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A NEW METHOD OF VALIDATING THE CLUSTER VALIDITY INDICES FOR INTERVAL TYPE-2 FUZZY BASED CLUSTERING ALGORITHM

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Abstract

In recent years several classification techniques have been proposed which are classified into supervised and unsupervised classifications. In unsupervised classification, fuzzy clustering analysis is a most common technique since it never needs training data for fuzzy clustering algorithm. Nevertheless, different clustering algorithms have different initial conditions to generate different partitions and use different parameters in order to produce different results. Thus, the partitions generated by fuzzy clustering algorithm are in need to validate. Many cluster validity indices have been proposed in the last three decades for validating type-1 fuzzy based FCM algorithm. Recently many type-2 fuzzy based applications were presented due to its extract degree of fuzziness. But its computational complexity is very high, so interval type-2 fuzzy system is widely used in many applications. After the updation of cluster centriods in type-2 fuzzy based FCM algorithm, the type-2 fuzzy membership function is taken as unreliability of type-1 membership function. Therefore there is a need for a new method to validate the cluster validity index for interval type-2 fuzzy system based applications. In this paper, we have presented a new approach of validating the 14 cluster validity indices and performed extensive comparison of the mentioned indices in conjunction with various interval type-2 fuzzy c-means clustering algorithms. For experimental analysis we have taken the number of widely used datasets and Berkely image database.

Keywords : Cluster validity indices, IT2FCM, Extended IT2FCM, IT2FCMα

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I. Introduction

Cluster analysis is a process of splitting the given dataset into similar groups. The partition should have two properties: homogeneity with in clusters and heterogeneity between clusters. There are two types of clustering mechanism such as hard clustering and fuzzy clusteringis available. In hard clustering, every point of dataset is assigned precisely into a single cluster, while in fuzzy clustering every point of dataset belongs to more than a cluster based on degree [0, 1].

The clustering technique is used to done the partition and then it is validated using validation function. In clustering algorithm the validation function is used to know whether they present accuracy over the structures of the data set or not. If the users want to check the validity of the results, two dimensional data are needed whereas the data sets cannot visualize effectively incase large multidimensional data sets are present. Thus, in order to verify a high volume data, the objective of the cluster validity was fully brought into use as to discover an optimal cluster count which do the validation work more effectively on clustered data. Several validity indices of clusters have been presented in the last three decades for validating type-1 fuzzy based FCM algorithm. Recently many type-2 fuzzy based applications [XIV,XLII, XLIX,XI, V] are proposed due to its extract degree of fuzziness. But its computational complexity is very high, so interval type-2 fuzzy system is widely employed in many applications[XXVII,XLIV,XXXIV,XXXV,XXIV,XV,XIII]. Therefore, there is a need, a new method of validating the validity index of cluster for interval type-2 fuzzy system based applications. In this paper, we presented a novel technique of validating the validity indices of cluster and we have taken the experiment with widely used artificial data sets [XLIII] and Berkely image database.

Review by Wang and Zhang [XLIII] and Kim et al. [XLVIII, III].on fuzzy cluster validity indices is plenty, which consequently resulted in proposing to distinctive measurement criteria to evaluate and select an optimal clustering design. They are namely identified as compactness and separation. Compactness brings the member the each cluster to be very close to the other whereas separation is meant to be separating the clusters widely. A general measurement of compactness is called as variance. The above said reviewers come out with three major and common approaches to calculate the distance of two clusters of different size, distance of closest and distant cluster members and distance of cluster centers.

Fuzzy c-mean is well known clustering algorithm, which is widely applied in many applications for the last three decades as it has many extended version. It also split into two types in which first typeis to use to extend the measure of dissimilarity $d(x_j,c_i)$ between cluster center c_i and data point x_j and the and the other is to add penalized term[VI]. At the same time we have to consider another significant factor which controls the efficiency of FCM is the weighting exponent 'm'. Pal and Bezdek[XX] and Kim et al. [III] have conducted many experiments for these components.

In past three decades count of cluster validity indices is proposed for fuzzy environment. Wang and Zhang [XLVII] and Kim et al. [III] have reviewed many fuzzy cluster validity indices. Validity indices in fuzzy environment can be split into three categories [XLIII]. (i) Indices including the membership measures alone which includes V_{PC} [XXII,X], V_{PE}[XVII,XIX], V_{MPC}[XL]and V_P[XXXI](ii) Indices including the data set and membership measures which includes V_{FS} [XLVI], V_{XB}[XLV], V_K[XLI],V_{FHV}[XVI], V_{PBMF}[XXX], V_{PCAES} [XXV], SC[XXXIII], V_{SCG}[XXVIII],V_{ECA}[XXIX] and DWSC[XII],V_W[XLVII] (iii) Other approaches for fuzzy cluster validity.

There are two techniques available todesign fuzzy logic systems (FLSs)[XXIII, XIV]:Type-1 FLSs (T1FLSs) and Type-2 FLSs (T2FLSs). Type-2 fuzzy is an extended form of type-1 fuzzy by using two fuzziness parameters m_1 and m_2 to build FOU which corresponds to the upper and lower fuzzy clustering measures. The membership functions of type-1 fuzzy are fixed whereas the membership functions are allotted by themselves in type-2 fuzzy. The membership grade of type-1 fuzzy is a crisp number lies between [0,1] and the membership grade of type-2 fuzzy is a subset lies in [0, 1]. This is known as primary membership. Moreover, every primary member contains a secondary membership measure. However, the measures are taken at the interval of [0, 1] by the secondary membership functions and is known as generalized T2FLSs, but the uniform function value 1 is taken in the interval of T2FLSs. Since the computation process of common T2FLSs are much difficult and its type-reduction is computationally expensive. Therefore the interval T2FLSs is usually applied in many applications.

The section II explains the preliminaries of Type-2 fuzzy logic and outlines the IT2FCM[II], Extended IT2FCM [XXXVIII], IT2FCMα [XXXVIII], IT2FPCM[IX], MKIT2FCM[VII]and GMKIT2FCM[VI]. The section III explains the new method of validating the cluster validity indices for IT2FCM based algorithm, also the modified cluster validity indices are listed. Section 4 demonstrates the experimental results of proposed technique.

II. Preliminaries

FCM is a commonly accepted cluster algorithm which indeed permits a single piece of information that belongs to more than clusters as it is mainly employed in patter recognition. In supervised learning, FCM is believed to be more powerful and an effective algorithm if it is meant to analyze a cluster.

The FCM algorithm, proposed by Dunn [XXI] and generalized by Bezdek [XXII], is used to depict the fuzzy classification for the pixels by computing the fuzzy membership measure. FCM is a data clustering algorithm where every data point in connection with a cluster, to a degree determined by a membership grade. The fundamental process of FCM is to split a finite collection of dataset $X==\{x_1, x_2 \ldots x_n\}$ into a collection of 'c' fuzzy clusters according to few given condition.When fuzzy clustering is attempted, fuzzy membership design comprisesseveral uncertainties, such as distance measure, fuzzifier and prototype. It minimizes an objective function J, concerning fuzzy membership U,

$$J = \sum_{i=1}^{N} \sum_{j=1}^{C} {u_{ij}}^{m} \left\| x_i - c_j \right\|^2 \tag{1}$$

where m is any real number greater than 1, u_{ij} is the membership degree of x_i in the cluster j, x_i is the i^{th} of d-dimensional measured data, c_j is the d-dimension center of the cluster, list of c cluster centers C and || * || is any norm showing the resemblance between any evaluated data and the center.

The issues in FCM are computationally expensive and highly dependent on the initial choice of U. If data-specific experimental values are not available, then m = 2 is the usual choice. Extensions that exist, simultaneously estimate the intensity in homogeneity bias field while producing the fuzzy partitioning.In IT2 fuzzy set the primary membership $J_{x_i}$of a pattern $x_i$can be depicted as a membership interval with all secondary grades of the primary membership value equal to 1. The membership value for x_i is illustrated by the interval between upper $\bar{u}(x_i)$ and lower $\underline{u}(x_i)$ membership values. Thus everyx_i has primary membership interval as [II, IV]

$$J_{x_i} = \left[\bar{u}(x_i), \underline{u}(x_i)\right] \tag{2}$$

Therefore the primary membership extends the pattern x_i by Interval type-2 fuzzy set and turns as

$$\overline{u}_j(x_i) = \begin{cases} \dfrac{1}{\sum_{k=1}^{c}\left(\left({d_{ji}}/{d_{ki}}\right)\right)^{2/(m_1-1)}}, & if\ \dfrac{1}{\sum_{k=1}^{c}\left({d_{ji}}/{d_{ki}}\right)} < \dfrac{1}{C} \\ \dfrac{1}{\sum_{k=1}^{c}\left(\left({d_{ji}}/{d_{ki}}\right)\right)^{2/(m_2-1)}}, & otherwise \end{cases} \tag{3}$$

$$\underline{u}_j(x_i) = \begin{cases} \dfrac{1}{\sum_{k=1}^{c}\left(\left(d_{ji}/d_{ki}\right)\right)^{2/(m_1-1)}}, & if\ \dfrac{1}{\sum_{k=1}^{c}(d_{ji}/d_{ki})} \geq \dfrac{1}{C} \\ \dfrac{1}{\sum_{k=1}^{c}\left(\left(d_{ji}/d_{ki}\right)\right)^{2/(m_2-1)}}, & otherwise \end{cases} \tag{4}$$

where

d_{ji} is the Euclidean distance between the x_j and the centroid v_i

d_{ki} is the Euclidean distance between the x_k and the centroid v_i

list of c cluster centers $C=\{c_1,c_2,\ldots c_c\}$

In equation (3) and (4) IT2 fuzzy set use the two unique measures of m ie. m_1 and m_2. m_1 and m_2 are fuzzifiers which represents the fuzzy degree. Using these fuzzifiers the objective function is to be reduced in IT2FCM as

$$J_{m_1(U,v)} = \sum_{i=1}^{N}\sum_{j=1}^{C} u_j(x_i)^{m_1}\, d_{ji}^2 \tag{5}$$

$$J_{m_2(U\,,v)} = \sum_{i=1}^{N}\sum_{j=1}^{C} u_j(x_i)^{m_2} d_{ji}^2 \tag{6}$$

