



Mixed mode crack KI, KII on pipe wall subjected to water hammer modeled by four equations fluid structure interaction

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Abstract

In this paper, we studied the failure of the pipe during the transient flow. The pipe is made of ductile cast iron. To simulate the flow, a model includes an upstream tank connected to pipe with a valve at the end is presented; the transient flow is caused by fast time closure of the valve. The governing equations of water hammer are given from the mass and movement continuity conservation laws for fluid and mechanical behaviors laws for pipe structure. This mathematical model is a system of nonlinear hyperbolic partial differential equations where have solved by the method of characteristic along finite difference schema. To understand the behavior of material against surge pressure, we introduce the strain energy density theory (SED) S. The available mechanical propriety of ductile cast iron is used from previous study to get the critical value of strain energy density S_c . At the variance of stress intensity factor KIC criterion, the benefit of strain energy density S; that it can predict the crack growth initiation and direction when the applied stress does not coincide with the crack plane.

Keywords: Water hammer, transient flow, method of characteristics, finite differences, strain energy density

1. Introduction

The pipe network used for fluid transportation is important in each installation especially for which in critical importance, the interruption of fluid supplying can occur and affect the work efficiency that will appear as leakage with a major aspect to environment.

To treat this problem, the understands of behavior of material is necessary; a pipe carried in this study is made from ductile cast iron, replacing the gray cast iron do to their résistance to several service condition; chemical (corrosion,..) and mechanical (high ultimate yield stress). The metallurgical and mechanical test are done by R. Lacalle et al[XII]; in other way to benchmark a flow transition, a case study is realized with an upstream tank connected to a pipe; with a valve characterized by a

fast time closure, the valve feed an downstream tank. The mathematical one-dimensional model has been developed in various researches. For instance, numerical models exist in previous papers of, Streeter and Wylie [XVI], Wylie and Streeter [XVIII]. The numerical solution of the governing equations is obtained by the method of characteristics [VII]. Some important phenomena, that exist in unsteady flows, such as attenuation, line packing, potential surge, pyramiding, and rarefaction are mentioned. Tijsseling AS [XVII] gave a mathematical model to describe the acoustic behavior of a thin pipe filled with liquid, based on fluid structure coupling equations.

II. Mathematical model

The equations describing the phenomenon of transient flow in pipes are based on the principles of mass, the quantity of movement conservation and the laws of mechanical behavior (hook law) [V], the final form of the system of equations is:

$$\rho_f \frac{\partial V}{\partial t} + \frac{\partial P}{\partial x} = \rho g(J - \sin \alpha_0) \quad (1)$$

$$\frac{\partial \sigma}{\partial x} - \rho_m \frac{\partial \dot{U}}{\partial t} = 0 \quad (2)$$

$$\frac{\partial \sigma}{\partial t} - \frac{\nu R}{e} \frac{\partial P}{\partial t} - E \frac{\partial \dot{U}}{\partial x} = 0 \quad (3)$$

$$\left[\frac{1}{K} + \frac{(2R(1-\nu^2))}{eE} \right] \frac{\partial P}{\partial t} - 2\nu \frac{\partial \dot{U}}{\partial x} + \frac{\partial V}{\partial x} = 0 \quad (4)$$

This partial derivative equations system is hyperbolic type, in which $P, V, (U); \sigma$ are respectively the pressure, fluid velocity, displacement of pipe structure and axial stress.

III. Resolution methods

III.i. Method of characteristic

Among the various methods to solving the system of differential equations with partial derivatives of hyperbolic type, we used the method of characteristics. We applying this method, the system of equations above is as follows:

$$C_f^+ : +BF.dp + DF.dt + FF.dV + GF.d\dot{U} - IF.d\sigma + LF.dx = 0 \quad (5)$$

$$C_f^- : - BF.dp + DF.dt + FF.dV + GF.d\dot{U} + IF.d\sigma - LF.dx = 0 \quad (6)$$

$$C_m^+ : + BM.dp + DM.dt + FM.dV + GM.d\dot{U} - IM.d\sigma + LM.dx = 0 \quad (7)$$

$$C_m^- : - BM.dp + DM.dt + FM.dV + GM.d\dot{U} + IM.d\sigma - LM.dx = 0 \quad (8)$$

C_f is the wave propagation speed of the pressure and velocity perturbations in the fluid medium, and C_m is the speed of propagation of the wave of the longitudinal stresses and deformations in the material.

