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ADAPTIVE PI-SLIDING MODE CONTROL OF NON-HOLOMONIC WHEELED MOBILE ROBOT

Iman Abdalkarim Hasan[1], Nabil Hassan Hadi[2]

[1,2] Mechanical Engineering Department, College of Engineering, Baghdad University, 10071, Baghdad, Iraq.

Corresponding Author Email: mona.ka93@yahoo.com

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Abstract

Tracking wheeled mobile robot control is a complicated problem encounter in robotic science. Many issues occurring that are affecting the control of nonlinear robot in actual application. The applications would include uncertainties parameter and internal disturbances. The factors restrict the study of mobile robot tracing control. In this study we modified adaptive sliding mode controller for nonholonomic wheeled mobile robot. The kinematic controller used to produce the desired tracking velocities as input term after that used suggested of the dynamic controller to overcome the uncertainties, disturbance and chattering effect of the sliding controller. according to stability of Lyapunov, the final controlled system is proven to be globally asymptotically stable. Proposed control system is verified and validated using MATLAB\SIMULINK to track the required WMR trajectory. A comparison between PI adaptive sliding mode and PI sliding mode is done. Simulated result portrays that in the presence of continuous disturbances and uncertainties and presented work with very good accuracy and fast error convergence and robustness.

Keywords : Wheeled mobile robot, kinematic control, dynamic control, sliding mode control, adaptive control.

I. Introduction

In recent years, nonholonomic mobile robot have wide real applications are used in many purposes Commercial, Military, medical and Industrial. Nonlinear control theory applies to more realistic systems, because all real-time control systems are nonlinear systems. The Target of the control systems is to find the best trajectory tracking for the mobile robot. In order to know the exact value of the parameters, there will be difficulties to find them therefore causing difficulties in scientific tasks. The sliding mode control theory is a common strategy to control nonlinear systems due to system durability and convergence on time and strong against disturbances. Many researchers have proposed different controllers to the method of trajectory tracing using kinematic model IX, the robust PID controller [XVIII], kinematic and dynamic models [XX], backstepping method [XIII,XXI] and fuzzy algorithm [XIX].

And others proposed trajectory tracing methods such neural network [X], sliding mode [XII], and adaptive motion control XVI. These schemas do not contain the dynamic of the mobile robot which are not a processed apparently. With development in research, field wheeled mobile robot work is focused on designing the wheeled mobile robot that can adapted with various environments by estimating the uncertainties using input output feedback linearization [III], sliding mode [IV, VII], neural network [II], adaptive control [XIV, XXII] and fuzzy logic [VI]. The system is based on dynamics of robot and needs that accurate parameter to mobile robot must be known a priori.

The most commonly used and efficient approach to nonlinear control design in latest literature for wheeled mobile robot is the sliding mode control [V]. Because of SMC's powerful robustness characteristics against model uncertainties and disturbances, it is gaining a lot of interest in the robotics Society and several SMC based systems are being suggested to control the trajectory tracking non-holonomic WMR. At first, they presented SMC method using chain form for trajectory tracking of nonholonomic WMR [I]. The authors of [XVII] designed novel sliding mode control law to stabilize mobile robot asymptotically to the required trajectory. In [VII], they developed a new SMC by representing a mobile robot's kinematic equation as two-dimensional polar coordinates.

We know to design conventional SMC creates serious chatter on the sliding surface which consequently deteriorates the systems efficiency through implementing unmodeled dynamics of high frequency [XI]. Due to the transgression of non-holonomic limitations, the dynamic model is time-varying and extremely non-linear, forcing robust stabilizing control to be applied. The authors of [VIII] proposed a method based on a second order sliding mode control to reach the desired trajectory. The authors of [II], presented adaptive neural sliding mode control by use Self recurrent wavelet neural networks (SRWNNs) reducing the effects of uncertainties and external disturbances for dynamics mobile robot. The aim of this work was to tackle the mobile robot trajectory problem by using the benefit of SMC characteristics.

Adaptive PI dynamic sliding mode controller is suggested in this work. Additionally, modified kinematic controller backstepping is used to generate kinematic velocities. in sliding mode dynamic control are used the velocities error and derivative. The Lyapunov stability theory proved that all stability analyses for the complete motion equations of the WMR. In this paper, Results are compared between the PI sliding mode control and adaptive PI sliding mode control.

This paper arranged as follows: WMR kinematic control designed in section 2. Section 3 discusses the modeling of WMR and ASMC. The fourth section discusses the acuter dynamic. in section 5, the MATLAB/Simulink results in molding. Lastly, section 6 concludes this paper.

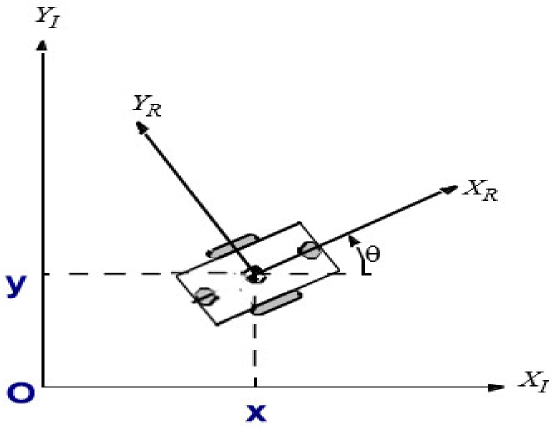


Fig1. model of WMR

II. Kinematic control design

The chassis wheeled mobile robot contain a pair of driving wheels with a caster wheel for supporting proposes. The driving wheel mounted on a DC motor at the same axis. The voltage is the control input for the many of the dc however the DC's motor torque is the input control term in WMR. To produce various path tracing we must control the input voltage for each DC motor as a function of path, time and feedback. So, the must importing part is integrate the mathematical model with output torque of DC motor. Figure 1 show a WMR model.