IT2FCM

IT2 fuzzy sets possess upper and lower membership functions. The IT2FCM have two fuzzifiersm_1 and m_2, thatdepict several fuzzy degrees that providesunique objective functions which is to be reduced in IT2FCM, which are shown in equation (5) and (6), andalso the IT2 represent (3) and (4).

Step:1 Compute $\underline{\mu}_{ij}$ and $\overline{\mu}_{ij}$ (7)

Step: 2 Calculate cluster centriods v_i using the following Equation (8). Each centriod of cluster is depicted by interval between v^L and v^R.

$$v_i = \frac{\sum_i^N (u_{ik})^m x_k}{\sum_i^N (u_{ik})^m} \tag{8}$$

After attainment of v_i^L ,v_i^R, type-reduction is employed to obtain cluster centriod using Equation (9)

$$v_i = \frac{v^L + v^R}{2} \tag{9}$$

Step:3 Get membership grades:

$$u_j(x_i) = \frac{u_j^R(x_i) + u_j^L(x_i)}{2}, \quad j = 1, \ldots \ldots . C \tag{10}$$

$$u_j^R(x_i) = \frac{\sum_{l=1}^M u_{jl}(x_i)}{M} \tag{11}$$

$$where u_{jl}(x_i) = \begin{cases} \overline{u}_j(x_i), & if x_{il} uses \overline{u}_j(x_i) for v_j^R \\ \underline{u}_j(x_i), & otherwise \end{cases} \quad and$$

$$u_j^L(x_i) = \frac{\sum_{l=1}^M u_{jl}(x_i)}{M} \tag{12}$$

$$where u_{jl}(x_i) = \begin{cases} \overline{u}_j(x_i), & if x_{il} uses \overline{u}_j(x_i) for v_j^L \\ \underline{u}_j(x_i), & otherwise \end{cases}$$

Step:4 (13)

If $\left(u_j(x_i) > u_k(x_i)\right)$,

$for k = 1, \ldots ., C$ and $j \neq k$ Then x_i, is assigned to cluster j.

Step: 5 If $max_i \left\| v_i^{(l+1)} - v_i^{(l)} \right\| \prec \varepsilon$ $then stop$;else l=l+1, and goto Step 2.

α - cut implemented IT2FCM (IT2FCMα) for segmentation

A significant concept in fuzzy set theory and applications is the α-cut decomposition theorem developed by Zadeh(1975). These cuts are crisp sets associated with certain α levels that represent distinct grades of membership. The idea behind the α-cut representation is to define a useful special fuzzy set that is associated with each α-cut. The equation (14) shows the α-cut of Type-1 fuzzy set[XXVI]

$$A_\alpha = \{A \epsilon X \mid A(x) \geq \alpha, \alpha \epsilon [0,1]\} \tag{14}$$

The two-dimensional α-plane[XXVI] , denoted as$\tilde{A}_\alpha$, the union of all primary membership shows secondary grades are greater than or equal to the special value α, i.e.,

$$\tilde{A}_\alpha = \bigcup_{x \epsilon X} (x,u) \mid \mu_{\tilde{A}}(x,u) \geq \alpha = \bigcup_{x \epsilon X} \left(\mu_{\tilde{A}}(x)\right)_\alpha \tag{15}$$

where$\left(\mu_{\tilde{A}}(x)\right)_\alpha$ is the α-cut of vertical slice $\mu_{\tilde{A}}(x)$.

The membership values are represent in interval as the upper$\overline{u}$and the lower $\underline{u}$,also eachcentroid of cluster is mentioned as theinterval between v^L and v^R.The centroid of an interval type-2 fuzzy set is represented in equation (16).

$$v_c(\alpha) = Centroid\left(\tilde{A}(\alpha)\right) = \sum_{u_1 \epsilon \alpha J x_1} \dots \sum_{u_N \epsilon \alpha J x_N} \alpha \Big/ \frac{\sum_{i=1}^{N} x_i u_i}{\sum_{i=1}^{N} u_i} \tag{16}$$
$$= \alpha / [\,^\alpha v^L\,,\,^\alpha v^R]$$

Fuzzy clustering algorithm along with cluster cores includes more good clustering property; which won't be able to specify the cluster cores for two overlapping areas. Therefore, to specify most common cluster cores for any overlapping areas in fuzzy partition, we have proposed α-cut implemented IT2FCM clustering algorithm[XXXVII] denoted by ITFCMα. The main contribution of this algorithm is to utilize α-plane representation concept in type-2 fuzzy for calculating centroid type reduction in IT2FCM and extended method presented in [II].The idea of cluster core can be elaboratedin more common manner that, if the given measure α is much smaller than the membership measureμ_{ij}at i^{th} cluster of data pointx_j, then it is known that this data point$x_j$will absolutely belongs to the i^{th} clusteralong with the membership measure of 1 and $\underline{\mu}_{i'j}$=0 &$\overline{\mu}_{i'j}$=0 for all $i \neq i'$.A basic rule to assure that any two of the c cluster cores will never overlapas0.5≤α≤1.The measure of a relative distanceis employed to determine the cluster cores produced by IT2FCMα is given as, The cluster cores generated by IT2FCMα can be determined by a relative distance measure with

$$\overline{u}_j(x_i) = \begin{cases} \dfrac{1}{\sum_{k=1}^{C}\left(d_{ji}/d_{ki}\right)^{2/(m_1-1)}}, & if \dfrac{1}{\sum_{k=1}^{C}\left(d_{ji}/d_{ki}\right)} < \dfrac{1}{C} \\ \dfrac{1}{\sum_{k=1}^{C}\left(d_{ji}/d_{ki}\right)^{2/(m_2-1)}}, & otherwise \end{cases} \quad (17)$$

$$\underline{u}_j(x_i) = \begin{cases} \dfrac{1}{\sum_{k=1}^{C}\left(d_{ji}/d_{ki}\right)^{2/(m_1-1)}}, & if \dfrac{1}{\sum_{k=1}^{C}\left(d_{ji}/d_{ki}\right)} \geq \dfrac{1}{C} \\ \dfrac{1}{\sum_{k=1}^{C}\left(d_{ji}/d_{ki}\right)^{2/(m_2-1)}}, & otherwise \end{cases} \quad (18)$$

Therefore, the cluster cores produced by the presented IT2FCMα algorithm ensurethe cluster cores in IT2FCMα have different overlapping areas.

Step :1 Compute $\underline{\mu}_{ij}$ and $\overline{\mu}_{ij}$

Step: 2 Calculate cluster centriods$v_i$in the same way of FCM. Everyclustercentroid is illustrated by theinterval between v^L and v^R.Update cluster centers with equation (19)

$$v_i\ (\alpha) = Centroid\left(\tilde{A}(\alpha)\right) = \sum_{u_1 \epsilon \alpha J x_1} \cdots \sum_{u_N \epsilon \alpha J x_N} \alpha \Big/ \frac{\sum_{i=1}^{N} x_i u_i}{\sum_{i=1}^{N} u_i} \quad (19)$$

$$= \alpha/\left[{}^{\alpha}v^L\ ,\ {}^{\alpha}v^R\right]$$

$$v_i\ (\alpha) = \frac{v_i^L + v_i^R}{2} \quad (20)$$

This center-updation contains type-reduction and defuzzification.