For BF,DF,FF,GF,IF,LF and BM,DM,FM,GM,IM,LM refer to appendix

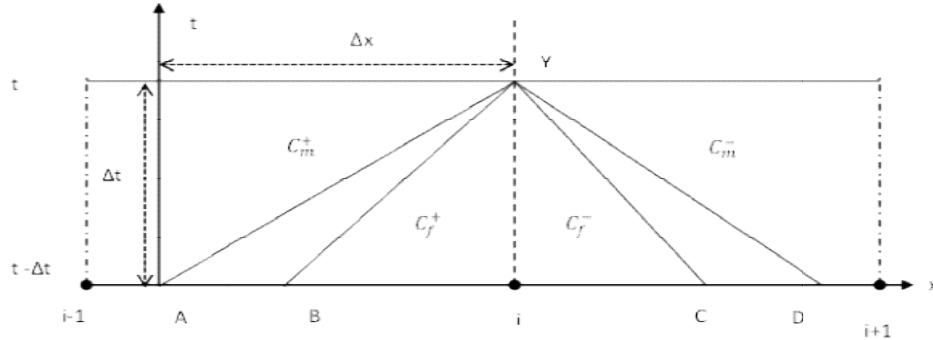


Fig. 1. Discretization and characteristics line in the (x, t) plan.

III.ii. Calculation scheme by the method of finite differences

The goal is to know at any times the pressure P , the velocity of flow V ; the velocity of displacement of the structure \dot{u} , and the axial stress σ , at any part of the pipe, we are led to only the relationship and solve simultaneously for each part of the pipe. The calculation of the pressure at the point (y) to the present time (t) figure (1), is based on points (i-1) at the time before (t-Δt). To do this we use the regular mesh in the (x, t) by division of the driving section to N segment of Δx and Δt step. The numerical scheme stability criterion (current-Hilbert criterion) satisfy, $\Delta T \leq \Delta x / (| \mp | C_m)$

$$BMP_y + FMV_y + GM\dot{u}_y - IM\sigma_y = Hm_A \quad (9)$$

$$-BMP_y + FMV_y + GM\dot{u}_y + IM\sigma_y = Hm_B \quad (10)$$

$$BFP_y + FFV_y + GF\dot{u}_y - IF\sigma_y = Hf_c \quad (11)$$

$$-BFP_y + FFV_y + GF\dot{u}_y + IF\sigma_y = Hf_D \quad (12)$$

III.iii. The friction terms formulations

The effect of friction of the fluid against the wall of the pipe is completely taken into account by the introduction of the end of the variation of the instantaneous speed and average speed change rate, also called the rate of change of frequency. The used model is the Brunone's, which is effective and meaningful. It is $J = J_S + J_U$, with J_S is the hydraulic gradient of the permanent flow and J_U is the hydraulic gradient of unsteady flow.

This term has the following algebraic form:

$$J = \frac{\lambda V |V|}{2gD} + \frac{K_B}{2g} \left(\frac{\partial V}{\partial t} + c_f \text{sign}(V) \left| \frac{\partial V}{\partial x} \right| \right) \quad (13)$$

IV. . Case studies

IV.i. Installations

The installation was studied by M. Dallali et al [X] for pipe with 50km length; the data of hydraulic system are the initial velocity V_0 of 1.29 m/s and initial pressure of 1,22MPa. The material properties are given in table 1. This installation is composed by an upstream tank connected to pipe with valve characterized by fast closure time at the end. The used material is ductile cast iron; the pipe is filled with water. The pipe has diameter 0.672 m connected to a tank of 124.88 m in elevation. The water has kinematic viscosity coefficient of $10^{-6} \text{ m}^2/\text{s}$. The fluid discharge in continuous service plan is $0.591 \text{ m}^3/\text{s}$. The installation shown schematically in Figure3 consists substantially of a tank, a pipe and a valve.