the kinematic model can be established by representing the linear and angular DDMR velocities in the robot structure.
Let defined posture vector as

$$q(t) = [x(t)\ y(t)\ \theta(t)]^T \tag{1}$$

Defined kinematic model in WMR in global coordinate;

$$\dot{q}(t) = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} = S(\theta)\, p(t) \tag{2}$$

Where $q(t), \dot{q}(t) \epsilon R^3$, $S(\theta)\ \epsilon R^{3\times 2}$ is the Jacobian matrix of the robot. $p(t) \epsilon R^2$ are the control input of the robot , $v(t) \epsilon R$ is the linear velocity, $w(t) \epsilon R$ is the angular velocity of the center of mass. $(x(t), y(t))$, $(\dot{x}(t), \dot{y}(t))$ are the real position and linear velocity of the WMR, $\theta(t) \epsilon R$ is the head angle of the WMR, and $\dot{\theta}(t)$ is the angular velocity of the WMR. Note that the Differential drive wheel mobile robot have the nonholonomic constraint, whereas the driving wheels roll purely and do not slip, i.e.

$$\dot{x}sin(\theta) - \dot{y}cos(\theta) - d\dot{\theta} = 0 \tag{3}$$

It has been we identified a requirement of mobile robot aiming to formulate the tracking control error, which produces a trajectory for the real one to direct:

$$\dot{q}_r = \begin{bmatrix} \dot{x}_r(t) \\ \dot{y}_r(t) \\ \dot{\theta}_r(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r(t) \\ w_r(t) \end{bmatrix} \tag{4}$$

$$= S_r(\theta) \cdot p_r(t)$$

Where $q_r(t) = [x_r(t)\ y_r(t)\ \theta_r(t)]^T \epsilon R^{3\times2}$define the time-varying reference trajectory, $S_r(\theta) \epsilon R^2$ is the Jacobian matrix of the desired robot,$p_r \epsilon R^3$are the control input of the desired robot, $v_r$is the linear velocity of desired robot, $w_r$is the angle velocity of the desired robot.

To find the velocity control law such $q \to q_r$ as $t \to \infty$. The tracking error between the desired and real robot propose $\bar{q} = q_r - q = [(x_r - x)\ (y_r - y)\ (\theta_r - \theta)]^T$,As in XV we also define the tracing error as shown in Figure2

$$q_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{5}$$

And the error rate becomes.

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = v \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + w \begin{bmatrix} y_e \\ -x_e \\ 1 \end{bmatrix} + \begin{bmatrix} v_r \cos\theta_e \\ v_r \sin\theta_e \\ w_r \end{bmatrix} \tag{6}$$

Where $[v_r(t)\ w_r(t)]^\wedge T \epsilon R^2$ are the reference linear and angular velocity which obtained by $v_r = \sqrt{(\dot{x}_r)^2 + (\dot{y}_r)^2}$ and $w_r = \frac{\dot{x}_r \ddot{y}_r - \ddot{x}_r \dot{y}_r}{\dot{x}_r + \dot{y}_r}$

The kinematic control and its structure are illustrated in Figure 3. equation 7 is kinematic controller of mobile robot can approximate the velocities which make the system asymptotically stable is XV

$$\begin{bmatrix} v_c \\ w_c \end{bmatrix} = \begin{bmatrix} v_r \cos\theta_e + k_1 x_e \\ w_r + k_2 v_r y_e + k_3 v_r \sin\theta_e \end{bmatrix} \tag{7}$$

where $k_1 . k_2$ *and* $k_3$are positive constants and $v_r > 0$

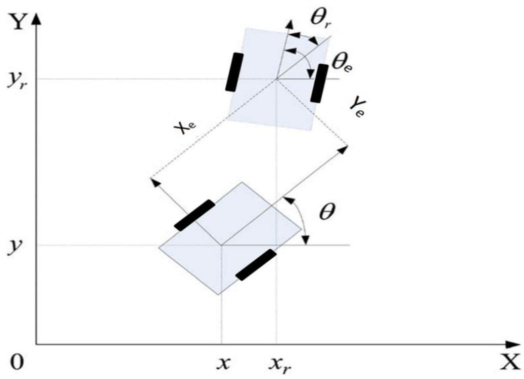


Fig2. Posture tracing error

III. Dynamic design of the controller

This section introduces two dynamic controllers. Firstly, we're addressing the WMR's dynamic model. Secondly, we design the dynamic sliding mode controller for the mobile robot torque level. finally, suggested adaptive sliding mode dynamic controller to solve system uncertainties and external disturbances.

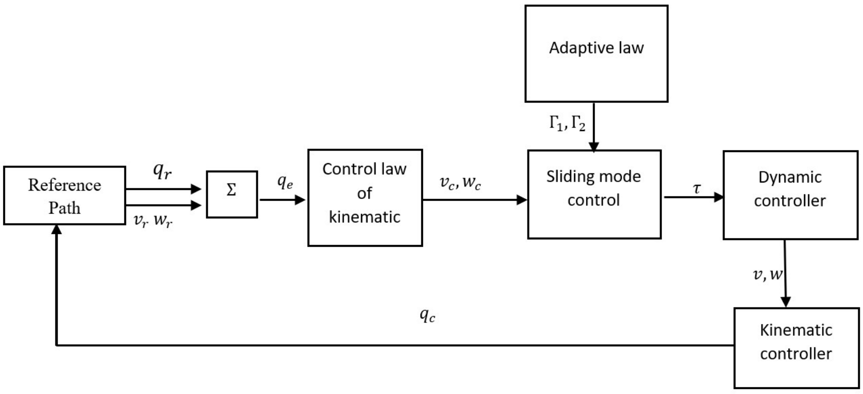


Fig 3. The proposed scheme of Control structure

III.i. Dynamic WMR model

This subsection, designs the dynamic model of the wheeled mobile robot by using sliding mode control (SMDC). Th aim of the dynamic controller is to make the tracking error velocity to zero.
General dynamic equation is defined for this mobile robot, XVI.