Step: 3 Perform type reduction and defuzzification, to compute ${}^{\alpha}v^L\ and\ {}^{\alpha}v^R$ (21)

$$u_j\ (x_i) = \frac{u_j^R(x_i) + u_j^L(x_i)}{2},\quad j = 1, \dots \dots . . C$$

$$u_j^R(x_i) = \frac{\sum_{l=1}^{M} u_{jl}(x_i)}{M} \quad (22)$$

$$\text{where } u_{jl}(x_i) = \begin{cases} \bar{u}_j(x_i), & \text{if } x_{il} \text{ uses } \bar{u}_j(x_i) \text{for } v_j^R \\ \underline{u}_j(x_i), & \text{otherwise} \end{cases} \quad (23)$$

And (24)

$$u_j^L(x_i) = \frac{\sum_{l=1}^{M} u_{jl}(x_i)}{M}$$

$$\text{where } u_{jl}(x_i) = \begin{cases} \bar{u}_j(x_i), & \text{if } x_{il} \text{ uses } \bar{u}_j(x_i) \text{for } v_j^L \\ \underline{u}_j(x_i), & \text{otherwise} \end{cases} \quad (25)$$

Step: 4

If $\left(u_j(x_i) > u_k(x_i)\right)$, (26)

$for k = 1, \ldots, C$ and $j \neq k$ Then x_i, is allotted to cluster j.

Step: 5 If $max_i \left\| {}^{\alpha}v_i^{(l+1)} - {}^{\alpha}v_i^{(l)} \right\| \prec \varepsilon, then stop$; else l=l+1, and goto Step 2. (27)

Interval Type-2 Fuzzy Possibilistic C-Means Algorithm (IT2FPCM)

The IT2FPCM algorithm [IX] is derived from FPCM algorithm and type-2 fuzzy logic approaches and itcarries both fuzziness and possibility weights asμ and m, which are represented by a range instead ofaaccuratemeasure; that is m = [m_1,m_2], where m_1 and $m_2$depictsthe weighting exponents of the lower limit and upper limit for fuzziness and $\mu = [\mu_1, \mu_2]$ where μ_1 and $\mu_2$depictweighting exponents of the lower limit and upper limit of for probability.

IT2FPCM algorithm comprises of the following steps:

Step 1:c, m_1 and m2 are initialized

Step 2: Centroidsareinitialized randomly for the lower bound and upper bound of the interval.

Step 3: By applying Equations (28) and (29) the fuzzy partition matrices are updated for lower bound and upper bound of the interval.

.

$$\underline{\mu}_i(x_k) = min\left\{\left(\sum_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{m_1-1}}\right)^{-1}, \left(\sum_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{m_2-1}}\right)^{-1}\right\} \quad (28)$$

$$\overline{\mu}_i(x_k) = max\left\{\left(\sum_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{m_1-1}}\right)^{-1}, \left(\sum_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{m_2-1}}\right)^{-1}\right\} \quad (29)$$

Step 4: By applying Equations (30) and (31) the matrices of the possibilistic partition is updated for lower bound and upper bound of the interval.

$$\underline{\tau}_i(x_k) = min\left\{\left(\sum_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{\eta_1-1}}\right)^{-1}, \left(\sum_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{\eta_2-1}}\right)^{-1}\right\} \quad (30)$$

$$\overline{\tau}_i(x_k) = max\left\{\left(\sum_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{\eta_1-1}}\right)^{-1}, \left(\sum_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{\eta_2-1}}\right)^{-1}\right\} \quad (31)$$

Step 5: Using Equations (32) & (33) number of centriods used for fuzzy partition matrix at lower and upper bounds are computed.

$$\underline{v}_i = \frac{\sum_{j=1}^{n}\left(\underline{\mu}_i(x_j)+\underline{\tau}_i(x_j)\right)^{m_1} x_j}{\sum_{j=1}^{n}\left(\underline{\mu}_i(x_j)+\underline{\tau}_i(x_j)\right)^{m_1}} \quad (32)$$

$$\overline{v}_i = \frac{\sum_{j=1}^{n}\left(\overline{\mu}_i(x_j)+\overline{\tau}_i(x_j)\right)^{m_1} x_j}{\sum_{j=1}^{n}\left(\overline{\mu}_i(x_j)+\overline{\tau}_i(x_j)\right)^{m_1}} \quad (33)$$

Step 6: Using Equations (34) & (35) type reduction is performed at the centriods and fuzzy partition matrix.

$$v_j = \frac{\underline{v}_j+\overline{v}_j}{2} \quad (34)$$

$$\mu_i(x_j) = \frac{\underline{\mu}_i(x_j)+\overline{\mu}_i(x_j)}{2} \quad (35)$$

Step 7: Repeat step 3-5 until $\left|\tilde{J}_{m,\eta}(t) - \tilde{J}_{m,\eta}(t-1)\right| < \varepsilon$

Multiple Kernel Interval Type-2 Fuzzy c-Means algorithm (MKIT2FCM)

Dzunget. al. presented theMKIT2FCM [VI] . The KIT2FCMemploy a nonlinear map limitedas $\emptyset: x \rightarrow \emptyset(x)\epsilon H, x\epsilon X\epsilon R^d$. Where X refers the feature space or the dataset and H is known as a kernel space. The algorithm of KIT2FCM employs double fuzzifiers m_1 and m_2 alike with IT2FCM in order to dealwith uncertainty. The lower and upper membershipmeasures, $\overline{u}_{ij}$ and $\underline{u}_{ij}$ are determined as follows:

$$\overline{u}_{ij} = \begin{cases} \frac{1}{\sum_{l=1}^{c}\left(\frac{d_{\emptyset ij}}{d_{\emptyset lj}}\right)^{2/(m_1-1}} \; if \; \frac{1}{\sum_{l=1}^{c}\left(\frac{d_{\emptyset ij}}{d_{\emptyset lj}}\right)} < \frac{1}{c} \\ \frac{1}{\sum_{l=1}^{c}\left(\frac{d_{\emptyset ij}}{d_{\emptyset lj}}\right)^{2/(m_2-1}} \; otherwise \end{cases} \quad (2) \qquad (36)$$

$$\underline{u}_{ij} = \begin{cases} \frac{1}{\sum_{l=1}^{c}\left(\frac{d_{\emptyset ij}}{d_{\emptyset lj}}\right)^{2/(m_1-1}} \; if \; \frac{1}{\sum_{l=1}^{c}\left(\frac{d_{\emptyset ij}}{d_{\emptyset lj}}\right)} \geq \frac{1}{c} \\ \frac{1}{\sum_{l=1}^{c}\left(\frac{d_{\emptyset ij}}{d_{\emptyset lj}}\right)^{2/(m_2-1}} \; otherwise \end{cases} \quad (3) \qquad (37)$$

in which $i = \overline{1,c}$, j= $\overline{1,n}$, $d_{\emptyset ij} = \left\|\emptyset(x_j) - \emptyset(v_i)\right\|$. If we employ the Gaussian kernel the $k(x,x) = 1 \; and \left\|\emptyset(x_j) - \emptyset(v_i)\right\|^2 = 2(1 - k(x_j, v_i))$.The main purpose of the MKIT2FCM is to obtain better results which are enhanced byuniting different kernels. The objective of the MKIT2FCM is to reduce the objective functionslike KIT2FCM, i.e.,

$$J_{m1}((U,V) = \sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^{m1}\|\emptyset_{com}(x_i) - v_i\|^2 \;\; (4) \qquad (38)$$

$$J_{m2}((U,V) = \sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^{m2}\|\emptyset_{com}(x_i) - v_i\|^2 (5) \qquad (39)$$

where,$\emptyset_{com}$ is known as the transformation determined by the combined kernels.

$$k_{com}(x,y) = (\emptyset_{com}(x), \emptyset_{com}(y))$$

Herek_{com} is the composite kernel which is the integration of multiple kernels. Lower/Upper degrees of membership, $\overline{u}_{ij}$ and $\underline{u}_{ij}$ are decided by the equation (36) and (37)along with $d_{\emptyset}$which is substituted by $d_{\emptyset com}$

$$d_{\emptyset com}(x_j, v_i)^2 = \left\|\emptyset_{com}(x_j) - v_i\right\|^2 = k_{com}(x_j, x_j) + \frac{2\sum_{h=1}^{n}(u_{ih})^m k_{com}(x_j, x_h)}{\sum_{h=1}^{n}(u_{ih})^m} + \frac{\sum_{h=1}^{n}\sum_{l=1}^{n}(u_{ih})^m (u_{il})^m k_{com}(x_h, x_l)}{\left(\sum_{h=1}^{n}(u_{ih})^m\right)^2} \qquad (40)$$

Given a set of n data points $= \{x_i\}_{i=1}^{n}$, a set of kernel functions $\{k_i\}_{i=1}^{l}$, parameters $m_1, m_2$and the suitable number of clusters c. Output of a membership matrix$U = \{u_{ic}\}_{i,c=1}^{n}, c$ and the weights $\{w_i\}_{i=1}^{l}$for the kernels.

