



## STUDY THE BAYESIAN APPROACH FOR COMPUTING RETURN LEVELS OF EXTREME RAINFALL AT KHYBER PAKHTUNKHWA (KPK), PAKISTAN

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<https://doi.org/10.26782/jmcms.2020.01.00009>

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### Abstract

*It has been observed that the extreme rainfall is an unusual and very essential hydrological parameter therefore probabilistic modeling is important for the analysis of such extreme weather events. Extreme rainfall analysis has much importance for a civil engineer and planning division of a country to take into account the capability of building structures for extreme weather conditions. To understand the extreme behavior of Khyber Pakhtunkhwa we use yearly maximum rainfall of four major cities of this province from 1960 to 2010. In this study, we have estimated the parameters of Generalized Extreme Value (GEV) distribution by using Bayesian approach. The Akaike Information Criteria and Acceptance Rate are used to check the reliability of the model. After getting ensured the reliability we find return levels against different return periods (10, 25, 50, 75 and 100 years) of Meteorological stations Peshawar, Dir, Parachinar and D I Khan of KPK province of Pakistan. Our result will be useful for policy makers, civil engineers, planning division, agricultural departments and research scholars, farmers for irrigation system and civil society of KPK, Pakistan for precautionary measures.*

**Keywords:** Extreme Rainfall, Bayesian approach, Return period, Return levels.

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### I. Introduction:

The irregular and unusual condition of weather intensity in a locality is said to be extreme weather events. One of the most important weather event is extreme rainfall which has pertinent socio-economic on a locality. It has been shown that the Bayesian

analysis is suitable to estimate the extreme value parameters in case of scarcity of data adding that this method also uses other information through a prior distribution [XVI]. The Earth's climate changes in twentieth century with rainfall pattern are caused by both natural and anthropogenic activities [II]. The relative roles and local impacts of climate change and rainfall are under exciting consideration [X and XVIII]. It has been known for a long time that the major extreme weather events can be commonly specified by one of the type of Generalized Extreme Value family i.e. type (i), type (ii) and type (iii) [VII]. [I] investigated rainfall and said that the flood of 2010 in Pakistan was due to the extreme rainfall in that year. In 1955 [XIII] urged to use extreme value modeling for calculation of climatic parameters. Generalized Extreme Value distribution is widely used to model extreme conditions of hydrology i.e. flood flow [XV], water level at coastal areas [III], extreme conditions of temperature [XX] and extreme rainfall [XI]. [V] claims that global warming let the extreme weather events increased in different places. As per report of Intergovernmental Panel on Climate Change heat waves and extreme rainfall will occur more frequently in future [XII]. Estimates of extreme weather events calculated by using historical data are used in civil engineering in order to minimize the failure probability [IV]. There are climatic changes which are interlinked with these extreme events which results negative impact on our society following landslides, flashy floods etc. United Nations Framework Convention on Climate Change (UNFCCC) claims that the water resources, ecological unit and Biodiversity, agricultural science, coastal areas and communal health are influenced by climate change in Asia. Developing countries like Pakistan will also be badly affected by its productions due to these natural disasters. [VIII] put forth his conclusion after analyzing climate change that paddy cultivation of rice is negatively affected by climate change. Now going towards the solution all these extreme weather events are predictable after careful analysis. In order to reduce the impact of extreme rainfall events precautionary measures may be used [XXII].

In extreme value modeling one must be conscious about all the other physical constraints, the extreme probable value or can be developed from an accompanying processes and maybe the same variable at a different locality. Sometimes we notice data, not to be absolutely illustrative for the whole period having some historic indications, although not in the form of data, but in the shape of behavior, considerably more extreme than that which has been considered. Due to these reasons an expert with knowledge of the physical processes can have information that is appropriate to extreme behavior, irrespective of the prevailing data which strongly leads Bayesian framework. Therefore Bayesian statistics is strongly recommended and used to analyze extreme weather events. [XXI] Shows the strong relationship between prior and posterior distribution of parameter of Weibull distribution. [XVII], tried to explain the theoretical behavior of Bayesian estimation with the significance of prior distribution. Markov Chain Monte Carlo (MCMC) method is used to solve the problem of computational features of Bayesian statistics for the analysis of extreme value theory [IX]. Thus in this analysis, we have used Bayesian method to compute the return levels of extreme rainfall against different return periods over Khyber Pakhtunkhwa.

