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NANO g*b-CLOSED SETS IN NANO TOPOLOGICAL SPACES

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Abstract

*The notion of the paper has to investigate new set named as nano g*b-closed sets(Ng*b) in nano topological spaces(NTS). Some importantresults of Ng*b-closed sets are analysed. Also, we examine the relationship of Ng*b-closed sets with other sets in NTS.*

Keywords: Nanog*b-closed, nanobclosure.

1. Introduction

The idea of gclosed sets in TS was first investigated by Levine. Andrijevic initiated and studied b-open sets. Vidhya discussed the studiedthe g*bclosed set in TS. Lellis Thivagar introduced a NTSin which he examined the terms of higher &lower approximation , boundary region. Nopen, Nclosed, Ninterior and Nclosure of a set are the elements of NTS. In this article, we introduced a new set on NTS called Ng*b-closed sets and analyzed the association of Ng*bclosed set with other sets. From the beginning to end of this paper (P,τ) and (Q,σ)express the non-empty TS.

II. Preliminaries

Definition 2.1:"Consider℧ be a non-empty finite set of objects called the universe and be an equivalence relation on ℧ named as the indiscernibility relation. Thus℧ is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are indiscernible with one another. The pair (℧,R) are called approximation space". Let $P \subseteq ℧$.

1. "The lower approximation of P with respect to R is the set of all objects, which classified as P with

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respect to R and given as$\mathcal{L}_R(P)$.

$L_R(P) = \mho_{p\in U}\{R(p) : R(p)\subseteq P\}$

where R(p) represents the equivalence class determined by p".

2. "The upper approximation of P with respect to R is the set of all objects, which classified as Pwith

respect to R and given as $\mho_R(P)$.

$\mho_R(P) = \mho_{p\in U}\{R(p) : R(p) \cap P \neq \phi\}$"

3. "The boundary region of P with respect to R is the set of all objects, which classifies neither as P

nor as notP with respect to R and it is given as $B_R(P)$.

$B_R(P) = \mho_R(P)- \mathcal{L}_R(P)$"

Property 2.2"Consider $P,Q\subseteq U$ and $(\mho,R)$ is an approximation space, then

(a) $\mathcal{L}_R(P)\subseteq P\subseteq \mho_R(P)$

(b)$\mathcal{L}_R(\phi) = \mho_R(\phi) =\phi$and $\mathcal{L}_R(\mho) = \mho_R(\mho) = \mho$

(c)$\mho_R(P\cup Q) = \mho_R(P)\cup\mho_R(Q)$

(d) $\mho_R(P\cap Q)\subseteq \mho_R(P) \cap\mho_R(Q)$

(e) $\mathcal{L}_R(P\cup Q)\supseteq\mathcal{L}_R(P) \cup\mathcal{L}_R(Q)$

(f) $\mathcal{L}_R(P\cap Q) = \mathcal{L}_R(P) \cap\mathcal{L}_R(Q)$

(g) $\mathcal{L}_R(P)\subseteq\mathcal{L}_R(Q)$ and $\mho_R(P) \subseteq \mho_R(Q)$ whenever $P\subseteq Q$

(h) $\mho_R(P^c) = [\mathcal{L}_R(P)]^c$ and $\mathcal{L}_R(P^c) = [\mho_R(P)]^c$

(i) $\mho_R\mho_R(P) = \mathcal{L}_R\mho_R(P) = \mho_R(P)$

(j) $\mathcal{L}_R\mathcal{L}_R(P) = \mho_R\mathcal{L}_R(P) = \mathcal{L}_R(P)$"

Definition 2.3 "Let $\mho$ be the universe, R be an equivalence relation on $\mho$ and $\tau_R(P) = \{\mho,\phi,\mathcal{L}_R(P),\mho_R(P),B_R(P)\}$ where $P\subseteq\mho$Then $\tau_R(P)$ satisfies the following three axioms

1. $\mho$ and $\phi\in\tau_R(P)$

2. The union of the elements of any subcollection of $\tau_R(P)$ is in $\tau_R(P)$

3. The intersection of the elements of any finite subcollection of $\tau_R(P)$ is in $\tau_R(P)$".

Thus$\tau_R(P)$ is considered as topology on $\mho$ called the NT on $\mho$with reference toP. Therefore $(\mho,\tau_R(P))$ is the NTS. The elements of $\tau_R(P)$ are Nopen sets and the complement of Nopen are Nclosed sets.