Step 1: Initiate the centroid matrix $V^0 = \{v_i\}_{i=1}^{c}$ by selectingarbitrary value from the membership matrix $U^0$and the datasetis updated using the Equation (41).

$$u_{ij} = \frac{1}{\sum_{l=1}^{c}\left(\frac{d_{ij}}{d_{ij}}\right)^{2/(m-1)}} \tag{41}$$

where m is a constant, m>1 and $d_{ij} = d(x_j - v_i) = \|x_j - v_i\|$

Step 2: Perform the following steps until the termination condition is satisfied or maximum iterations are reached:

2.1 Calculate Interval membership values $\overline{u}_{ij}$ and $\underline{u}_{ij}$using the Equation (36) ,(37) and (40)

2.2 Upgrade the centriod matrix adopted the iterative algorithm to discover centroids at KIT2FCM using Equation (42)

$$v_i = (v_i^R + v_i^L)/2 \tag{42}$$

2.3 Update the membership matrix using Equation (43)

$$u_i(x_k) = \left(u_i^R(x_k) + u_i^L(x_k)\right)/2\ , j = 1, \ldots, C \tag{43}$$

in which

$$u_i^L = \sum_{l=1}^{M} u_{il}/M\ , u_{il} = \begin{cases} \overline{u}_i(x_k) & if\ x_{il} uses\ \overline{u}_i(x_k) for\ v_i^L \\ \underline{u}_j(x_k) & otherwise \end{cases}$$

$$u_i^R = \sum_{l=1}^{M} u_{il}/M\ , u_{il} = \begin{cases} \overline{u}_i(x_k) & if\ x_{il} uses\ \overline{u}_i(x_k) for\ v_i^R \\ \underline{u}_j(x_k) & otherwise \end{cases}$$

2.4 Assign data x_j to cluster c_i if data $\left(u_j(x_i) > u_k(x_i)\right), k = 1, .., c\ and\ j \neq k$

Genetic Multiple Kernel Interval Type-2 Fuzzy c-Means algorithm (GMKIT2FCM)

Dzunget. al. [VI] proposed GMKIT2FCM cluster algorithm. Which consists two main stages:

(i) To discover the primarycentriods and the count of the clusters using the coefficients average of the multiple kernel $w_i = \frac{1}{l}$.

(ii) To enhance the multiple kernel coefficients, Genetic Algorithm is used.

Stage 1: The objective of stage 1 is to discover the optimal count of clusters and primary centroids automatically. Here the average MKIT2FCM is used along with $w_0 = w_i = \frac{1}{l}$ wherel is the count of kernels.

$$k_{com} = {w_0}^b k_1 + {w_0}^b k_{2+\cdots+{w_0}^b k_l}$$

The stage 1 includes all the six steps to discover the optimal count of clusters and the stage 1 process returns the bestchromosomewiththe optimalcountofclusters andtheprimarycentroidsof theclusters.

Stage 2: The main objective of the stage 2 is to set the multiple kernel's coefficients w_i to produce the best clustering result. Similar to Stage 1, the Stage 2 process also uses the GA to get best results through the fitness value of 21 and it includesall the six main steps. After thecompletion of these two stages, the algorithm achievesbetter clusteringperformance with the cluster count C_{opt}, the cluster centroids V (v_1 ...v_{copt}) and the multiple kernel's coefficientsw_1...$w_i$with the fitness measure f_0 .

III. Validating the Cluster Validity Indices for Interval Type-2 Fuzzy Based Clustering Algorithms

The type-2 fuzzy set has the membership measures areattained as follow [XXXVI]:

$$a_{ij} = \mu_{ij} - \frac{1 - \mu_{ij}}{2}$$

Where$\mu_{ij}$and $a_{ij}$are the initial membership and type-2 membership respectively. After cluster center updation[XLIX],the function area of type-2 membership can be taken as the unreliability of type-1 membership. Therefore, in this paper it is proposed to apply the cluster validity indices for upper and lower centriods separately, after that the cumulative cluster validity index is calculated; because to perform accurate validation in interval type-2 fuzzy based clustering algorithm in type-2 fuzzy membership area. The cluster validity indices are defined either making use of these membership values or membership values with data set. The 14 cluster validity indices values have been calculated which are proposed for Type-1 based FCM clustering algorithm by various researchers.

Table 1 lists the cluster validation indices which are modified for IT2FCM, based on type-1 FCM cluster validity indices.Here x_j is j^{th} data point,c_i is the cluster count,$\tilde{v}_i$ is the center of the cluster, $\bar{\tilde{v}}$ is the grand mean of given data and $\tilde{u}_{ij}$ is the membership measure of data $x_j$atcluster c .In Table 1, thefirst and second columnsshowthe validity index of cluster and the cluster validity index description respectively.The last column indicates the cluster validity index should search for the resultant value of the function.

Table 1. List of modified cluster validity indices

Validity index	**Function description**	**Search for**
Validity involving only the membership values		
Interval Type-2PartitionCoefficient(PC)	$\tilde{V}_{PC} = \frac{1}{n}\sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij}^{2}$	Max
Interval Type-2 Partition Entropy (PE)	$\tilde{V}_{PE} = -\frac{1}{n}\sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij} log\, \tilde{u}_{ij}$	Min
Interval Type-2 Modification of Partition Coefficient (MPC)	$\tilde{V}_{MPC} = 1 - \frac{c}{c-1}(1-\tilde{V}_{PC})$	Max
Interval Type-2 Chen and Linkens. (Validity index P)	$\tilde{V}_{P} = \frac{1}{n}\sum_{k=1}^{n}\max_{i}\tilde{u}_{ik} - \frac{1}{k}\sum_{i=1}^{c-1}\sum_{j=i+1}^{c}\left[\frac{1}{n}\sum_{k=1}^{n}\min(\tilde{u}_{ik},\tilde{u}_{jk})\right]$	Max
Validity including the dataset and the membership measures		
Interval Type-2 Fukuyama and Sugeno(FS)	$\tilde{V}_{FS} = \sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij}^{m}\left\|x_j-\tilde{v}_i\right\|^2 - \sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij}^{m}\left\|\tilde{v}_i-\bar{\tilde{v}}\right\|^2$	Min
Interval Type-2 Xie and Beni(XB)	$\tilde{V}_{XB} = \frac{\sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij}^{m}\left\|x_j-\tilde{v}_i\right\|^2}{n\min_{i\neq j}\left\|\tilde{v}_i-\tilde{v}_j\right\|^2}$	Min
Interval Type-2 Kwon(K)	$\tilde{V}_{K} = \frac{\sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij}^{2}\left\|x_j-\tilde{v}_i\right\|^2 + \frac{1}{c}\sum_{j=1}^{n}\left\|\tilde{v}_i-\bar{\tilde{v}}\right\|^2}{\min_{i\neq j}\left\|\tilde{v}_i-\tilde{v}_j\right\|^2}$	Min
Interval Type-2 Pakhira-Bandyopadhyay-Maulik (PBMF)	$\tilde{V}_{PBMF} = \left(\frac{1}{c}\times\frac{\sum_{j=1}^{n}\left\|x_j-\tilde{v}_i\right\|}{\sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij}^{m}\left\|x_j-\tilde{v}_i\right\|^2}\times \max_{i,j=1}^{c}\left\|\tilde{v}_i-\tilde{v}_j\right\|^2\right)^2$	Min

Interval Type-2 Partition Coefficient and Exponential Separation(PCAES)	$\tilde{V}_{PCAES} = \sum_{i=1}^{c}\sum_{j=1}^{n} \tilde{u}_{ij}^2/\tilde{u}_M - \sum_{i=1}^{c} exp\left(-\min_{k\neq i}\{\lVert\tilde{v}_i - \tilde{v}_k\rVert^2/\tilde{\beta}_T\}\right)$ $\tilde{u}_M = \min_{1\le i\le c}\left\{\sum_{j=1}^{n} \tilde{u}_{ij}^2\right\}$ $\tilde{\beta}_T = \frac{\sum_{l=1}^{c}\lVert\tilde{v}_l - \bar{\tilde{v}}\rVert^2}{c}$	Max
Interval Type-2 Zahid. Separation and Compactness(SC) SC_1 and SC_2 evaluate the separation ratio and compactness.	$\widetilde{SC} = \widetilde{SC}_1(c) - \widetilde{SC}_2(c)$ $\widetilde{SC}_1(c) = \frac{\sum_{i=1}^{c}\lVert\tilde{v}_i - \bar{\tilde{v}}\rVert^2/c}{\sum_{i=1}^{c}\left(\sum_{j=1}^{n}(\tilde{u}_{ij}^m)\lVert x_j - \tilde{v}_i\rVert^2/\sum_{j=1}^{n}\tilde{u}_{ij}\right)}$ $\widetilde{SC}_2(c) = \frac{\sum_{i=1}^{c}\sum_{l=i+1}^{n}\left(\sum_{j=1}^{n} min\left(\tilde{u}_{ij},\tilde{u}_{lj}\right)\right)^2/\sum_{j=1}^{n} min(\tilde{u}_{ij},\tilde{u}}{\sum_{j=1}^{n}\left(max_{1\le i\le c}\tilde{u}_{ij}\right)^2/\sum_{j=1}^{n} max_{1\le i\le c}\tilde{u}_{ij}}$	Max
Interval Type-2 W-index(W)	$\tilde{V}_w = \left(\frac{\widetilde{Var}(U,V)}{\widetilde{Var}_{max}}\right)/\left(\frac{\widetilde{Sep}(c,U)}{\widetilde{Sep}_{max}}\right)$ $\widetilde{Var}(U,V) = \left[\sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij}\left(1 - exp\left(-\frac{\lVert x_j - \tilde{v}_i\rVert^2}{\tilde{\beta}}\right)\right)/n(i)\right] \times \left(\frac{c+1}{c-1}\right)^{1/2}$ $\tilde{\beta} = \left(\frac{\sum_{j=1}^{n}\lVert x_j - \bar{\tilde{v}}\rVert^{-1}}{n}\right)$ $\widetilde{Sep}(c,U) = \max_{i\neq j}\left[\max_{x_k \epsilon X}\min(\tilde{u}_{ik},\tilde{u}_{jk})\right]$ $\widetilde{Var}_{max} = \max_{c}\widetilde{Var}(U,V)$ $\widetilde{Sep}_{max} = \max_{c}\widetilde{Sep}(c,V)$	Min
Interval Type-2 Gath and Geva	$\tilde{V}_{FHV} = \sum_{i=1}^{c}\left[det(\tilde{F}_i)\right]^{1/2}$	Min

