m + k_3 A L_1 T_m - k_3 L_1 T_1 + AB L_1 L_2 T_m - k_2 AB L_3 T_m + L_1 L_2 T_2 - k_2 L_3 T_2 \\ -k_1 k_3 A T_m + k_1 k_3 T_1 - k_1 AB L_2 T_m - k_1 L_2 T_2 \\ k_1 k_2 AB T_m + k_1 k_2 T_2 \end{bmatrix} \#(66)$$

Compared to T_m, both $T_1 and T_2$ could be neglected, so:

$$F^t = \frac{1}{k_1 k_2 k_3} \begin{bmatrix} k_2 k_3 T_m + k_3 L_1 A T_m + AB L_1 L_2 T_m - k_2 AB L_3 T_m \\ -k_1 k_3 A T_m - k_1 AB L_2 T_m \\ k_1 k_2 AB T_m \end{bmatrix} \# \tag{67}$$

Extracting the common term:

$$F^t = \begin{bmatrix} \frac{1}{k_1} + \frac{L_1 A T_m}{k_1 k_2} + \frac{AB L_2 L_3}{k_1 k_2 k_3} - \frac{AB L_3}{k_2 k_3} \\ -\frac{A}{k_2} - \frac{AB L_2}{k_2 k_3} \\ \frac{AB}{k_3} \end{bmatrix} T_m \# \tag{68}$$

Or

$$F^t = \begin{bmatrix} \mathrm{F}_1 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \end{bmatrix} = \begin{bmatrix} \frac{k_2 k_3 + A L_1 k_3 + AB L_1 L_2 - AB L_3}{k_1 k_2 k_3} \\ \frac{-A h_3 - AB L_2}{k_2 k_3} \\ \frac{AB}{k_3} \end{bmatrix} T_m \# \tag{69}$$

Hence:

$$\mathrm{F}_1 = \frac{k_2 k_3 + A L_1 k_3 + AB L_1 L_2 - AB L_3}{k_1 k_2 k_3} \times T_m \# \tag{70}$$

$$\mathrm{F}_2 = \frac{-A k_3 - AB L_2}{k_2 k_3} \times T_m \# \tag{71}$$

$$\mathrm{F}_3 = \frac{AB}{k_3} \times T_m \# \tag{72}$$

After neglecting $T_1$and T_2, the resulted equations are the same of [X].

II.iii. Optimization

The goal of the optimization is to find the lengths of the bars that gives equal force distribution over the three phalanxes i.e. (F_1=F_2=F_3) , for this purpose a matlab script will be utilized and using (fmincon) optimization algorithm to find the solution of nonlinear multivariable function [V].

It should be noted that the three forces must be positive, and this criterion will be included in the nonlinear constraints of the optimizationfunction.

The values of the lengths that make any of the three forces negative are not accepted.

Physically, since there is not any adhesion material between the object and any of the phalanxes so the negative values will be physically meaningless, and it represents a slipping of the object from the contact surface.

The upper and lower limits of the length are taken with considering the size of an adult male hand.

The objective function that needs to be minimizedwill be:

$$objectivefunction = |F_1 - F_2| + |F_1 - F_3| + |F_2 - F_3|$$

Lower and upper bounds (lb., ub.) will be assumed to be:

Table :1 lower and upper bounds for the optimization function

link	Upper bound(mm)	Lower bound(mm)
a	30	10
b	75	25
c	30	10
d	75	25
e	75	25
f	22.5	7.5
g	45	15

The GRASHOF's theorem will be used as linear inequality for the optimization function. to ensure that $Q_2$and Q_3 can take any value from 0 to 90 degrees , this purpose can only be achieved if the links (c) and (f) are able to make a full revolution, so depending of GRASHOF's theorem [I] , the linear inequality will be :

$$a + d \leq b + c$$

$$c + e \leq f + g$$

Or in matlabform:

$$A = \begin{bmatrix} -1, -1, 1, 0, 0, 1, 0 \\ 0, 0, -1, -1, 1, 0, 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For nonlinearconstraints:

$$nonlinearinequality = \begin{bmatrix} -F\_1 \\ -F\_2 \\ -F\_3 \end{bmatrix} \geq 0$$

Ensuring that each of the three forces must be positive.

Each of the three forces is a function of three groups of variables:

The lengths of the links that form the geometric structure of the finger.And this group of variables will be determined by the optimization function.

(k_1,k_2, andk_3) which is the point of action of the three forces, and it will be taken to be (15 ,20 , 25)mm respectively .

(Q_2, and Q_3) these two angles will be taken as five combinations, and find the optimized lengths of the links for each combination so the result will be five sets of the lengths,and then calculate the average of each length, that will be done using (While – End) loop in matlab.