## II. Data description and Methodology:

Two kinds of data series i.e. “Annual Maximum Series (AMS) and Peak Over Threshold (POT)” [VIII]. Peak over threshold series is used to incorporate maximum number of data points. Keeping in view the complex situation sometimes in the selection of suitable threshold in POT series [XIV] prefer AMS for extreme value analysis. They, claims that annual maximum series are quite easier to get information from the past pattern of the rainfall series of parameters. [XIX] Showed that the non-stationary atmospheric parameters can be better analyzed and modeled if Bayesian statistical method is used. In our analysis we use daily rainfall data of four major cities of Khyber Pakhtunkhwa (Dir, Parachinar from 1961 to 2010 and Peshawar, D I Khan from 1981 to 2010), taken from Computerized Data Processing Centre, Pakistan Meteorological Department Karachi. We have extracted the yearly maximum rainfall from the daily observations and plotted the scatter diagrams shown in Fig.1. No obvious trends indicated by these figures suggesting fitting probabilistic model for this data set. Due to the vast applications of Bayesian approach current study uses Bayesian method to calculate return levels of extreme rainfall of KPK.

## III. Bayesian Approach

It is known for a long time that the Bayesian approach examines the uncertainty about the unknown parameters in terms of probability. Therefore unknown parameters are treated here as random variables. The density of posterior distribution ( $f$ ) for  $Z_1, Z_2, \dots, Z_n$  data points, is directly proportional to the product of Prior ( $g$ ) and Likelihood distribution ( $L$ ) as;

$$f(\mu, \phi, \xi | Z_1, \dots, Z_n) \propto L(\mu, \phi, \xi; Z_1, \dots, Z_n) \times g(\xi) \quad (1)$$

Normally the output of the analysis of extreme rainfall is to get idea about the expected amount of rainfall in future which is termed as return levels. If ‘y’ denotes the future rainfall values with probability density function (pdf):

$$h(y | Z_1, \dots, Z_n) = \iiint f(y | \mu, \phi, \xi) f(\mu, \phi, \xi | Z_1, \dots, Z_n) d\mu d\phi d\xi \quad (2)$$

Then the estimates of probability of ‘n’ year return levels for sample  $\theta_1, \theta_2, \dots, \theta_R$  will be

$$\Pr[y \leq q_n | Z_1, \dots, Z_n] \approx \frac{1}{R} \sum_{i=1}^R \Pr(y \leq q_n | \theta_i) \quad (3)$$

The posterior density is calculated by MCMC simulation technique for which the acceptance rate must be in between 10% to 40% [XIII – VIII]. So Bayesian approach is suitable for the above said stations of rainfall data having acceptance rates 0.317, 0.31, 0.316 and 0.308 respectively. The Fig. 2 represents the trace plots of GEV parameters for 40000 iterations, showing that there is no periodicity in the data. Fig.3 reveals the posterior density plots for parameters of parent distribution ( $\mu$ ,  $\phi$  and  $\xi$ ) indicating the identical shape of parameters.

Due to complications in solving eq. (2), MCMC simulation technique is used to find the posterior distribution. In this communication, we have utilized standard software R-package for extreme value modeling.

#### IV. Results and Discussion:

Fig. 1 shows that there are several numbers of extreme values having magnitude more than 30 years annual normal values of above said stations i.e. (64.9mm, 87.2mm, 60.2mm and 50.8mm respectively). D I Khan shows eleven extreme values with highest rainfall of 150mm occurred on August 4, 2010 throughout the study period. [VI] shows three categories of heavy rainfall events i.e. significant rainfall, heavy rainfall and very heavy rainfall as rainfall exceeds 10mm, 15mm and 25mm respectively. In our analysis the event is said to be extreme when it exceeds the 30 years normal value of rainfall in a locality. The Table: 1 shows the summary of frequency analysis of above stations.