$$M(q)\ddot{q} + V(q.\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau - A^T\lambda \quad (8)$$

Where $M(q) \epsilon R^{n\times n}$ is a Symmetrical positive definite matrix of inertia, $V(q.\dot{q}) \epsilon R^{n\times n}$ is the matrix of Centripetal and Coriolis, $F(\dot{q}) \epsilon R^{n\times 1}$ is the matrix of the surface friction, $G(q) \epsilon R^{n\times 1}$ is the vector of gravity, $\tau_d \epsilon R^{n\times 1}$is a limited unknown disturbance, $B(q) \epsilon R^{n\times(n-m)}$ the input transformation matrix , $\tau \epsilon R^{(n-m)\times 1}$ is the control input vector, $A \epsilon R^{m\times n}$ is a matrix of non-holonomic constraints, $\lambda \epsilon R^{m\times 1}$ is a Lagrange multiplier linked to the limitations and $\dot{q}, \ddot{q}$ indicate vectors of velocity and acceleration. The variables can be described in Equation (8) as:

$$M(q) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}, A^T(q) = \begin{bmatrix} -sin\theta \\ cos\theta \\ 0 \end{bmatrix}$$

$$B(q) = \frac{1}{r}\begin{bmatrix} cos\theta & cos\theta \\ sin\theta & sin\theta \\ R & -R \end{bmatrix}, \tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}, G(q)=0, V(q.\dot{q}) = 0$$

$$\lambda = -m(\dot{x}_c cos\theta + \dot{y}_c sin\theta)\,\dot{\theta}.$$

Where m is the WMR mass, I is the moment of the WMR inertia, there are distance between two wheels and wheel
Radius which are 2R and r, respectively. The τ_l and τ_r are the left and right DC motor torque control inputs, respectively.
In terms of inner velocities, to convey the dynamic equations of movement. Replacement of equation (2) and its derivative in equation (8) and pre-multiplication with $S^T(\theta)$, following is obtained:

$$\bar{M}(q)\,\zeta + \bar{V}(q.\dot{q})\zeta + \bar{F}(\dot{q}) + \bar{\tau}_d = \bar{B}(q)\tau \tag{9}$$

Where $\bar{M}(q) = S^T(q)\ M(q)S(q)\ \epsilon R^{2\times2}$, $\bar{V} = S^T(q)\left[M(q)\dot{S}(q) +\ V(q,\dot{q})S(q)\right]\epsilon R^{2\times2}$,
, $\bar{F}(\dot{q}) = S^T\ F(\dot{q})\ \epsilon R^{2X1}$, $\bar{\tau}_d\ = S^T\ \tau_d$, $\bar{B} = S^T(q)\ B(q)$.

Due to the distance that are between the mass center and the WMR coordinate center is zero, it can be neglected from (9). The factors identified in (9) are as;

$\bar{M}(q) = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}$ and $\bar{B}(q) = \frac{1}{r}\begin{bmatrix} 1 & 1 \\ R & -R \end{bmatrix}$.

Assuming all the disturbances and uncertainties are zero, the dynamic equation can be reduced (9)

$$\dot{v} = E\tau \tag{10}$$

Where there is a system matrix E

$$E = \bar{M}^{-1} * \bar{B} = \frac{1}{m \cdot r \cdot I}\begin{bmatrix} I & I \\ Rm & -Rm \end{bmatrix} \tag{11}$$

III.ii. The control design of the sliding mode dynamic (SMDC):

The sliding mode control method has been implemented in this subsection, as the dynamic tracking control of the wheeled mobile robot is design using the Sliding mode control method, which allows the real velocity to converge with the produced control velocities from the kinematic. Firstly, introduce the tracing error velocity and its derivative to discover the torque input

$$e_c(t) = \begin{bmatrix} e_{c1} \\ e_{c2} \end{bmatrix} = \begin{bmatrix} v_c(t) - v(t) \\ w_c(t) - w(t) \end{bmatrix} \tag{12}$$

$$\dot{e}_c(t) = \begin{bmatrix} \dot{e_{c1}} \\ \dot{e_{c2}} \end{bmatrix} = \begin{bmatrix} \dot{v}_c(t) - \dot{v}(t) \\ \dot{w}_c(t) - \dot{w}(t) \end{bmatrix} \tag{13}$$

Let's select the PI-type sliding surface as:

$$s = \begin{bmatrix} s1(t) \\ s2(t) \end{bmatrix} = e_c(t) + \beta\int_0^t e_c(\tau)d\tau \tag{14}$$

where β is positive integral sliding surface constant, and $\beta > 0$. However, If the sliding surface of the system is $s(t) = 0.\, e_c(t) = -\beta \int_0^t e_c(\tau)d\tau$, the tracking error $e_c(\infty) \to 0$ since $\beta > 0$.

The sliding surface derivative is provided as;

$$\dot{s}(t) = \dot{e}_c(t) + \beta\, e_c \tag{15}$$

Positive definite function can be easily defined as;

$$V = \frac{1}{2} S^2 \tag{16}$$

The time derivative

$$\dot{V} = SS \tag{17}$$

To obtain V converges to zero, it is sufficient that

$$S\dot{S} = -k|S| \tag{18}$$

where $\alpha > 0$. in the words

$$\frac{S}{|S|}\dot{S} = -k \tag{19}$$

$$\dot{S}\, sgn(S) = -k \tag{20}$$

Considering (9) in a decentralized structure and equation (15), we can rewrite (20) as

$$[\dot{e_c} + \beta e_c] sgn(S) = -k \tag{21}$$

Then

$$(\dot{v_c}(t) - E\tau) + \beta e_c(t) + k sgn(s) = 0 \tag{22}$$

$$\tau = \tau_{eq} + \tau_{sw} = E^{-1}[\dot{v}_c(t) + \beta e_c(t) + k \cdot sgn(S)] \tag{23}$$