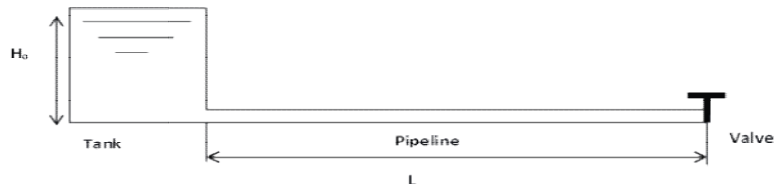


Fig. 2. Scheme of installation

IV.ii. Material propriety

The ductile cast iron pipes are supplied in 6-meter length and in general, conformance to EN 545, belonging to the K9 class, has a nominal diameter (ND) of 700mm, corresponding to a 762 mm external diameter, and 10.2 mm of wall thickness. The pipe was composed of 6000 mm long stretches connected through push-on joints, according to the manufacturer certification. In this case, K9 class pipes are subjected to, and must sustain, a maximum internal pressure of 32 bars (3.2 MPa) [II].

Table 1. Mechanical proprieties of ductile cast iron

E(GPa)	σ_y (MPa)	σ_u (MPa)	ν	K_{Ic} (MPa $\sqrt{\text{m}}$)	ρ_m (kg/m ³)
170	300	420	0,28	14,9	7050

IV.iii. Pressure and stress results

A developed program is used in FORTRAN language, to simulate the pressure and stress result. In the figure (4), it is given the value of pressure at the valve. At time $t = 0$, the pressure P value is that of the steady state, in this case the pressure is equal to 1, 22 Mega pascal. When closing the valve, we see the gradual rise in pressure until the maximum value $P = 2.48$ MPa, which corresponds to the value of the traditional hammer $\Delta p = \rho C_f V_0$. The speed of wave propagation of pressure disturbances is $C_f = 959.57$ m / s. The steps lasts a time of $(T = 2L / C_f)$ equal to 1.73 min, corresponds to the time of wave return. From that time, the pressure starts to decrease to the initial pressure.

When the wave is propagated to the second time to the tank, pressure further decreases to a value $p = 0$ MPa. The cavitation phenomenon is avoided, because it refers to an initial pressure of 1,22MPa. From that time which corresponds to the arrival of the returned pressure wave from tank , a pressure increase is observed until the initial value of the steady state at $t = 3.47$ min which is equal $4L / C_f$.



Fig. 3. Pressure at the valve ($x=L$)

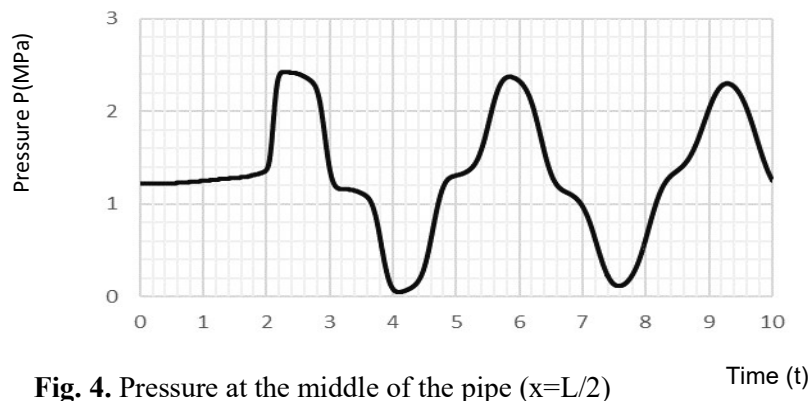


Fig. 4. Pressure at the middle of the pipe ($x=L/2$)

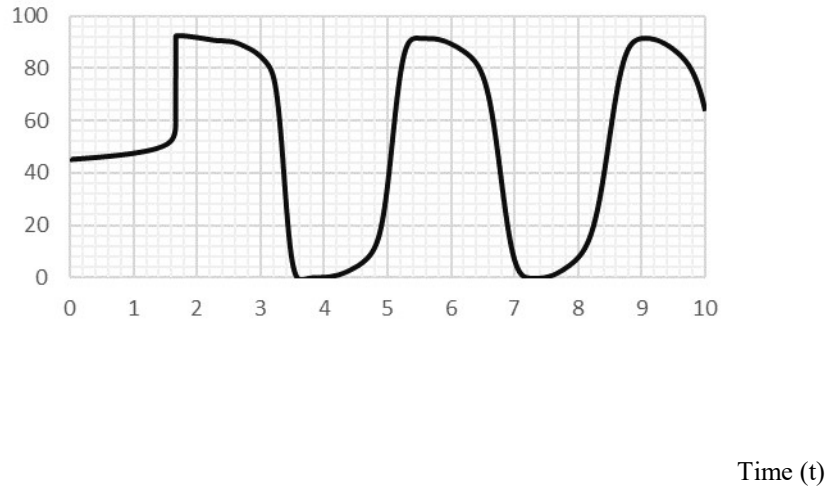


Fig. 5. Hoop stress σ_{θ} at the valve ($x=L$)

Eventually the phenomenon is repeated almost periodically but with mitigation if we take into account a high value fluid friction with the wall of the pipe, if not the phenomenon is repeated without reducing the value of ΔP .