These extreme values have shown that there are spatial variations in extreme values i.e. extreme value for a station may be normal for another station. Table: 2 illustrate the Bayesian parametric estimation of GEV distribution. Analysis of Table 3 depicts that acceptance rates of above stations are 0.49, 0.124, 0.126 and 0.435 which are within range of 10% to 40% suggests the suitability of the Bayesian approach for observations of study region. Akaike Information Criteria (AIC) is used for comparison of model among the observed meteorological stations. The Bayesian statistics in Table 4 depicts that the 50 years return levels of Peshawar, Dir, Parachinar and D I Khan of KPK are 152mm, 213mm, 139mm and 205mm, 75 years return levels are 168mm, 243mm, 153mm, 241mm and those of 100 years return levels are 179mm, 261mm, 163mm, and 270mm respectively. These extreme rainfalls are dangerous if remains continuous for few days and may lead destructive flood in a society. Thus the calculation of above return levels helps policy makers, planning divisions, civil engineers, forecasters and higher authorities of KPK in precautionary measures to save our country from different kinds of losses.

**Table:1** Summary of frequency of extreme event in study stations

Stations	30 years Normal Value (mm)	No.of Extreme Values	Magnitude of extreme value (mm)	Date	Heaviest rainfall (mm)	Date
Peshawar,	64.9mm	10	142mm, 119 mm	3 <sup>rd</sup> , Oct 1996, 5 <sup>th</sup> , Apr 2008	274 mm	29 <sup>th</sup> , July 2010
Dir,	87.2mm	14	149mm, 147mm and 125mm	29 <sup>th</sup> Jul 2010, 31 <sup>st</sup> Jul 1989 and 12 <sup>th</sup> Mar 1993	166 mm	10 <sup>th</sup> , Oct 2004
Parachinar	60.2mm	9	106.5 mm and 103 mm	21 <sup>st</sup> Aug 2004 and 31 <sup>st</sup> Mar 2007	113.6 mm	11 <sup>th</sup> , Mar 1993
D I Khan	50.8mm	11	128.5 mm,	21 <sup>st</sup> Mar	150	4 <sup>th</sup>

			112.5 mm and 116 mm	1987, 23 <sup>th</sup> Aug 1989 and 4 <sup>th</sup> July 1994		August 2010
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**Table 2:** Bayesian estimation of GEV distribution parameters