Fuzzy Hypervolume Validity (FHV)	$\tilde{F}_i = \frac{\sum_{j=1}^{n}(\tilde{u}_{ij})^m (x_j - \tilde{v}_i)(x_j - \tilde{v}_i)^T}{\sum_{j=1}^{n}(\tilde{u}_{ij})^m}$	
Interval Type-2 Bouguessa and wang (Separation and Compactness as Global)	$\tilde{V}_{SCG}(U,V,X) = \frac{\sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij}^m(\tilde{v}_i - \bar{\tilde{v}})(\tilde{v}_i - \bar{\tilde{v}})^T}{\sum_{i=1}^{c}\left(\sum_{j=1}^{n}\tilde{u}_{ij}^m (x_j - \tilde{v}_i)(x_j - \tilde{v}_i)^T / \sum_{j=1}^{n}\tilde{u}_{ij}^m\right)}$	Max
Interval Type-2 Zarandi Exponential Compactness And Separation (ECAS) index. Exponential Separation (ESsep) Exponential Compactness(ECcomp)	$\tilde{V}_{ECAS} = \frac{\widetilde{EC}_{comp}(c)}{\max_c(\widetilde{EC}_{comp}(c))} - \frac{\widetilde{ES}_{sep}(c)}{\max_c \widetilde{ES}_{sep}(c)}$ $\widetilde{EC}_{comp}(c) = \sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij}^m exp\left(-\left(\frac{\lVert x_i - \tilde{v}_j\rVert^2}{\sum_{k=1}^{n}\lVert x_k - \bar{\tilde{v}}\rVert^2/n(i)} + \frac{1}{c+1}\right)\right)$ $\widetilde{ES}_{sep}(c) = \sum_{i=1}^{c} exp\left(-\min_{i\neq k}\left\{\frac{(c-1)\lVert\tilde{v}_i - \tilde{v}_k\rVert^2}{\sum_{l=1}^{c}\lVert\tilde{v}_l - \bar{\tilde{v}}\rVert^2/c}\right\}\right)$	Max
where $x_j$is j[th] data point, c_i is the cluster count, $v_i$is thecenter of the cluster, $\bar{\tilde{v}} = \frac{\sum_{i=1}^{n}\bar{\mu}(x_i)+\underline{\mu}(x_i)}{n}$ and $\tilde{u}_{ij} = \frac{(\overline{u_{ji}}+u_{ji})}{2}$		

Methodology to calculate cluster validity indices

The validation process of the interval type-2 fuzzy c-means is given as,

Step1: Fix the clustering parameters depends upon the clustering algorithm for the given dataset X. Usually, except number of cluster c, all the parameters are defined.

Step 2: Assign the minimal (c_{min}) and maximal (c_{max}) numbers.

Step 3: For each c = c_{min} to c_{max}

Initialize the centers of the cluster

Implement the interval type-2 fuzzy c-mean clustering algorithm.

For each v_i

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Compute the validity indices for upper and lower centorids separately.

Compute the cumulative validity index.

End for

Return the cumulative validity index.

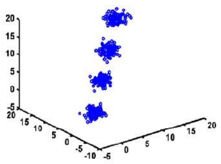
End for

IV. Experimental Results

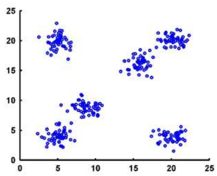
To test validity indices extensive comparisons have been made with various data sets with above mentioned validity indices in IT2FCM, Extended IT2FCM and IT2FCMα, IT2FPCM,MKIT2FCM,GMKIT2FCM. In all experiments the fuzzifier m_1 and $m_2$are set into (6,8) and α=.7 the test form convergence in the IT2FCM algorithm using $\varepsilon=10^{-5}$ and $\|.\|$isthe function of distance specified as Euclidean distance. We have performed the experiment in two ways: (i) Calculating the cluster validity indices in interval type-2 fuzzy membership (i.e. calculating the validity indices upper and lower centriod separately) (ii) Calculating the cluster validity indices in interval type-1 fuzzy membership.For the experiments we have taken the widely used data set and Berkely image database.

Data set

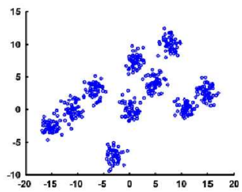
We have evaluate the validity indices of cluster by eight commonartificial datasets which are used by Wang and Zhang[XLIII](Table 2). The eight data sets are Butterfly, Data_4_3,Data_6_2,Data_10_2,Data_15_2, Data_4noise, Example_1, and Example_2. The dataset name itself represents the details of data set, for example Data_4_3 means there are four clusters and three dimensions. Also we have used Butterfly dataset comprises of 3 clusters and 15two dimensional data-points. The datasets Example_1 and Example_2 comprises of 16 two dimensional data-points and 3 & 4 clusters respectively. Fig 1 shows these data sets.



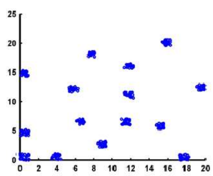
Data_4_3



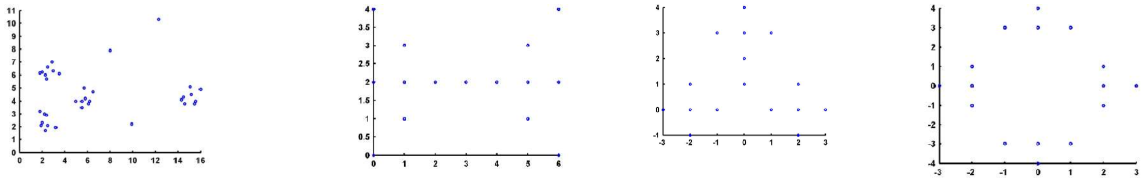
Data_6_2



Data_10_2



Data_15_2



Data_4noise Butterfly Example_1 Example_2

Fig 1. The common datasets

Table 2. Common datasets with its characteristics

Data set	Number of clusters	Dimension
Data_4_3	4	3
Data 6_2	6	2
Data_10_2	10	2
Data_15_2	15	2
Data_4noise	2	4
Butterfly	3	2
Example_1	3	2
Example_2	4	2

Image dataset

Fig 2 demonstrates the sample images taken from the Berkely image database. Literature shows that HSV color space gives the best color differences among the different color spaces. Hence the images are converted into HSV color space. In HSV color space the color is presented in terms of three components: Hue (H), Saturation (S) and Value (V). So a color image of size m X n pixels corresponds to an array of size m X n and three dimensions.

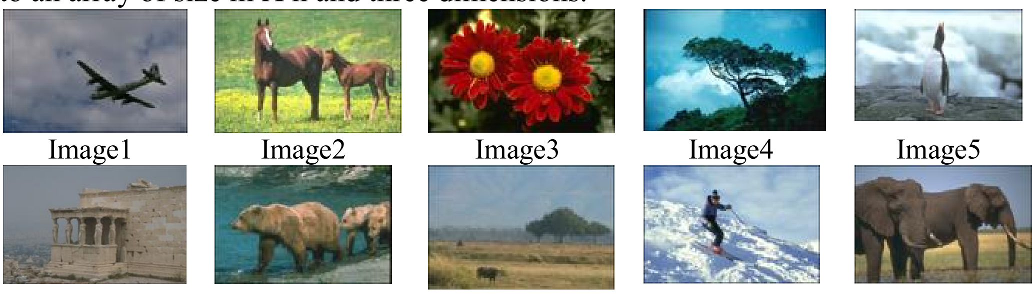


Image1 Image2 Image3 Image4 Image5

Image6 Image7 Image8 Image9 Image10

Fig 2. Sample imput images

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Comparison of cluster Validity indices: Calculating the cluster validity indices in interval type-2 fuzzy membership

In Table 3 it is clearly seen that fourunlike validity indices (includingmembership measures alone) are used to the eight common artificial datasets by implementing IT2FCM, Extended IT2FCM, IT2FCMα, IT2FPCM, MKIT2FCM and GMKIT2FCM respectively. In Table 3, the column c*affords the expectedclusters count for every dataset, and the rest of the columns indicates the optimal count of clusters obtained by applying every index. V_{PC} and V_{MPE} and V_P correctly identifies the optimum number of clusters in the Data_4_3, Data_6_2, Data_10_2, Data_15_2, Butterfly, Example_1 and Example_2 datasets except Data_10_2.V_{PE} fails to recognize$c$*in the Data_10_2data set for the implementation of IT2FCM,Extended-IT2FCM,IT2FCMα, IT2FPCM, MKIT2FCM and GMKIT2FCM.