In fig. 5 it is shown, the values of hoop stress σ_{θ} , at the distance $x=L$; in which it take his maximum value of 94, 48 MPa. It is evident that the pipe system will be not fail by the limit charging or load, because σ_{θ} is less than σ_u .

Table 2. Pressure and stress results

x	0	L/5	2L/5	3L/5	4L/5	L
P_{\max} (MPa)	1,23	2,32	2,41	2,44	2,46	2,48
σ_{\max} (MPa)	46,86	88,39	91,82	92,96	93,72	94,48

V. Failure analyses

V.i. Failure description

The modes and causes of pipe failures are corrosion, pitting, manufacturing defects, human error and unexpected levels of loading, all of those, play a role in the large number of pipe failures. We focused in this study on the case of unexpected load. This type of failure is Longitudinal cracking, once the crack has initiated, it may travel the length of the pipe. The end result has been the removal of a section of the top of the pipe, producing a hole that may be as long as the pipe and taking up a third of its circumference [VIII].

V.ii. Failure criterion

V.ii.a. The K_{IC} - criterion

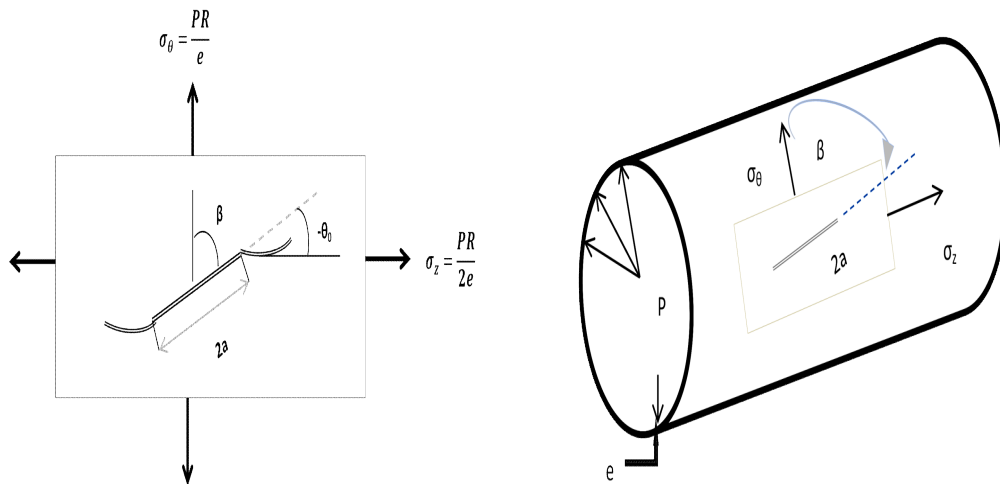
The K_{IC} criterion in failure analysis is applied only for crack aligned to the plane of principal load or stress and is not used in combined load situations fig. 6. A misalignment between the crack and applied load, if ignored, lead to serious errors in the predictions of failure load and design with dimensioning of pipe network. For this reason, it should be used only to facilitate use and has limited application in structure design. The wall structure in the immediate vicinity of the crack tip will behave differently from that of the bulk. The behaviors of the structure over to the crack tip, where the stresses are exceedingly high, are not known [VI].

In most of the structural components the cracks have has a random position to the direction of main loading. That misalignment invalidates the concept of KI criterion. The crack tip stress state in a structural component must be described by at least two parameters, KI and KII.

The strain intensity factor K_I is calculated by the equation $K_I = \sigma_0 \sqrt{\pi a}$, if we put equation $\sigma_0 = PR/e$ we get $K_I = PR\sqrt{\pi a}/e$ and therefore, the allowable pressure is as fellow $P = K_{IC} e / R \sqrt{\pi a}$, in the same manner, the critical pressure is:

$$P_C = K_{IC} e / R \sqrt{\pi a} \quad (14)$$

Where K_{IC} is the fracture toughness of material.