Where $k = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$,and k_i is a positive constant, $sgn(s) = [sgn(s_1) \quad sgn(s_2)]^T$.The dynamic equation (10) It becomes in the presence of uncertainty and disturbance

$$\dot{v}(t) = E\tau(t) + \tau_d(t) = \bar{E}(t)\tau + \Delta E \cdot \tau(t) + \tau_d(t) \tag{24}$$

Where $\bar{E}$ is designated as the system's nominal part, m, r, I, R And ΔE denoted system matrix uncertainties. Thus, we can present $\delta(t)$as the upper limited of uncertain tie;

$$\delta(t) = \begin{bmatrix} \delta_1(t) \\ \delta_2(t) \end{bmatrix} = \Delta E\, \tau(t) + \tau_d(t) \tag{25}$$

Then can be write the dynamic equation as

$$\dot{v} = \bar{E}\tau(t) + \delta(t) \tag{26}$$

Therefore, the sliding mode dynamic control (23) can be rewritten as

$$\tau = \tau_{eq} + \tau_{sw} = \bar{E}^{-1}[\dot{v}_c(t) + \beta e_c(t) + k \cdot sgn(S)] \tag{27}$$

To compensate for system uncertainties and disturbances, switching gain k should be selected. The most commonly known method to decrease the chattering phenomenon is to use the saturation function $sat\ (S.\varepsilon)$.So, replacing sgn (S) by $sat\ (S.\varepsilon)$ in (27) implies

$$\tau = \tau_{eq} + \tau_{sw} = \bar{E}^{-1}[\dot{v}_c + \beta e_c + k \cdot sat(S.\varepsilon)] \tag{28}$$

Where

$$sat(s.\varepsilon) = \begin{cases} sgn(s_i). & |s_i| > \varepsilon > 0 \\ \frac{s_i}{\varepsilon}. & |s_i| \leq 0 \end{cases} \qquad i = 1.2 \tag{29}$$

and ε is a small positive constant.

III.3. Adaptive sliding mode control design (ASMDC)

in order to caster with variations in the system parameters for example mass and inertia and unknown disturbances owing to the dynamic surroundings, we are proposing an ASMDC system for estimating the upper limit of$|\delta_i(t)|$. Suppose that the optimum limits of the $\delta^*$and Γ^*exist. The algorithm adaptive to the bound of Γ^* is

$$\hat{\Gamma}(t) = \begin{bmatrix} \hat{\Gamma}_1 & 0 \\ 0 & \hat{\Gamma}_2 \end{bmatrix} = \begin{bmatrix} \rho_1 s_1 sat(s_1.\varepsilon) & 0 \\ 0 & \rho_2 s_2 sat(s_2.\varepsilon) \end{bmatrix} \tag{30}$$

Where $\hat{\Gamma}(t)$ is the value estimated of Γ^*. The ρ_i(i = 1,2) is referred as a positive gain in adaptation. The WMR ASMDC is conceived as follows:

$$\tau = \tau_{eq} + \tau_{sw} = \bar{E}^{-1}[\dot{v}_c(t) + \beta e_c(t) + \hat{\Gamma}(t) \cdot sat(S.\varepsilon)] \tag{31}$$

And we describe the error of estimation as

$$\tilde{\Gamma}(t) = \hat{\Gamma}(t) - \Gamma^* \tag{32}$$

The essential objective is to select an adaptive law to update the estimate $\hat{\Gamma}$such that $S(t)$converge to a zero vector.

Theorem 1. If the ASMDC (31) and the kinematic controller (7) are given. Both the posture tracking error and the velocity tracking error of the WMR's complete motion equations with uncertainties and disturbances (26) converge to zero vectors asymptotically.

It can be defined as candidate forthe Lyapunov function as

$$V = V_1 + V_2 \tag{33}$$

$$V_1(x_e.y_e.\theta_e) = \frac{1}{2}(x_e^2 + y_e^2) + \frac{1 - cos\theta_e}{k_2} \tag{34}$$

$$V_2\left(S(t).\tilde{\Gamma}(t)\right) = \frac{1}{2}S^T(t)S(t) + \frac{1}{2}\left(\left(\frac{1}{\rho_1}\right)\tilde{\Gamma}_1^2(t) + \left(\frac{1}{\rho_2}\right)\tilde{\Gamma}_2^2(t)\right) \tag{35}$$

Clearly, $V \geq 0$. Substituting (6) and (7) for the time derivative of in (34), we obtain

$$\dot{V}_1 = -k_1 x_e^2 - \frac{k_3 v_r sin\theta_e^2}{k_2} \leq 0 \tag{36}$$

Thus, if the desired velocity $v_r \geq 0$ then $\dot{V}_1 \leq 0$. Differentiating (35), one can obtain

$$\dot{V}_2\left(S(t).\tilde{\Gamma}(t)\right) = S^T(t) \cdot \dot{S}(t) + \left(\frac{1}{\rho_1}\right)\tilde{\Gamma}_1 \dot{\tilde{\Gamma}}_1 + \left(\frac{1}{\rho_2}\right)\tilde{\Gamma}_2 \dot{\tilde{\Gamma}}_2$$
$$= S^T \cdot [(\dot{v}_c(t) - \dot{v}(t)) + \beta\, e_c] + (1\backslash\rho_1)\tilde{\Gamma}_1 \dot{\tilde{\Gamma}}_1 + \left(\frac{1}{\rho_2}\right)\tilde{\Gamma}_2 \dot{\tilde{\Gamma}}_2 \tag{37}$$