Table 4 to Table 6 covers the results achieved whiletenunlike validity indices (includingdataset and membership measures)are applied to the abovementioned 8 datasets using IT2FCM, Extended-IT2FCM, IT2FCMα,IT2FPCM, MKIT2FCM and GMKIT2FCMalgorithm. Hence the ideal clustering algorithm is attained when the measureof$\tilde{V}_{FS}$,$\tilde{V}_{XB}$,$\tilde{V}_{K}$,$\tilde{V}_{PBMF}$, ,$\tilde{V}_{FHV}$,$\tilde{V}_{w}$, these cluster validity indices are minimal. Also, $\tilde{V}_{PCAES}$,$\widetilde{SC}$,$\tilde{V}_{SCG}$,$\tilde{V}_{ECAS}$, the value of these cluster validity indices is maximum.

The observation also made from the obtained results that $\tilde{V}_{FS}$,$\tilde{V}_{XB}$,$\tilde{V}_{K}$,$\tilde{V}_{PBMF}$, ,$\tilde{V}_{FHV}$,$\tilde{V}_{w}$, these cluster validity index values are minimum in IT2FCMα,Extended-IT2FCM,IT2FPCM, GMKIT2FCM algorithm compare with IT2FCM and KIT2FCM algorithm. Also$\tilde{V}_{PCAES}$,$\widetilde{SC}$,$\tilde{V}_{SCG}$,$\tilde{V}_{ECAS}$, these cluster validity index values are maximum in IT2FCMα and Extended-IT2FCM IT2FPCM and GMKIT2FCM algorithm compare with IT2FCM and MKIT2FCM algorithm. Out of the experimental results it is shown that the proposed IT2FCMα and Extended-IT2FCM IT2FPCM and GMKIT2FCM algorithmsidentify the optimal cluster count for 8 datasets in a correct manner.

Table 7outlines the results got while the four unlike validity indices (including the membership measures alone), are implemented to Berkely image database (shown in Figure 2) by implementing IT2FCM, Extended IT2FCM, IT2FCMα, IT2FPCM, MIT2FCMand GMIT2FCMrespectively.In Table 7, the column c* provides the exact cluster counts expected for every dataset, and the remaining columns demonstrate the optimal cluster count obtained by applying every index.Table 8 to Table 10 summarize the results got while the tenunlike validity indices(includingthe dataset and the membership measures)are used to the abovementioned 8 datasets using IT2FCM, Extended-IT2FCM, IT2FCMα, IT2FPCM, MIT2FCM and GMIT2FCM algorithm. The obtained results show that $\tilde{V}_{FS}$,$\tilde{V}_{XB}$,$\tilde{V}_{K}$,$\tilde{V}_{PBMF}$, ,$\tilde{V}_{FHV}$,$\tilde{V}_{w}$,these cluster validity index values are minimum in IT2FCMα, Extended-IT2FCM,IT2FPCM and GMKIT2FCM algorithm compare with IT2FCM and MKIT2FCMalgorithm. Also$\tilde{V}_{PCAES}$,$\widetilde{SC}$,$\tilde{V}_{SCG}$,$\tilde{V}_{ECAS}$ these cluster validity index values are maximum in IT2FCMα, Extended-IT2FCM, IT2FPCM and GMKIT2FCM algorithm compare with IT2FCM and MKIT2FCMalgorithm. Therefore, the experimental results demonstrate that the proposed algorithmsIT2FCMα and Extended-IT2FCM IT2FPCM and GMKIT2FCM correctly recognize the optimal cluster counts for Berkely image database.

Comparison of cluster Validity indices: Calculating the validity indices of cluster in interval type-1 fuzzy membership

In order to gain a performance analysisover theproposed method of validating the validity index of cluster, a comparison is formulatedby usingvalidity indices of the existing fuzzy cluster between the type-2 fuzzy membership and type-1 fuzzy membership.In Table 11one can see the summary of the results got while the four unlike validity indices (including the membership measures alone), are applied to Berkely image database (shown in Figure 2) by implementing IT2FCM, Extended IT2FCM , IT2FCMα, IT2FPCM, MIT2FCM and GMIT2FCM respectively.

In Table 11, the column c^* provides the actual cluster count for every dataset, and the other columns show the optimal number of clusters obtained using each index.Table 12 to Table 14 summarize the results got while the ten unlike validity indices(including the dataset and membership measures) are applied to the abovementioned 8 datasets using IT2FCM, Extended-IT2FCM, IT2FCMα, IT2FPCM, MIT2FCM and GMIT2FCMalgorithm.Comparison between the data values of Table 7 to Table 10 and Table 11 to Table 14, calculating cluster validity indices in type-2 fuzzy membership is found more desirable for validating the interval type-2 fuzzy based clustering algorithm.

V. Conclusion

In this paper we have applied the cluster validity indices for upper and lower centriods separately, after that the cumulative cluster validity index is calculated; because to perform accurate validation in interval type-2 fuzzy based clustering algorithm in type-2 fuzzy membership area. The cluster validity indices are defined either making use of these membership values or membership measures with dataset. The measure of 14 cluster validity indices has been calculated which are only proposed for Type-1 based FCM clustering algorithm by various researchers. Extensive comparisons have been done with artificial and Berkely Image Database. The results show that the new way of validating the validity indices outer performs the IT2FCM based algorithms. Also the cluster validity indices are able to handle membership measures with datasets. From the experimental analysis it is noted that calculating cluster validity indices in type-2 fuzzy membership is more desirable for validating the interval type-2 fuzzy based clustering algorithm. Further experiment is required to evaluate the other overlapping clusters and noise points.

Table 3. Values of c preferred by validity involving only the membership values for 8 dataset

		IT2FCM				Extended-IT2FCM				IT2FCMα				IT2FPCM				MKIT2FCM				GMKIT2FCM			
	c*	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_{P}$	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_{P}$	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_{P}$	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_{P}$	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_{P}$	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_{P}$
Data_4_3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
Data 6_2	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
Data_10_2	10	5	6	10	10	6	8	10	10	7	8	10	10	5	6	10	10	6	8	9	9	6	9	9	10
Data_15_2	15	15	16	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	14	15	15	15	15	15
Data_4 noise	4	4	5	4	4	4	4	4	4	4	4	4	4	4	5	4	4	4	4	4	4	4	4	4	4
Butterfly	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Example_1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Example_2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

Table 4. Values of c preferred by validity involving the membership values and data set for 8 dataset (IT2FCM, Extended-IT2FCM)

	IT2FCM										Extended-IT2FCM									
	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$
Data_4_3	.128	.119	.115	.124	.721	.825	.121	.128	.835	.921	.112	.109	.105	.104	.822	.895	.116	.112	.885	.931
Data 6_2	.125	.101	.128	.119	.786	.845	.126	.113	.789	.832	.115	.099	.118	.098	.896	.915	.112	.093	.819	.912
Data_10_2	.125	.119	.122	.129	.821	.901	.132	.122	.825	.912	.105	.109	.120	.119	.879	.925	.122	.112	.845	.932
Data_15_2	.132	.112	.139	.122	.799	.923	.125	.116	.875	.925	.121	.097	.129	.102	.845	.935	.092	.096	.915	.945
Data_4noise	.142	.132	.135	.128	.629	.122	.133	.125	.721	.820	.132	.125	.119	.128	.821	.859	.116	.103	.878	.889
Butterfly	.118	.102	.119	.103	.801	.889	.138	.125	.892	.924	.108	.095	.109	.099	.898	.921	.128	.098	.922	.924
Example_1	.119	.102	.122	.111	.825	.912	.121	.112	.901	.932	.099	.092	.112	.102	.925	.919	.103	.102	.941	.942
Example_2	.122	.110	.112	.123	.729	.852	.125	.122	.856	.921	.112	.102	.092	.118	.859	.952	.120	.112	.952	.931

Table 5. Values of c preferred by validity involving the membership values and data set for 8 dataset ((IT2FCMα, IT2FPCM)