V.ii.b. The strain energy density criterion

The Strain energy density criterion S allow us to treat several cases of mechanical problems of failure, because of its ability to treat various types of crack or flaw of material structure and in each crack position to load mode [XIV]. The strain energy density factor S is a function of the stress intensity factor, and take in account the two parameter K_I and K_{II} .

$$S = a_{11} \cdot K_I^2 + 2a_{12} K_I K_{II} + a_{22} K_{II}^2 + a_{33} K_{III}^2 \quad (15)$$

Where $a_{11}, a_{12}, a_{22}, a_{33}$ are as follow:

$$a_{11} = \frac{1}{16\mu} [(3 - 4\nu - \cos \theta)(1 + \cos \theta)] \quad (16)$$

$$a_{12} = \frac{1}{16\mu} (2\sin \theta) [\cos \theta (1 - 2\nu)] \quad (17)$$

$$a_{22} = \frac{1}{16\mu} [4(1 - \nu)(1 - \cos \theta) + (1 + \cos \theta) (3 \cos \theta - 1)] \quad (18)$$

$$a_{33} = \frac{1}{4\nu} \quad (19)$$

The direction of crack tip growth is unknown, because of the misalignment between the crack and the planes of the principal stress. The strain energy density (S) concept can provide us information about the direction of crack tip. In this case of the plane strain, the stress intensity factors for this problem are

$$K_I = \frac{PR}{2E} \sqrt{a} (1 + \sin 2\beta) \quad (20)$$

$$K_{II} = \frac{PR}{2E} \sqrt{A} \cos B \sin \beta \quad (21)$$

The fundamental hypotheses on unstable crack growth in the Sih theory [6] are as follows:

Crack initiation takes place in a direction determined by the stationary value of the strain energy density factor:

$$dS/d\theta = 0 \text{ when } \theta = \theta_0 \quad (22)$$

Crack extension occurs when the strain energy density factor reaches a critical value:

$$S_C = S(K_I, K_{II}, K_{III}), \text{ for } \theta = \theta_0 \quad (23)$$

The basic concept is to keep the use of the pipe even if crack occur under a limited size. The advantage of the S theory is that it can define the value of permissible load, in this case the pressure (P) for a virtually crack without causing it. The difference between S and SC is analog to K and KC and thus SC is also a measure of the resistance of material against sudden raise of failure.

The angle θ_0 in Eqs (22) and (23) is zero when the crack is orientated normal to the direction of applied tension and the crack extends in a self-similar manner. In this case $k_{II} = k_{III} = 0$, $a_{11} = \frac{1}{4\mu} (1-2\nu)$ and equation (15) simplifies to:

$$S = \frac{1-2\nu}{4M} K_I^2 \quad (24)$$

Here, k_I is associated with the same crack tip stress field as K; they differ only by a factor of $\sqrt{\pi}$, $K_I = K_I / \pi$, the equation (24), will be

$$S = \frac{1-2\nu}{4\pi M} K_I^2 = \frac{(1+\nu)(1-2\nu)}{4\pi E} K_I^2 \quad (25)$$

Sc can be computed from Kc.

V.III. a. Crack angle versus crack tip angle

The directions of crack propagation defined by the angles θ_0 are obtained for all crack positions described by $\beta = 10^\circ, 20^\circ, \dots, 90^\circ$. If we put the equations (20) and (21) to into equation (15) and satisfy the hypotheses of S theory (Eq (22) and (23)) we will get the strain energy density factor as following:

$$S = \left(\frac{PR}{2E} \right)^2 A F(\beta, \theta) \quad (26)$$

Where the function $F(\beta, \theta)$ stands for:

$$F(\beta, \theta) = a_{11} (1 + \sin 2\beta)^2 + a_{12} (1 + \sin 2\beta) \sin 2\beta + a_{22} \sin 2\beta \cos 2\beta \quad (27)$$