$\dot{\tilde{\Gamma}}(t)$ is equal to $\dot{\hat{\Gamma}}(t)$ because the Γ^* is constant. Substituting (26) and (31) for (37) imply

$$\dot{V}_2\left(S(t),\tilde{\Gamma}(t)\right) = S^T(t) \cdot [-\tilde{\Gamma}(t) \cdot sat(S,\varepsilon) - \delta(t))] + \left(\frac{1}{\rho_1}\right)\tilde{\Gamma}_1 \dot{\hat{\Gamma}}_1 + \left(\frac{1}{\rho_2}\right)\tilde{\Gamma}_2 \dot{\hat{\Gamma}}_2$$
$$= S^T \cdot [-\left(\Gamma^* + \Gamma^{\sim}(t)\right) \cdot sat(S,\varepsilon) - \delta(t)] + \left(\frac{1}{\rho_1}\right) \cdot \tilde{\Gamma}_1 \dot{\hat{\Gamma}}_1 + \left(\frac{1}{\rho_2}\right) \cdot \tilde{\Gamma}_2 \dot{\hat{\Gamma}}_2$$
$$= S^T \cdot [-\Gamma^* \cdot sat(S,\varepsilon) - \delta(t)]] + \sum_{i=1}^{2} \tilde{\Gamma}_i(t) \times [\left(\frac{1}{\rho_i}\right)\dot{\hat{\Gamma}}_i - S_i \cdot sat(S_i,\varepsilon) \tag{38}$$

From adaptive law (30), one can get

$$\left[\left(\frac{1}{\rho_i}\right)\dot{\hat{\Gamma}}_i - S_i \cdot sat(S_i,\varepsilon)\right] = 0. t \geq 0 \tag{39}$$

Substituting (39) for (38) implies

$$\dot{V}_2\left(S(t),\tilde{\Gamma}(t)\right) = S^T(t) \cdot [-\Gamma^* \cdot sat(S,\varepsilon) - \delta(t)]$$
$$= -\left[(\Gamma_1^* S_1(t) sat(S_1,\varepsilon) + \Gamma_2^* S_2 sat(S_2,\varepsilon) + \left(\delta_1(t) S_1(t) + \delta_2(t) S_2(t)\right)\right]$$
$$\leq \sum_{i+1}^{2} (|\delta_i(t)| \cdot S_i\, sat(s_i,\varepsilon) - \Gamma_i^* S_i sat(S_i,\varepsilon)) \tag{40}$$
$$= -\sum_{i+1}^{2} (S_i sat(S_i,\varepsilon)(\Gamma_i^* - |\delta_i(t)|)) \leq 0$$

The adaptive law $\dot{\hat{\Gamma}}$ ensures $\dot{V}_2$is negative semi definite. With the adaptive law (30), $V_2\left(S(t), \tilde{\Gamma}(t)\right)$ as a function of it does not increase, that is

$$V_2\left(S(t), \tilde{\Gamma}(t)\right) \leq V_2\left(S(0), \tilde{\Gamma}(0)\right), \forall t \geq 0 \tag{41}$$

Thus, the $S(t)$ and $\tilde{\Gamma}(t)$ are limited. The execution of adaptive law (30) is

$$\hat{\Gamma}_i(t) = \rho_i \int_0^t S_i\, sat(s_i, \varepsilon) d\tau + \hat{\Gamma}_i(0) \tag{42}$$

where$\hat{\Gamma}_i(0)$ Is the original upper limit estimate of Γ_i^*. From (36) and (40) we can conclude that $\dot{V}$ is negative semi-definite. That is, the posture tracking error q_e and the sliding surface S approach zerovectors. It has been noted in (14) that once S = 0, then $e_c(t) = -\beta \int_0^t e_c(\tau) d\tau$ and it is obvious that$e_c(\infty) \to 0$. This method completes the proof of the theorem.

IV. Actuator dynamics

This section includes the dynamics of the actuator for wheeled mobile robot. It is turned on by two of dc motors the WMR (left and right). Torques are used as control inputs by the control law established in Equation (31). In fact, the wheels are controlled by actuators, and it is convenient to transformation from torque to actuator voltage. The mechanical and electrical equations representing a dc motor can be given as

$$V(t) = R_a i(t) + V_e(t) \tag{43}$$

$$\tau_m(t) = k_t i(t) \tag{44}$$

Where the $V(t)$ is actuator voltage, R_a is resistance of armature, $i(t)$is current of armature, $V_e(t)$ back emf, $\tau_m(t)$is the torque generated by motor and k_t is the torque constant. The back emf is defined as

$$V_e = k_b w_m \tag{42}$$

Where k_b is the velocity constant and $w_m$is the motor angular velocity. The wheel torque is defined as;

$$\tau_\omega = N \tau_m \tag{43}$$

Where N is the gear ratio.

The relation between the wheel angular velocity and the motor angular velocity is defined as

$$\omega_\omega = \frac{1}{N} \omega_m \tag{44}$$

By using the torque and velocity values of DC motor in Equation (43) obtained

$$\mathbf{V(t)} = \frac{R_a}{Nk_t}\tau_\omega(t) + k_b N\omega_\omega \quad (48)$$

The angular wheel velocities and the velocity vector $v(t)$ relation is defined as

$$\begin{bmatrix}\omega_R\\ \omega_L\end{bmatrix} = \begin{bmatrix}\frac{1}{r} & \frac{R}{r}\\ \frac{1}{r} & -\frac{R}{r}\end{bmatrix}\begin{bmatrix}v(t)\\ \omega(t)\end{bmatrix} \quad (45)$$

The wheels torque and the actuator input voltage vector $V(t)$ relation is given by