Remark 2.4[VI]The set B = {Ʊ,$\mathcal{L}_R$(P), B_R(P)} is called as basis of τ_R(P),where τ_R(P) is the NT on Ʊconsidering P.

Definition 2.5[VI]Consider (Ʊ,τ_R(P)) is a NTS with reference to P, where P⊆Ʊ and C⊆Ʊ, then

(i) The union of every Nopen subsets contained in Cwhich is represented as Nint(C) is known astheNinterior of the set C.

(ii) The intersection of everyNclosed subsets containing Cwhich is represented asNcl(C) is known as the Nclosure of C.

Definition 2.6 Consider (Ʊ, τ_R(P)) be a NTS and C⊆Ʊ. ThusC is called as

(i) Nsemiopen if C⊆Ncl(Nint(C)

(ii) Npreopen if C ⊆Nint(Ncl(C))

(iii) Nregularopen ifC = Nint(Ncl(C))

(iv) Nsemi preopen if C⊆Ncl(Nint(Ncl(C)))

(v) Nbopen if C⊆Ncl(Nint(C))∪Nint(Ncl(C))

Definition 2.7 Let (Ʊ,τ_R(P)) be a NTS and C ⊆Ʊ. Then C is said to be Nsemi-closed (Nregular closed, Npre-closed, Nclosed, Nsemi pre-closed, Nb-closed respectively) if its complement is Nsemi-open(Nregular open, Npre-open, Nopen, Nsemi pre-open, Nb-open respectively)

Definition 2.8 Let (Ʊ,τ_R(P)) be a NTS and C⊆Ʊ. ThusC is called as

(i) Ngclosed if Ncl(C) ⊆S whenever C⊆S and S is Nopen in Ʊ.

(ii) Nwgclosed if Ncl(Nint(C))⊆S whenever C⊆S and S is Nopen in Ʊ.

(iii)Ngpclosed if Npcl(C) ⊆S whenever C⊆S and S is Nopenin Ʊ.

(iv)Ngprclosed if Npcl(C) ⊆S whenever C⊆S and S is Nregularopen in Ʊ.

(v)Ngbclosed if Nbcl(C) ⊆S whenever C⊆S and S is Nopen in Ʊ.

III. Nano g*b-Closed Sets

Definition 3.1: A subset C of NTS (Ʊ,τ_R(P)) is known as"nanogeneralized star bclosed" (briefly, Ng*bclosed), if Nbcl(C)⊆S whenever C⊆SwhereS is Ngopen in Ʊ.

Theorem 3.2: Every Nclosed set is Ng*bclosed.

Proof: HereC be a Nclosed set in Ʊ,S is Ng-open in Ʊ∍ C⊆S, Nbcl(C) = C⊆S.

Thus we obtainNbcl(C)⊆S. ThereforeNclosed set C is Ng*bclosed set in Ʊ.

Remark 3.3: The next example explains that converse partof theorem 3.2 may not be true.

Example 3.4: ConsiderƱ = {e,f,g,h}with Ʊ/R = {{e},{g},{f,h}} and P = {e,f}$\subseteq$Ʊ. Finally nano topology is defined as $\tau_R(P)$ = {Ʊ,ϕ,{e},{f,h},{e,f,h}}. Hence C = {e} is Ng*bclosed set and not Nclosed.

Remark 3.5: The proof of the theorems 3.6 and 3.9can be proved similarly as theorem 3.2.

Theorem 3.6: Let (Ʊ,τ_R(P)) be a NTSandC$\subseteq$S, then

(i) Every Nregularclosed is Ng*bclosed.

(ii)Every Nsemiclosed is Ng*bclosed.

(iii) Every Nαclosed is Ng*bclosed.

(iv) EveryNpreclosed is Ng*bclosed.

(v) Every Ngclosed is Ng*bclosed.

(vi) EveryNgpclosed is Ng*bclosed.

(vii) Every Ngprclosed is Ng*bclosed.

Remark 3.7: The next example explains that converse part of theorem 3.6may not be true.

Example 3.8: ConsiderƱ = {α,β,γ,δ}with Ʊ/R = {{α },{γ},{β,δ}}and P = {α,β}$\subseteq$Ʊ. Hence NT is given as τ_R(P) = {Ʊ,ϕ,{α},{β,δ},{α,β,δ}}.Therefore {β} and {δ} are Ng*bclosed sets but not Nregularclosed and Nsemiclosedsets.Also the set {β,δ} is Ng*bclosed set but not Nclosed and Npreclosed set. Hence the set {α} is Ng*bclosed set but not Ngclosed and Ngpclosed set. Also the set {γ} is Ng*bclosed but not Ngprclosed.