	IT2FCMα										IT2FPCM									
	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PB}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECA}$	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PB}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$
Data_4_3	.102	.098	.095	.094	.925	.925	.103	.102	.925	.942	.101	.094	.093	.091	.952	.915	.101	.101	.914	.931
Data 6_2	.102	.094	.108	.093	.921	.945	.102	.089	.929	.933	.122	.096	.104	.089	.921	.935	.100	.085	.917	.911
Data_10_2	.099	.092	.112	.109	.952	.945	.110	.102	.935	.942	.095	.090	.110	.104	.925	.923	.112	.104	.932	.920
Data_15_2	.110	.091	.119	.112	.945	.925	.082	.092	.945	.952	.101	.092	.117	.110	.953	.915	.086	.090	.923	.931
Data_4noise	.118	.102	.115	.111	.892	.921	.099	.092	.924	.922	.115	.101	.112	.109	.896	.910	.093	.091	.921	.920
Butterfly	.098	.089	.092	.093	.921	.935	.118	.083	.942	.944	.096	.085	.090	.091	.911	.925	.112	.085	.941	.939
Example_1	.095	.083	.092	.092	.945	.939	.093	.092	.952	.952	.093	.084	.091	.090	.915	.914	.091	.091	.932	.942
Example_2	.102	.092	.085	.103	.951	.956	.118	.093	.964	.941	.104	.091	.083	.102	.931	.931	.115	.089	.924	.911

Table 6. Values of c preferred by validity involving the membership values and data set for 8 dataset (MKIT2FCM, GMKIT2FCM)

	MKIT2FCM										GMKIT2FCM									
	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_K$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_w$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_K$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_w$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$
Data_4_3	.118	.114	.112	.122	.711	.814	.120	.124	.853	.911	.098	.099	.103	.106	.792	.875	.096	.102	.875	.911
Data6_2	.123	.101	.122	.115	.746	.834	.122	.112	.797	.822	.095	.081	.112	.101	.866	.895	.092	.083	.809	.892
Data_10_2	.122	.116	.120	.122	.811	.900	.123	.120	.822	.902	.095	.099	.118	.109	.849	.905	.102	.102	.835	.912
Data_15_2	.122	.112	.135	.118	.753	.912	.119	.114	.852	.905	.102	.092	.129	.105	.815	.915	.072	.086	.905	.925
Data_4noise	.132	.123	.131	.124	.621	.121	.131	.127	.701	.810	.112	.112	.119	.122	.791	.839	.096	.093	.868	.869
Butterfly	.108	.112	.115	.101	.799	.872	.132	.123	.889	.914	.088	.082	.109	.097	.868	.901	.108	.088	.912	.904
Example_1	.109	.122	.120	.109	.824	.901	.118	.111	.900	.922	.089	.082	.112	.102	.895	.899	.083	.092	.931	.922
Example_2	.124	.108	.108	.121	.719	.825	.121	.120	.852	.911	.092	.090	.092	.116	.829	.932	0.10	.102	.942	.911

Table 7. Values of c preferred by validity involving only the membership values for 8 dataset

		IT2FCM				Extended-IT2FCM				IT2FCMα				IT2FPCM				MKIT2FCM				GMKIT2FCM			
	2	2	3	2	2	2	2	2	2	2	2	2	2	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_P$	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_P$	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_P$
Image 1	7	6	6	7	7	7	7	7	7	7	7	7	7	2	3	2	2	2	2	2	2	2	2	2	2
Image 2	5	5	6	5	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	7
Image 3	7	8	7	7	7	7	7	7	7	7	7	7	7	5	5	5	6	6	6	6	6	6	6	6	6
Image 4	5	5	6	5	5	5	5	5	5	5	5	5	5	8	7	7	7	6	7	7	7	6	7	6	7
Image 5	10	11	12	10	11	10	10	10	10	10	10	10	10	5	4	5	5	5	5	5	5	5	5	5	5
Image 6	13	12	14	13	12	13	13	13	13	13	13	13	13	11	12	10	11	10	10	10	10	10	10	10	10
Image 7	22	21	24	22	23	22	22	22	22	22	22	22	22	12	13	13	12	13	13	13	13	13	13	13	13
Image 8	10	11	9	10	12	10	10	10	10	10	10	10	10	21	24	21	23	22	22	21	22	22	22	21	22
Image 9	10	9	10	11	10	10	10	10	10	10	10	10	10	11	9	9	12	10	10	10	10	10	10	9	10
Image 10	2	2	3	2	2	2	2	2	2	2	2	2	2	9	9	10	10	10	10	10	9	10	10	10	10

Table 8. Values of c preferred by validity involving the membership values and data set for 8 dataset (IT2FCM, Extended-IT2FCM)

	IT2FCM										Extended-IT2FCM									
	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$
Image1	.178	.189	.165	.145	.154	.138	.621	.725	.835	.901	.121	.135	.149	.145	.114	.128	.721	.893	.913	.923
Image2	.165	.151	.158	.146	.149	.123	.686	.745	.789	.826	.125	.128	.111	.146	.129	.113	.822	.824	.828	.856
Image3	.185	.179	.162	.152	.159	.142	.721	.801	.825	.892	.125	.142	.159	.152	.129	.121	.828	.901	.921	.923
Image4	.157	.162	.159	.155	.152	.136	.699	.723	.875	.923	.117	.119	.142	.155	.122	.116	.795	.934	.924	.923
Image5	.168	.152	.149	.148	.143	.145	.701	.789	.892	.901	.128	.129	.112	.148	.133	.136	.813	.912	.925	.921
Image6	.159	.152	.142	.141	.139	.132	.725	.812	.901	.923	.129	.112	.122	.141	.119	.142	.825	.932	.921	.925
Image7	.162	.160	.151	.155	.153	.142	.629	.721	.856	.901	.122	.121	.140	.155	.123	.122	.792	.915	.925	.912
Image8	.163	.161	.152	.153	.149	.131	.721	.792	.901	.923	.143	.122	.121	.153	.109	.125	.821	.924	.934	.923
Image9	.169	.163	.153	.162	.155	.142	.625	.789	.892	.921	.149	.113	.143	.142	.115	.112	.725	.920	.910	.925
Image10	.162	.158	.146	.156	.152	.146	.721	.865	.910	.931	.142	.126	.118	.136	.132	.122	.821	.921	.931	.926

Table 9. Values of c preferred by validity involving the membership values and data set for 8 dataset ((IT2FCMα, IT2FPCM)

	IT2FCMα										IT2FPCM									
	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECA}$	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$
Image1	.118	.125	.129	.125	.102	.113	.801	.898	.933	.933	.168	.169	.135	.135	.134	.148	.611	.735	.815	.881
Image2	.105	.118	.091	.126	.119	.092	.898	.912	.858	.856	.155	.131	.128	.136	.129	.133	.676	.755	.769	.806
Image3	.115	.132	.139	.122	.112	.110	.921	.923	.951	.933	.175	.159	.132	.142	.139	.152	.711	.811	.805	.872
Image4	.102	.109	.122	.135	.092	.102	.845	.942	.944	.943	.147	.142	.129	.145	.132	.146	.689	.733	.855	.903
Image5	.115	.119	.106	.128	.123	.126	.893	.922	.935	.931	.158	.132	.119	.138	.123	.155	.691	.799	.872	.881
Image6	.112	.102	.112	.115	.104	.122	.915	.952	.931	.955	.149	.132	.112	.131	.119	.142	.715	.822	.881	.903
Image7	.108	.099	.119	.131	.113	.112	.895	.925	.935	.922	.152	.14	.121	.145	.133	.152	.619	.731	.836	.881
Image8	.123	.112	.110	.123	.097	.099	.921	.914	.944	.933	.153	.141	.122	.143	.129	.141	.711	.802	.881	.903
Image9	.129	.093	.133	.124	.105	.092	.825	.925	.930	.928	.159	.143	.123	.152	.135	.152	.615	.799	.872	.901
Image10	.122	.112	.098	.126	.122	.112	.891	.922	.941	.929	.152	.138	.116	.146	.132	.156	.711	.875	.89	.911

Table 10. Values of c preferred by validity involving the membership values and data set for 8 dataset (MKIT2FCM, GMKIT2FCM)