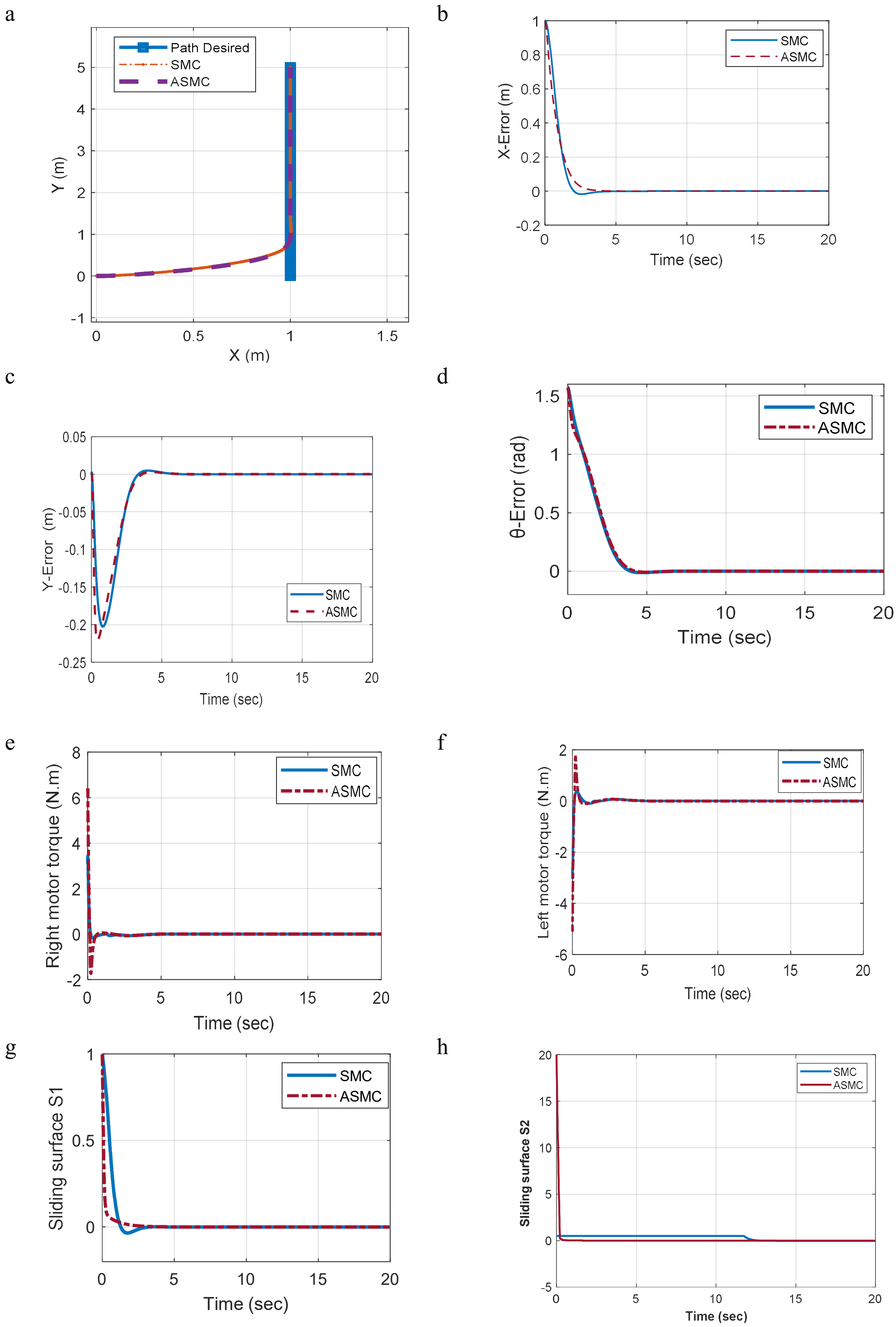
$$V(t) = \begin{bmatrix}V_R\\ V_L\end{bmatrix} = \begin{bmatrix}\frac{R_a}{Nk_t} & 0\\ 0 & \frac{R_a}{Nk_t}\end{bmatrix}\begin{bmatrix}\tau_{\omega R}(t)\\ \tau_{\omega L}(t)\end{bmatrix} + k_b\begin{bmatrix}\frac{1}{r} & \frac{R}{r}\\ \frac{1}{r} & -\frac{R}{r}\end{bmatrix}\begin{bmatrix}v(t)\\ \omega(t)\end{bmatrix} \quad (50)$$

V. Simulation Results and Discussion

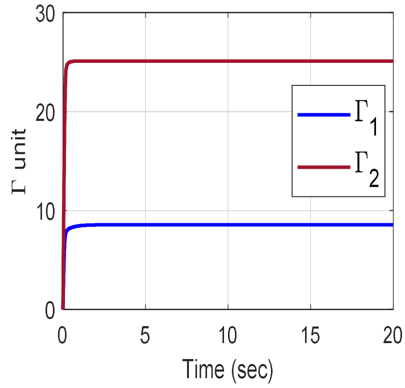
To show the proposed controller result we use MATLAB\Simulink for verification and validation. We used two kinds of the trajectory tracing (straight line and eight-shape). the parameter of the mobile robot is;

M =5 Kg, I =2.5 kg.m^2, R =0.03 m and r = 0.15 m.