Theorem 3.9: Consider (Ʊ,τ_R(P)) be a NTS and C$\subseteq$S, then every Ng*bclosed set is Nwgclosed, Ngbclosed and Nsemipreclosed.

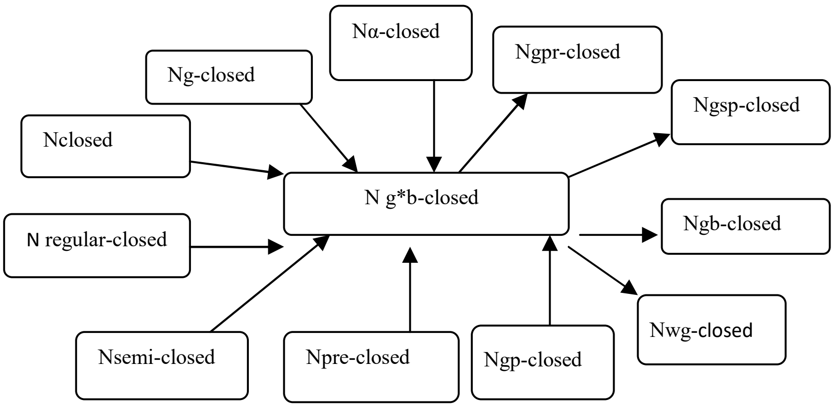
Remark 3.10: The following examples 3.11, 3.12 and 3.13 describes the converse part of the above theorem 3.9.

Example 3.11: ConsideringƱ = {e,f,g,h}along withƱ/R = {{e},{g},{f,h}} and P= {e,f}$\subseteq$ Ʊ. Hence we obtain NTasτ_R(P) = {Ʊ,ϕ,{e},{f,h},{e,f,h}}. Hence the set {e,f,h} is Nwgclosed set but not Ng*bclosed.

Example 3.12: ConsideringƱ = {α,β,γ}along with Ʊ/R = {{α},{β,γ}} and P = {α}$\subseteq$ Ʊ. Hence we obtain nano topology asτ_R(P) = {Ʊ,ϕ,{α }}. Finally the sets{α,β} and {δ} are Ngbclosed sets and not Ng*bclosed set.

Example 3.13: ConsideringƱ = {α,β,γ,δ}along with Ʊ/R = {{α},{γ},{β,δ }}and P = {α,β}$\subseteq$Ʊ. Hencewe obtain nano topology asτ_R(P) = {Ʊ,ϕ,{α},{β,δ},{α,β,δ}}. Therefore the set {α,β} is Nsemipreclosedand not Ng*bclosed.

Corollary 3.14: Thus we conclude that none of the implications are reversible.



Remark 3.15: If M and N are Ng*bclosedin Ʊ, then union of M and N is not Ng*b-closed in Ʊ.

Example 3.16: The proof for the above remark is given below.

ConsiderƱ = {α,β,γ,δ} with Ʊ/R = {{α },{γ},{β,δ}}and P = {α,β}⊆Ʊ. Thusτ_R(P) = {Ʊ,ϕ,{α},{β,δ},{α,β,δ}}. The Ng*bclosed sets in Ʊ are Ʊ,ϕ,{α},{β},{γ},{δ},{α,γ},{β,γ},{β,δ},{γ,δ},{α,β,γ},{α,γ,δ} and {β,γ,δ}. If M = {α} and N = {β}, then M and N are Ng*bclosed but M∪N = {α,β} is not Ng*bclosed in Ʊ.

Theorem 3.17: If K and L are two Ng*bclosed subsets, then K∩L is Ng*bclosed.

Proof: HereK and L be the two Ng*bclosedsubsets in Ʊ ∍K⊆S and L⊆S is Ngopen in Ʊ and so K∩L⊆ S. As KandL are Ng*bclosed, we getK⊆Nbcl(K) &L⊆Nbcl(L) ,finallyK∩L⊆Nbcl(K) ∩Nbcl(L) ⊆Nbcl(K∩L). Thus K∩L is a Ng*b-closed set in (Ʊ,τ_R(P)).

Theorem 3.18: SupposeC is a Ng*bclosed subset ofƱ∍C⊆D⊆Nbcl(C), then D is Ng*bclosed.