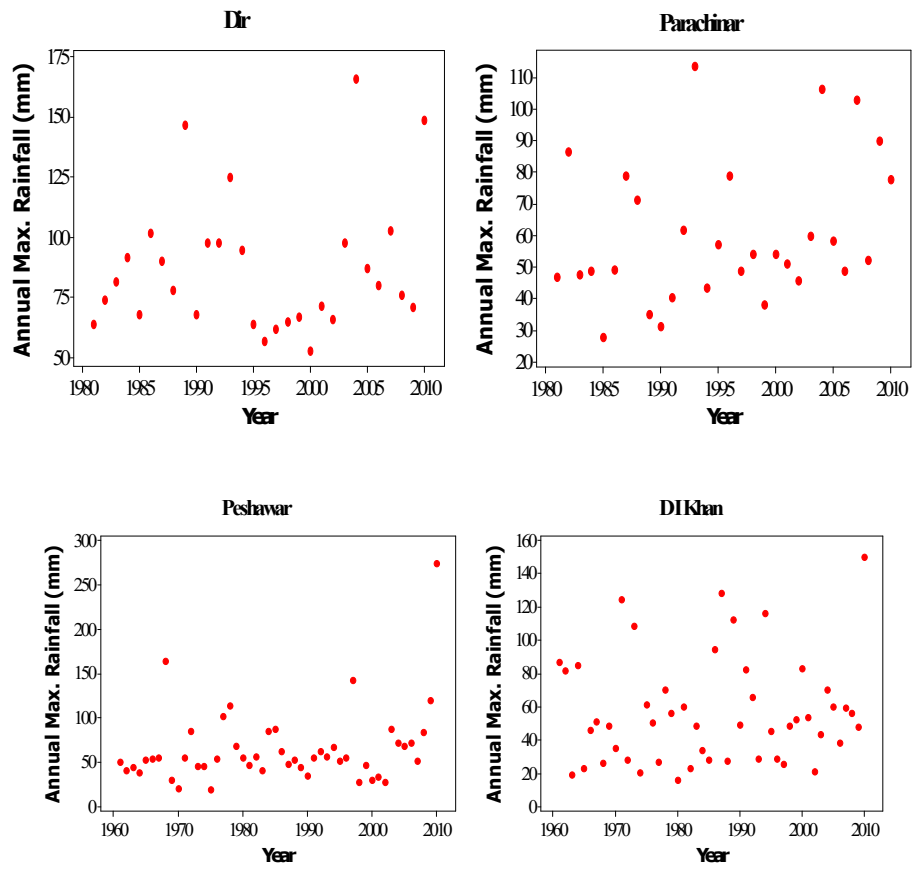
GEV Parameters	Peshawar (1960-2010)	Dir (1981-2010)	Parachinar (1981-2010)	D I Khan (1960-2010)
Location ( $\mu$ )	46.76	73.25	49.59	40.13
Scale ( $\Phi$ )	3.02	2.86	2.85	3.05
Shape ( $\xi$ )	0.12	0.272	0.097	0.274

**Table 3:** Reliability tests for estimation of GEV distribution parameter

S.NO	Station's Name	WMO Number	Latitude	Longitude	AIC	MSE	Acceptance Rate
1	Peshawar	41530	34° 02'	71° 56'	478	0.49	0.317
2	Dir	41508	35° 12'	71° 51'	277	0.124	0.310
3	Parachinar	41560	33° 52'	70° 05'	270	0.126	0.316
4	D I Khan	41624	31° 49'	70° 56'	479	0.435	0.308

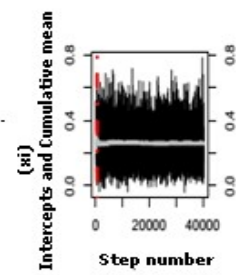
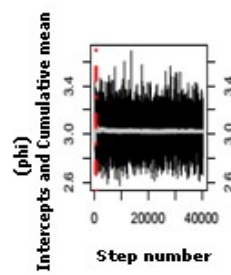
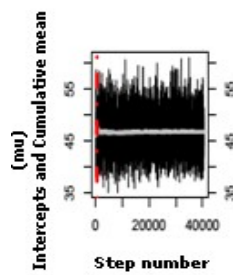
**Table 4:** Computation of Return Levels against different return periods using Bayesian Technique

Station Name	Estimation Method	Return Levels (mm)				
Return Period	Bayesian	10 year	25 year	50 year	75 year	100 year
Peshawar		100	129	152	168	179
Dir		130	171	213	243	261
Parachinar		94.5	118	139	153	163
D I Khan		108	157	205	241	270