The straight-line path is given by equations $x_r = 1$, $y_r = 0.25\,t$ and $\theta_r = \pi\backslash 2$ where the t is the time in second and the reference velocity $v_r = 0.25$ and $\omega_r = 0$. The initial reference point are $q_r(0) = [x_r(0)\ y_r(0)\ \theta_r(0)]^T = [1\ 0\ \pi\backslash 2\ ]^T$.The WMR initial path vector is $q(0) = [x(0)\ y(0)\ \theta(0)]^T = [\ 0\ 0\ 0\ ]^T$ for both case sliding mode and adaptive sliding as shown at figure 4-a. the initial errors for both case sliding mode and adaptive sliding is indicated in figures 4-b,c and d where $q_e(0) = [x_e(0)\ y_e(0)\ \theta_e(0)]^T = [1\ 0\ \pi/2]^T$ is eliminated to a very small errors in both case. the adaptive sliding mode converge to zero before the sliding mode has been shown in figure 4-b, c &d. In addition, the sliding surface s_1 and s_2 is eliminated to zero as shown in figure4-g, h. Figure4- e, f shown the torque produced by dynamic sliding mode and adaptive dynamic sliding mode controller. The torque produced by adaptive sliding mode shown with red color in figures.4 e, f is increased rapidly due to initial error and varied to constant value. The adaption parameter in equation (42) is change rapidly to steady value as shown in figure4-i to obtain optimum stable value for $\Gamma_1 = 200$ and $\Gamma_2 = 200$. in table 1 the x_e is smaller in ASMC, but the y_e and θ_e is smaller in SMC. In contrast the ASMC mean square error is smallest than SMC. In addition, chattering in this case reduced from 0.024049 in SME to 0.007298 in ASMC. Figure 4-j shown the actuator voltage.


a
Path Desired
SMC
ASMC
Y (m)
X (m)
b
SMC
ASMC
X-Error (m)
Time (sec)
c
SMC
ASMC
Y-Error (m)
Time (sec)
d
SMC
ASMC
θ-Error (rad)
Time (sec)
e
SMC
ASMC
Right motor torque (N.m)
Time (sec)
f
SMC
ASMC
Left motor torque (N.m)
Time (sec)
g
SMC
ASMC
Sliding surface S1
Time (sec)
h
SMC
ASMC
Sliding surface S2
Time (sec)

i



j

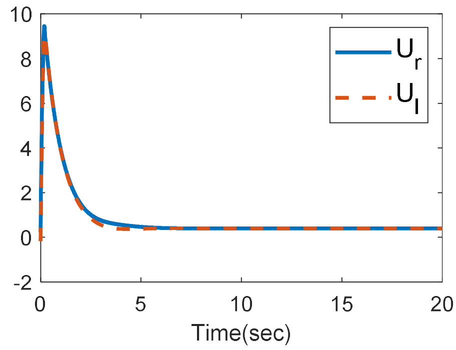
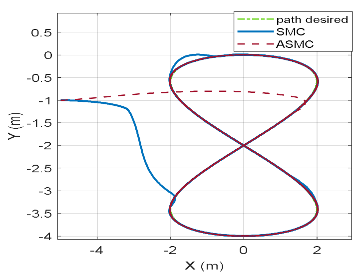


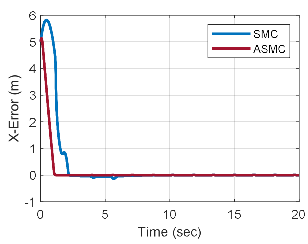
Fig 4. the comparisons of the tracing between SMDC and ASMC. (a) line trajectories of the two controllers. (b)The trajectory tracing error in x-direction. (c)The trajectory tracing error in y-direction. (d) The trajectory tracing error in θ -direction. (e) Right motor torque. (f) Left motor torque. (g) sliding surface S_1. (h) sliding surface S_2.(i) The adaption parameter. (j) actuator voltage.

The second case steady eight-shape is given by the initial reference point are $q_r(0) = [x_r(0)\ y_r(0)\ \theta_r(0)]^T = [0\ 0\ \pi\backslash 2\ ]^T$.The WMR initial path vector is $q(0) = [x(0)\ y(0)\ \theta(0)]^T = [-5\ -1\ 0\ ]^T$ for both case sliding mode and adaptive sliding as shown at figure 5-a .The initial errors for both case sliding mode and adaptive sliding is indicated in figures 5-b,c &d where $q_e(0) = [x_e(0)\ y_e(0)\ \theta_e(0)]^T = [5\ 1\ 0]^T$ is eliminated to a very small errors in both case. The adaptive error vanishes faster in compared with the SMD as shown in figure5-g&h for s1 and s2. The adaptive parameters increased rapidly to a constant stable value due to the initial error where $\Gamma_1 = 200$ and $\Gamma_2 = 200$ as shown in figure5-i. In table 1 the x_e, y_e and θ_e is smaller in ASMC. ASMC produces smallest mean square error compared to SMC. Furthermore, chattering in this case reduced from 0.614466 in SME to 0.061435 in ASMC. Figure 6-j shown the actuator voltage