The results are plotted in figure7. When we insert the crack angles β into equation (26), SC is the value of S, when the pressure reach the value of crack growth or crack extension, and it is:

$$S_c = \left(\frac{P_c R}{2e} \right)^2 a F(\beta, \theta), \quad (28)$$

This can be rearranged to solve the quantity:

$$\frac{P_c R}{2e} \sqrt{a} = \sqrt{\frac{S_c}{F(\beta, \theta)}} \quad (29)$$

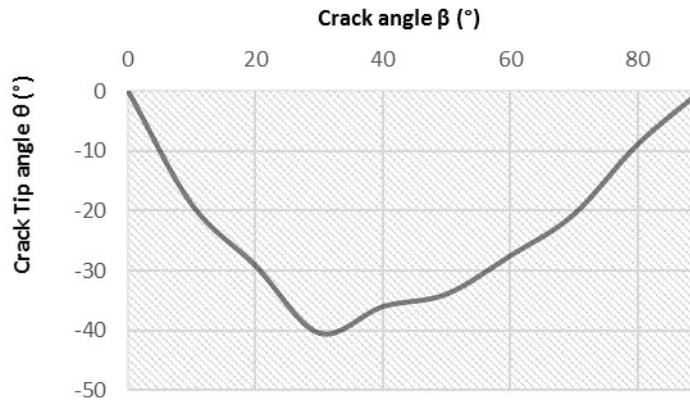


Fig. 7. Crack tip angle versus fracture angle

V.iii.b. Critical pressure versus crack length

A semi-elliptical crack (Figure6) which entails one-half thickness of pipe as depth ($c/e = 0.5$) is considered. The initial crack length $2a$ is very important for the critical pressure P_c values. In the current work, for semi-elliptical cracks, the ratio of crack depth over one-half length of elliptical crack is equal to 0.2, i.e. $c/a = 0.2$. This ratio value signifies the most critical semi-elliptical geometry subject to pitting corrosion in water pipelines. Consider a pipe of the following dimensions: $R = 672$ mm, $e = 10.2$ mm and $2a$ varied from 50 mm to 130 mm. The characteristics of the material are: $K_{IC} = 14.9 \text{ MPa} \sqrt{m}$, $\sigma_u = 420 \text{ MPa}$ and $E = 170000 \text{ MPa}$. The critical pressure P_c for the pipe with no initial crack is calculated by the equation $P_c = \sigma_u e / R$.

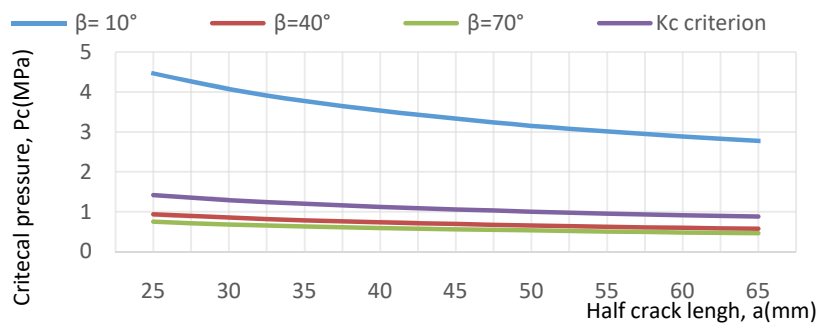


Fig. 8. Critical pressure versus half crack length

V.iii.c. Result and Discussion

In figure 8 we give the values of the critical pressure P_C , calculated by Eqs (29) and (14) for the strain energy density criterion S and K_C criterion respectively. For the angles 10° , 40° , 70° , The results shows that each criterion has a deferent shape, the line when the angle θ equal 10° the critical pressure varied from 4,46 to 2,77 MPa, and for the K_C criterion the critical pressure varied from 1,42 to 0,88 MPa. It is clear that there are deference between the two criterions that explain the behavior of the crack tip against the sudden rise of pressure; more, the criterions approximate when the crack angle tends to the 90° angle. In this case the behavior is known as the rapid crack propagation or the fast crack propagation and is the most vulnerable case in the rupture of the pipe.

VI. Conclusion

The analysis of obtained result for piping system during transient flow caused by water hammer phenomenon; let us know the behavior of the structure. The mathematical model represented by a system of partial differential equations, with introducing of a friction term. The equations were solved in the time domain by the characteristic method using linear integration. To ensure the accuracy of the solution, small- time increments in the constructed computer program were used. This program permits to get some systematic indications on the evolution and the damping of the head pressure waves due to the fast valve closure.

The failure of the pipe, characterized by a virtually circumferential crack, the strain energy density criterion is used to determine the fracture toughness of material in the zone of crack tip. It is also possible to predict the initiation and the direction of crack growth (θ) when we determine the value of the maximum potential energy density (SC).

The results are useful for maintenance and inspection of installation during revision plan.

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