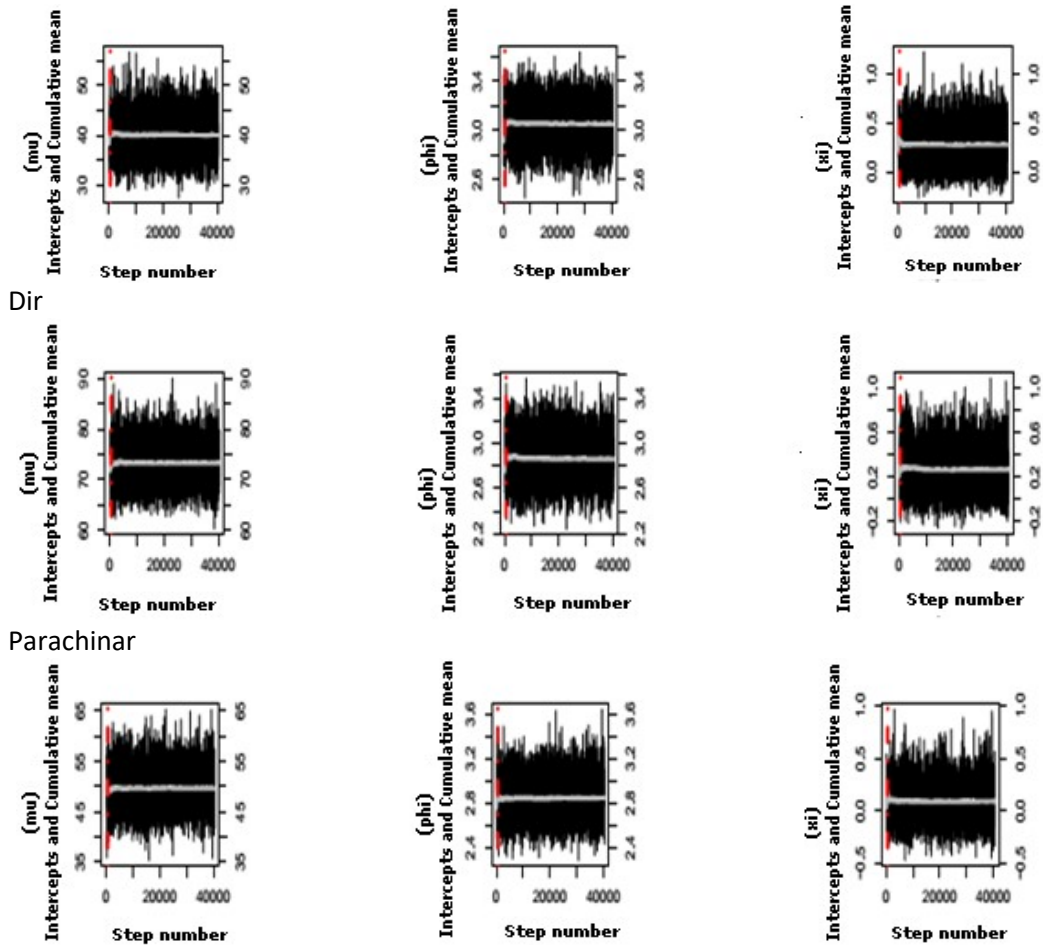


**Fig. 1:** Annual Extreme Rainfall of major cities of Khyber Pakhtunkhwa (KPK).

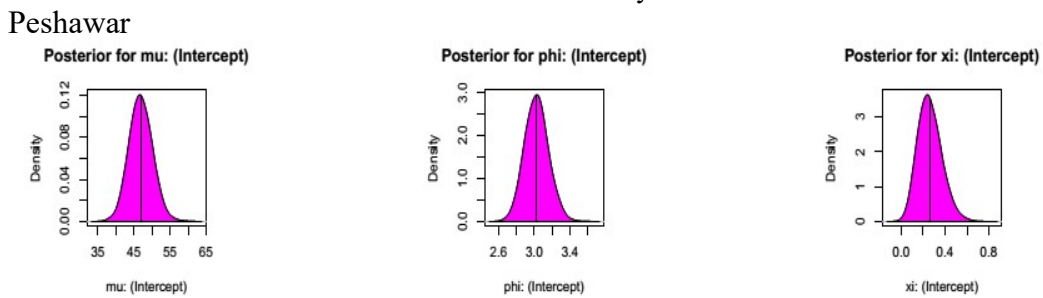
Peshawar



D I Khan

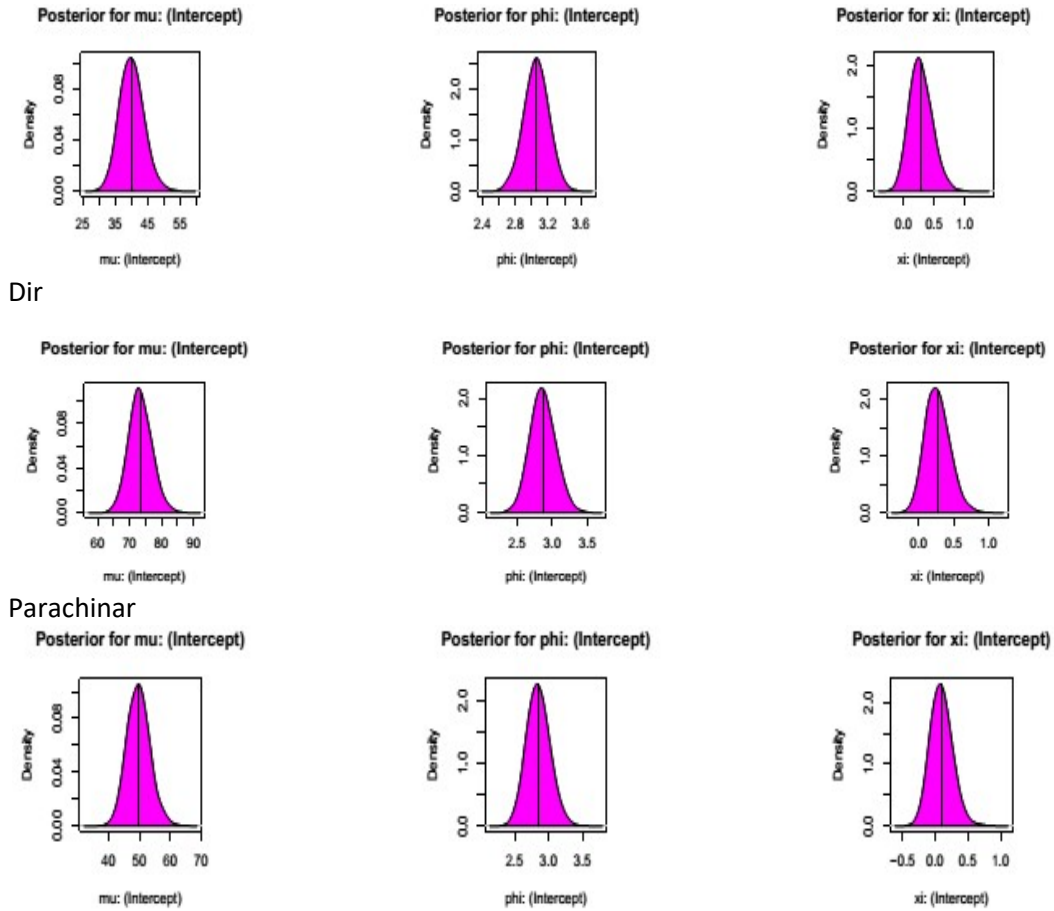


**Fig. 2:** Trace plots for location, scale and shape parameters of GEV distribution for 40000 iterations for Khyber Pakhtunkhwa



D I Khan





**Fig. 3:**Density plots for different cities of Khyber Pakhtunkhwa (KPK).

## V. Conclusions

Current analysis aims to apply Bayesian technique to estimate return levels of extreme annual rainfall of four major cities of Khyber Pakhtunkhwa Pakistan. The estimated acceptance rates of Peshawar, Dir, Parachinar and D I Khan are within the range 10% to 40% i.e. 0.317, 0.31, 0.316 and 0.308 which demands to use Bayesian technique for the said data. Markov Chain Monte Carlo simulation techniques have been used up to 40,000 iterations to calculate the parameters of GEV distribution. The 50 years return levels of Peshawar, Dir, Parachinar and D I Khan are 152mm, 213mm, 139mm and 205mm while those of 100 years return levels are 179mm, 261mm, 163mm, and 270mm respectively. So these expected values of rainfall are useful to engineers, government officials and planning division of KPK to take serious action and manage valuable precautionary measures in time.



## **VI. Acknowledgement:**

We appreciate the cooperation of Pakistan Meteorological department (PMD), especially Computerized Data Processing Center (CDPC) for provision of required rainfall data. The authors declare that they have no conflict of interest.

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