a



b



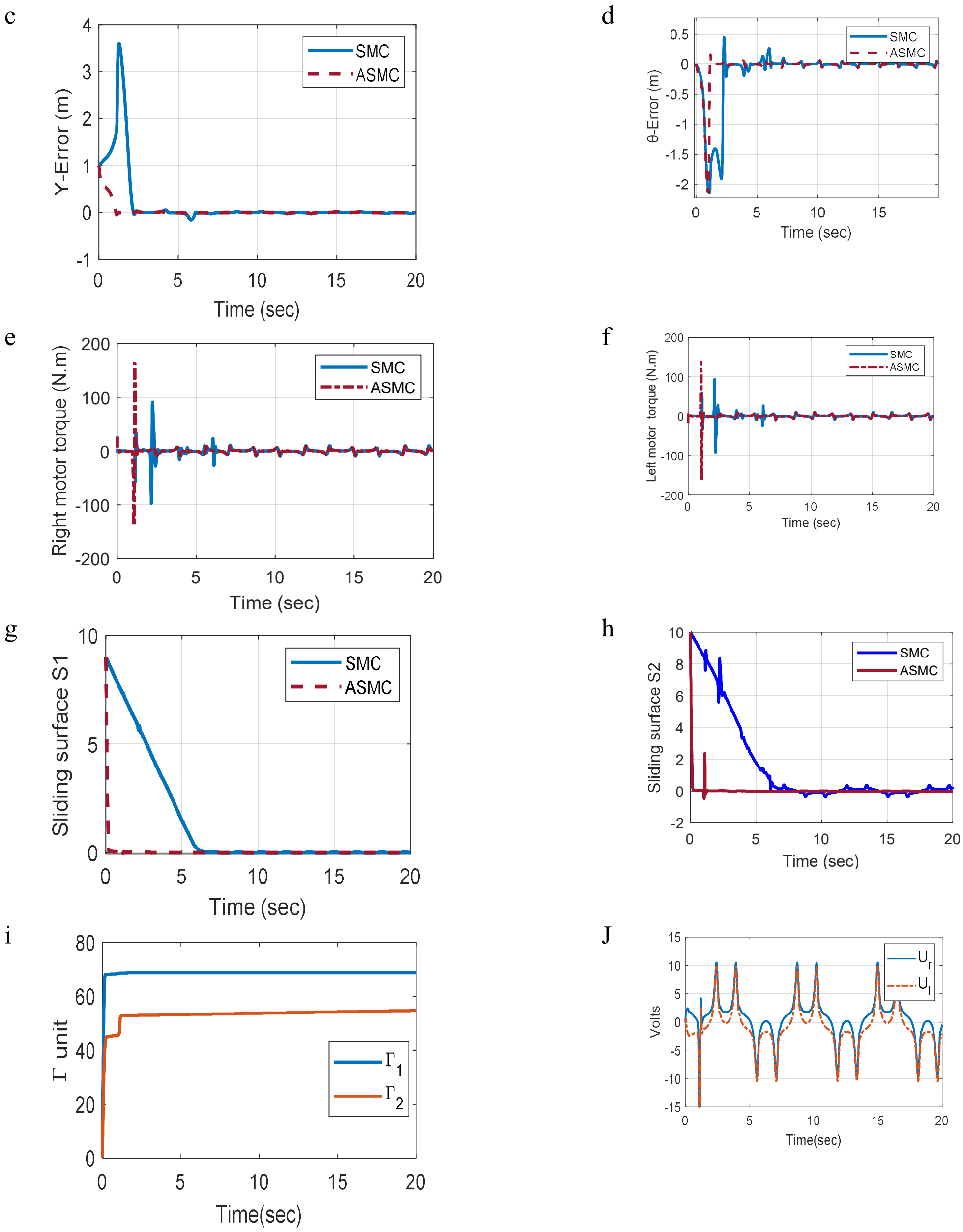


Fig5. the comparisons of the tracing between SMDC and ASMC. (a)eight-shape trajectories of the two controllers. (b)The trajectory tracing error in x-direction. (c)The trajectory tracing error in y-direction. (d) The trajectory tracing error in θ -direction. (e) Right motor torque. (f) Left motor torque. (g) sliding surface S_1. (h) sliding surface S_2.(i) The adaption parameter. (j) actuator voltage.

When you increase it $m \rightarrow 50\%$ and $I = 50\%$ There will be an increase in torque only after 10 sec as shown in Figure

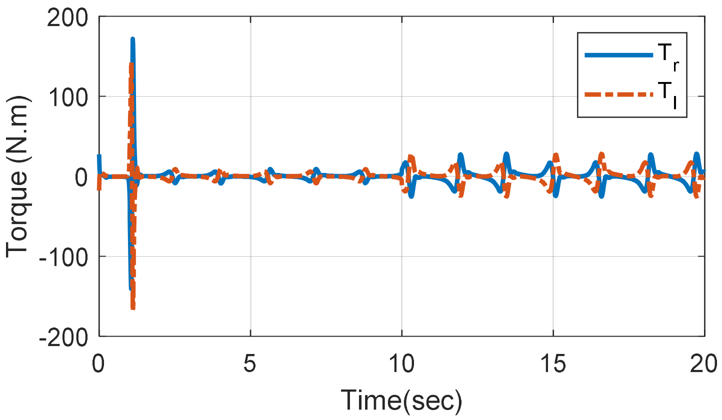


Fig 6. show increase in torque right and left

Table 1 shows the result comparison between sliding mode and adaptive sliding mode controller by equations the trajectory tracing error

$$x_{e-total} = \frac{1}{N}\sum_0^n |x_e| ,\quad y_{e-total} = \frac{1}{N}\sum_0^n |y_e| ,\ \theta_{e-total} = \frac{1}{N}\sum_0^n |\theta_e| \tag{51}$$

$$MSE = \frac{1}{N}\sum_0^n \sqrt{x_e^2 + y_e^2 + \theta_e^2} \qquad chatter = \frac{1}{N}\sum_0^n |v_r - v| = \frac{1}{N}\sum_0^n |v_e|$$

MSE is define the mean square error, N is number of total samples and n is the index of error samples and Chatter is defined chattering on the linear velocity error.

Table 1 shows the result comparison between SMDC and ASMDC.

For straight line

error	$x_e(m)$	$y_e(m)$	θ_e	**MSE**	**Chatter**
SMC	0.044352	0.017311	0.120494	0.387196	0.024049
ASMC	0.04212	0.0177	0.121187	0.375637	0.007298

For eight-shape

Error	$x_e(m)$	$y_e(m)$	θ_e	**MSE**	**Chatter**
SMC	0.38378	0.186752	0.155788	1.550757	0.614466
ASMC	0.134293	0.030149	0.054614	0.748006	0.061435

VI. Conclusion

In conclusion, an adaptive sliding mode dynamic control for Wheeled mobile robot trajectory tracing was proposed. Moreover, using modified kinematic controller to generate the kinematic velocity and then ASMC is designed to achieve the desired velocity order determined by the kinematic controller to reach the actual velocity for WMR. The proposed controller reduces the chattering phenomena. Using computer simulation, the efficacy of the proposed algorithm is demonstrated and it is shown that the suggested system has better transient efficiency on different trajectories with acceptable steady state error. Furthermore, the scheme proposed shows fast convergence and robustness in the presence of uncertainties and disturbances.

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