Proof: HereS be a Ng-open set of Ʊ∍ D⊆S, then C⊆S. SinceC is Ng*b-closed, we have Nbcl(A)⊆S. Hence Nbcl(C) ⊆Nbcl(Nbcl(C)) = Nbcl(C)⊆S. ThusD is Ng*bclosed.

Theorem 3.19: A subset C of Ʊ is Ng*bclosed if only ifNbcl(C) - Chas no non-empty Ngclosed set.

Proof:

Necessary part: ConsiderD be aNgclosed set of Ʊ∍ D⊆Nbcl(C)-C. The C ⊆Ʊ-D .SinceC is Ng*bclosed and Ʊ – D is Ng-open the Nbcl(C) ⊆Ʊ – D. This givesD⊆Ʊ – Nbcl(C). So D⊆ (Ʊ – Nbcl(C)) ∩ (Nbcl(C) - C)⊆(Ʊ -Nbcl(C)) ∩Nbcl(C) =ϕ.

Sufficient part: ConsiderNbcl(C) - Chas no non-empty Ngclosed set. HereC$\subseteq$S, where S isgopen. If Nbcl(C) $\not\subseteq$S, then Nbcl(C) $\cap S^c$ is a non-empty Ngclosed set ofNbcl(C) –C, condradicts the result.ThusNbcl(C) $\subseteq$S,whereS is Ng*bclosed.

Corollary 3.20: If D be a Nclosed set and C be Ng*bclosed, then C $\cap$ D isg*b-closed set which have been proved below.

Example 3.21: Let℧ = {e,f,g}along with ℧/R = {{e},{f,g}}&P = {e}$\subseteq$℧. Thereforeτ_R(P) = {℧,ϕ,{α}} and $\tau_R{}^c$(P) = {℧,ϕ,{β,γ}}. ThereforeNg*bclosed= {℧,ϕ,{β},{γ},{β,γ}}. If we takeC = {f}&D = {f,g}, then C$\cap$D will be aNg*bclosed set.

IV. Conclusion

Finally, we had constructed a set called Ng*b-closed and studied some of its basic properties.

References

I. A. Dhanis Arul Mary and I. Arockiarani, Onnanogb-closed sets in nano topological spaces, International Journal of Mathematical Archieve, 6(2), (2015), 54-58.

II. A. Dhanis Arul Mary, I. Arockiarani, On Semi pre-closed sets in nano topological spaces, Mathematical Sciences Int. Research Journal, 3(2)(2014), ISSN 2278- 8697.

III. C.R. Parvathy and , S. Praveena, On nano generalized pre regular closed sets in nano topological spaces, IOSR Journal of Mathematics (IOSR-JM), 13(2)(2017), 56-60.

IV. D. Andrijevic, On b-open sets, Mat. Vesink, 48 (1996), 59 - 64.

V. D. Vidhya and R. Parimelazhagan , g*b-closed sets in topological spaces,Int.J. Contemp. Math. Sciences, 7(2012), 1305-1312.

VI. D. Vidhya and R. Parimelazhagan , g*b-Continuous Maps and Pasting Lemma in Topological spaces, Int. J. Math. Analysis, 6(2012), 2307-2315.

VII. I. L. Reilly and Vamanamurthy, On -sets in topological spaces, Tamkang J. Math., 16(1985), 7-11.

VIII. K. Bhuvaneshwari and K. MythiliGnanapriya, Nano generalized closed sets, International Journal of Scientific and Research Publications, 4(5)(2014), 1-3.

IX. K. Bhuvaneswari and K. MythiliGnanapriya,Onnanogeneralised pre closed sets and nano pre generalised closed sets in nano topological spaces, International Journal of Innovative Research in Science, Engineering and Technology, 3(10)(2014), 16825-16829.

X. M. LellisThivagar and Carmel Richard, Note on Nano topological spaces - communicated.

XI. M. LellisThivagar and Carmel Richard, On nano forms of weakly open sets, International Journal of Mathe-matics and statistics Invention, 1(1),(2013),31-37.

XII. M. Parimala , C. Indirani. C and S. Jafari, On Nano b-open sets in Nano topological spaces, Jordan Journal of Mathematics and Statistics,9(3), (2016), 173-184.

XIII. N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo (2) 19 (1970), 89-96.

XIV. N. Nagaveni and M.Bhuvaneswari, On nano weakly generalized closed sets, International Journal of Pure and Applied Mathematics, 106(7),(2016),129-137.