It should be noted that in each combination Q_3 is greater that Q_2 and that because of the coupling mechanism that has been used to give pre-determined circular envelop of the finger.

III. Results and Discussion:

As Table 2shows, five combinations of (Q_2,Q_3) have been used {(10,20),(20,30),(30.40),(40,50)

Table 2: matlab results

Q_2(mm)	10	20	30	40	50	average_ length(m
Q_3(mm)	20	30	40	50	60	
a(mm)	10	10	13.49950671	11.20411084	12.63458832	11.4676
d(mm)	75	75	55.64721006	41.03995387	38.12387943	56.9622
c(mm)	11.80609808	12.30100545	19.75459686	16.83737501	25.62877128	17.2656
g(mm)	23.68276575	25.01782121	41.21307991	29.20120419	24.905997	28.8042
f(mm)	9.100407723	9.406131533	10.36288872	**15.15930275**	22.5	13.3057
b(mm)	73.19390192	72.69899455	49.36314696	35.13631229	25	51.0785
e(mm)	25	25	40.65432001	27.29123228	28.03476827	29.1961
F_1(N)	171.220761	166.4502988	180.8103757	173.5511887	169.1302475	
F_2(N)	80.29740408	100.3149436	180.8099189	173.5491242	169.0782799	
F_3(N)	80.2974029	100.3149407	180.8078217	173.5491572	103.5684739	
objective_f unction(N)	181.8467161	132.2707163	0.005108054	0.004128925	131.1235471	

(50,60)}, The optimization function in matlab has been putted into a loop toper form the same constraints for each combination of (Q_2) and (Q_3). For each combination of (Q_2) and (Q_3),the optimized lengths (a, b, c, d, e, f, g) which satisfy the constraints have been found in each of the five loops. For each combination of those lengths the three forces (F_1, F_3andF_3) have been found and in the last row of the same table, the objective function for each combination has been determined. Finally in the last column, the average length has been calculated for each of the links (a, c, d, e, f, and g). These average lengths will be taken for further development within this section.

The ranges of the two angles (Q_2) and (Q_3) which are ($Q_2 \in [10 \; to \; 50]$) and ($Q_3 \in [20 \; to \; 60]$) have been taken to be as similar aspossible to the real activeconfiguration of the human finger.

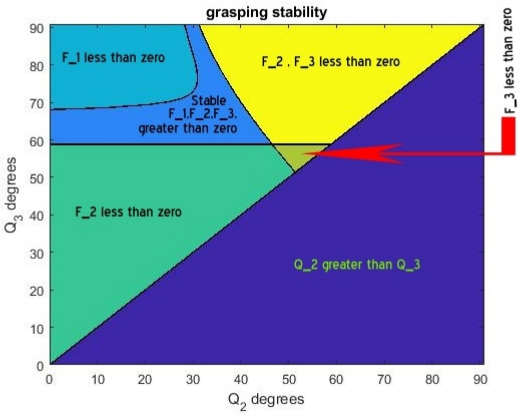

Fig. 2 :stability from the matlab code

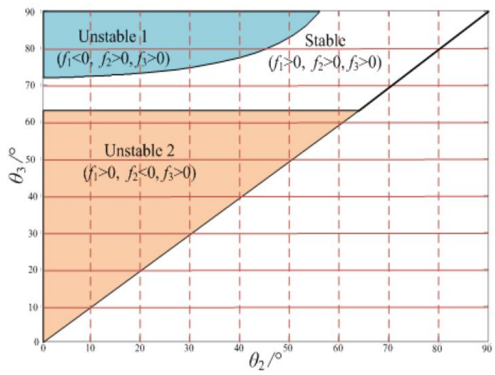

Fig. 3: Grasping stability of the three-phalanx COSA finger [X]

Furthermore, the average lengths from table 2 have been taken into consideration to demonstrate the stability of the finger mechanism, itshould be noted that (stable) means that all of the three forces are positive, and (unstable) means that at least one of the forces is negative. In other words, if all the forces that are exerted by the finger on the object are directed toward the object itself, then the configuration is stable, On the other hand if one of the phalanxes starts to slip on the object perimeter, then the configuration is unstable, and it tends to lose the object.

Another point is to be taken into consideration is that the stability criterion has an importance when dealing with a single finger (i.e. when the object is in contact with only one finger), but when the object is in contact with the whole hand, the stability will take another meaning, because the thumb will try to keep the object in contact with the other four fingers (pointer finger , middle finger , ring finger and pinky finger), in this case even if one of the forces get a negative value, the object may still in contact and not to lose.

Using the average lengths from (table_2), figure2 shows the stability of the mechanism.