	MKIT2FCM										GMKIT2FCM									
	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$
Image1	.131	.125	.169	.125	.124	.118	.741	.873	.893	.933	.108	.119	.133	.096	.772	.895	.106	.082	.865	.921
Image2	.135	.118	.131	.126	.139	.103	.842	.804	.808	.866	.105	.101	.142	.091	.846	.915	.102	.063	.799	.902
Image3	.135	.132	.179	.132	.139	.111	.848	.881	.901	.933	.105	.119	.148	.099	.829	.925	.112	.082	.825	.922
Image4	.127	.109	.162	.135	.132	.106	.815	.914	.904	.933	.112	.112	.159	.095	.795	.935	.082	.066	.895	.935
Image5	.138	.119	.132	.128	.143	.126	.833	.892	.905	.931	.122	.132	.149	.112	.771	.859	.106	.073	.858	.879
Image6	.139	.102	.142	.121	.129	.132	.845	.912	.901	.935	.098	.102	.139	.087	.848	.921	.118	.068	.902	.914
Image7	.132	.111	.16	.135	.133	.112	.812	.895	.905	.922	.099	.102	.142	.092	.875	.919	.093	.072	.921	.932
Image8	.153	.112	.141	.133	.119	.115	.841	.904	.914	.933	.102	.11	.122	.106	.809	.952	.110	.082	.932	.921
Image9	.159	.103	.163	.122	.125	.102	.745	.901	.890	.935	.108	.119	.133	.096	.772	.895	.106	.082	.865	.921
Image10	.152	.116	.138	.116	.142	.112	.841	.901	.911	.936	.105	.101	.142	.091	.846	.915	.102	.063	.799	.902

Table 11. Values of c preferred by validity involving only the membership values for 8 dataset -interval type-1 fuzzy membership

		IT2FCM				Extended-IT2FCM				IT2FCMα				IT2FPCM				MKIT2FCM				GMKIT2FCM			
	2	2	3	2	2	2	2	2	2	2	2	2	2	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_{P}$	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_{P}$	$\tilde{V}_{PC}$	$\tilde{V}_{PE}$	$\tilde{V}_{MP}$	$\tilde{V}_{P}$
Image 1	7	6	6	7	7	7	7	7	7	7	7	7	7	2	3	2	2	2	2	2	2	2	2	2	2
Image 2	5	5	6	5	6	6	6	6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	7	7	7
Image 3	7	8	7	7	7	7	7	7	7	7	7	7	7	5	5	5	6	6	6	6	6	6	6	6	6
Image 4	5	5	6	5	5	5	5	5	5	5	5	5	5	8	7	7	7	6	7	7	7	6	7	6	7
Image 5	10	11	12	10	11	10	10	10	10	10	10	10	10	5	4	5	5	5	5	5	5	5	5	5	5
Image 6	13	12	14	13	12	13	13	13	13	13	13	13	13	11	12	10	11	10	10	10	10	10	10	10	10
Image7	22	21	24	22	23	22	22	22	22	22	22	22	22	12	13	13	12	13	13	13	13	13	13	13	13
Image 8	10	11	9	10	12	10	10	10	10	10	10	10	10	21	24	21	23	22	22	21	22	22	22	21	22
Image 9	10	9	10	11	10	10	10	10	10	10	10	10	10	11	9	9	12	10	10	10	10	10	10	9	10
Image 10	2	2	3	2	2	2	2	2	2	2	2	2	2	9	9	10	10	10	10	10	9	10	10	10	10

Table 12. Values of c preferred by validity involving the membership values and data set for 8 dataset (IT2FCM, Extended-IT2FCM)- interval type-1 fuzzy membership

	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PB}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PC}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{EC}$	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PB}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PC}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$
Image1	.148	.159	.135	.115	.124	.108	.591	.695	.805	.871	.091	.105	.119	.115	.084	.098	.691	.863	.883	.893
Image2	.135	.121	.128	.116	.119	.093	.656	.715	.759	.796	.095	.098	.081	.116	.099	.083	.792	.794	.798	.826
Image3	.155	.149	.132	.122	.129	.112	.691	.771	.795	.862	.095	.112	.129	.122	.099	.091	.798	.871	.891	.893
Image4	.127	.132	.129	.125	.122	.106	.669	.693	.845	.893	.087	.089	.112	.125	.092	.086	.765	.904	.894	.893
Image5	.138	.122	.119	.118	.113	.115	.671	.759	.862	.871	.098	.099	.082	.118	.103	.106	.783	.882	.895	.891
Image6	.129	.122	.112	.111	.109	.102	.695	.782	.871	.893	.099	.082	.092	.111	.089	.112	.795	.902	.891	.895
Image7	.132	.13	.121	.125	.123	.112	.599	.691	.826	.871	.092	.091	.11	.125	.093	.092	.762	.885	.895	.882

Image8	.133	.131	.122	.123	.119	.101	.691	.762	.871	.893	.113	.092	.091	.123	.079	.095	.791	.894	.904	.893
Image9	.139	.133	.123	.132	.125	.112	.595	.759	.862	.891	.119	.083	.113	.112	.085	.082	.695	.890	.88	.895
Image10	.132	.128	.116	.126	.122	.116	.691	.835	.880	.901	.112	.096	.088	.106	.102	.092	.791	.891	.901	.896

Table 13. Values of c preferred by validity involving the membership values and data set for 8 dataset ((IT2FCMα, IT2FPCM)- interval type-1 fuzzy membership

	IT2FCMα										IT2FPCM									
	$\tilde{V}_{F}$	$\tilde{V}_{X}$	$\tilde{V}_{K}$	$\tilde{V}_{PB}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PC}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{EC}$	$\tilde{V}_{F}$	$\tilde{V}_{X}$	$\tilde{V}_{K}$	$\tilde{V}_{PB}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PC}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$
Image1	.088	.095	.099	.095	.072	.083	.771	.868	.903	.903	.138	.139	.105	.105	.104	.118	.581	.705	.785	.851
Image2	.075	.088	.061	.096	.089	.062	.868	.882	.828	.826	.125	.101	.098	.106	.099	.103	.646	.725	.739	.776
Image3	.085	.102	.109	.092	.082	.080	.891	.893	.921	.903	.145	.129	.102	.112	.109	.122	.681	.781	.775	.842
Image4	.07	.07	.09	.105	.062	.07	.815	.912	.914	.913	.11	.112	.09	.115	.102	.11	.659	.703	.825	.873

	2	9	2			2					7		9			6				
Image5	.085	.089	.076	.098	.093	.096	.863	.892	.905	.901	.128	.102	.089	.108	.093	.125	661	.769	.842	.851
Image6	.082	.072	.082	.085	.074	.092	.885	.922	.901	.925	.119	.102	.082	.101	.089	.112	.685	.792	.851	.873
Image7	.078	.069	.089	.101	.083	.082	.865	.895	.905	.892	.122	.11	.091	.115	.103	.122	.589	.701	.806	.851
Image8	.093	.082	.080	.093	.067	.069	.891	.884	.914	.903	.123	.111	.092	.113	.099	.111	.681	.772	.851	.873
Image9	.099	.063	.103	.094	.075	.062	.795	.895	.901	.898	.129	.113	.093	.122	.105	.122	.585	.769	.842	.871
Image10	.092	.082	.068	.096	.092	.082	.861	.892	.911	.899	.122	.108	.086	.116	.102	.126	.681	.845	.86	.881

Table 14. Values of c preferred by validity involving the membership values and data set for 8 dataset (MKIT2FCM, GMKIT2FCM) -interval type-1 fuzzy membership

	MKIT2FCM										GMKIT2FCM									
	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$	$\tilde{V}_{FS}$	$\tilde{V}_{XB}$	$\tilde{V}_{K}$	$\tilde{V}_{PBM}$	$\tilde{V}_{FH}$	$\tilde{V}_{w}$	$\tilde{V}_{PCA}$	$\widetilde{SC}$	$\tilde{V}_{SC}$	$\tilde{V}_{ECAS}$
Image1	.101	.095	.139	.095	.094	.088	.711	.843	.863	.903	.078	.089	.103	.066	.742	.865	.076	.052	.835	.891
Ima	.1	.0	.1	.0	.1	.0	.81	.7	.7	.8	.0	.0	.1	.0	.8	.8	.07	.0	.7	.8

ge2	05	88	01	96	09	73	2	74	78	36	75	71	12	61	16	85	2	33	69	72
Image3	.105	.102	.149	.102	.109	.081	.818	.851	.871	.903	.075	.089	.118	.069	.799	.895	.082	.052	.795	.892
Image4	.097	.079	.132	.105	.102	.076	.785	.884	.874	.903	.082	.082	.129	.065	.765	.905	.052	.036	.865	.905
Image5	.108	.089	.102	.098	.113	.096	.803	.862	.875	.901	.092	.102	.119	.082	.741	.829	.076	.043	.828	.849
Image6	.109	.072	.112	.091	.099	.102	.815	.882	.871	.905	.068	.072	.109	.057	.818	.891	.088	.038	.872	.884
Image7	.102	.081	.131	.105	.103	.082	.782	.865	.875	.892	.069	.072	.112	.062	.845	.889	.063	.042	.891	.902
Image8	.123	.082	.111	.103	.089	.085	.811	.874	.884	.903	.072	.081	.092	.076	.779	.922	.081	.052	.902	.891
Image9	.129	.073	.133	.092	.095	.072	.715	.871	.861	.905	.078	.089	.103	.066	.742	.865	.076	.052	.835	.891
Image10	.122	.086	.108	.086	.112	.082	.811	.871	.881	.906	.075	.071	.112	.061	.816	.885	.072	.033	.769	.872

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