The range in which ($Q_2 > Q_3$) has been ignored, because of the coupling feature in which the movement of the distal phalanx is coupled with the movement of the middle phalanx.

In (figure 2),there are six areas, the first one is where ($Q_2 > Q_3$) which will be ignored. The second largest area is where F_2(orF_2 as in the figure) is negative which represents an unstable area, and in an unstable area each combination of Q_2 and Q_3 make unstable configuration.

The same thing for the areas where F_1 is negative,F_3 is negative, and finally F_2along with F_3

Both are negative.

Only one area represents a stable configuration of Q_2andQ_3, in which, all the thee forces are positive (F_1 , F_2 ,and F_3).

Another figure (figure 3) from [X] in which different values have been given for the lengths which are (a=10 mm,b=50 mm,c=8 mm,d=52 mm,e=35 mm,f=7 mm,g=42 mm) with the same arms of forces (k_1=15 mm,k_2=20 mm,k_3=25 mm).

Comparing these two figures (figure 2 and figure 3), changing the lengths of the links (a ,b ,c ,d ,e ,f ,g) results in changing the area of stability within the working range of Q_2 and Q_3, this change in the stable area has a significant rule in the design of a –four bar mechanism- prosthetic finger.

In general the objects that is required to be held by the human hand have no pre-determined shape or perimeter, and as a result there is no pre-determined working range for Q_2 and $Q_3$to be lied in the stable range or area.

But in some cases the prosthetic hand could be specified to manipulate specific

objects or to do some oriented tasks in which the finger configuration is predetermined by a specific range for Q_2 and Q_3.

In this case, the lengths of each link could be modified to make the working range of Q_2 and Q_3 lies in the stable area as much as possible to lower the possibility of slipping or loose contact with the object.

Also this result is very important in designing a robotic hand for a factory or a production line in which the required object to be held has a specificouter perimeter and hence, the working range of Q_2andQ_3, is predetermined by just one or two combinations, in this case the lengths of the links could be modified to get these one or two combinations inside the stable area, and hence the robotic hand will be stable as much as possible.

To be more specific in designing the prosthetic finger and determining the lengths of the links , another feature should be taken into consideration, which is the absolute difference between the three forces or ($objectivefunction = |F_1 - F_2| + |F_1 - F_3| + |F_2 - F_3|$) .

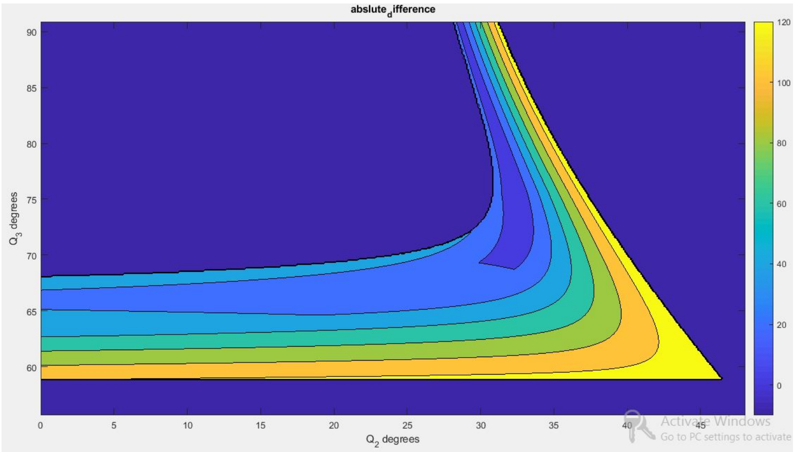

Fig. 4: absolute difference of the stable area.

This difference is illustrated in figure (4):

Noticing the range of $Q_2$and Q_3, in figure(4) , it is the stable area in figure(2) but enlarged and has a key feature which is the absolute difference as the color bar shows at the right of the figure.

This will add another criterion to be considered in designing the robotic or prosthetic finger, in which the working range not just to lie within a stable area but also to have a low value of absolute difference

This will reduce the difference between the forces on the phalanxes, and have their values to be close to each other as possible.

Having equal forces on each of the phalanxes have a significant importance in the stress distribution on the finger parts and reduce the wear of the joints.

IV. Conclusions

An optimization function along with the optimization code has been developed to get equal forces on the three phalanxes of the Coupling and Self-Adaptive finger mechanism based on five iterations with a different geometric position in each iteration, to get the iso-forced finger then the force distribution in each iteration has been found along with the optimization function value, the average length of each bar (or link) has been determined. Then the stability of the new iso-forced finger has been demonstrated and compared to the stability of the ordinary Coupling and Self-Adaptive mechanism.A new stable range has been obtained from the new configuration, that is closer to the working range of the iso-forced finger, so that to lower the possibility of slipping the object.

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