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A Comprehensive Explanatory Derivation from an Equation of the Special Theory of Relativity; Doppler Effect is a Property of Space – Time

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Abstract

Einstein was pioneer in his work of the theory of special relativity and the theory of general relativity. This paper took a famous equation from the theory of special relativity to have a comprehensive explanatory derivation from the equation of the theory of special relativity. This paper also explains why there is always constancy of the speed of light, the universal speed limit of the Universe, disregard of movement in or away of source that transmits light and the movement in or away of body that receives light. This paper also shows that Doppler Effect is a property of space – time. The Doppler Effect can explain why there is the constancy in the speed of light.

Keywords : The theory of special relativity, the theory of general relativity, the space – time, Doppler Effect, the speed of light – the Universal speed limit of the Universe.

I. Introduction

According to the Einstein's Special Theory of Relativity, the relationship between rest mass and relativistic mass is [I], [II]

$$m_r = \frac{m}{\sqrt{1-\frac{V^2}{C^2}}} \quad (1)$$

Where,m_r is the relativistic mass of an astronomical body having non-zero velocity$V$
$m$isthe rest mass of an astronomical body having no velocity($V = 0$).
Cis speed of light, the universal speed limit of the Universe
Equation 1 explains that if the velocity of a body becomes the velocity of light ($V = C$), then, the relativistic mass $m_r$attains infinity. From equation 1, we can subsequently derive, [III],[IV]

$$m_r{}^2 = \frac{m^2}{(1-\frac{V^2}{C^2})} \quad (2)$$

$$(\frac{m}{m_r})^2 = 1 - \frac{V^2}{C^2} \quad (3)$$

Given an astronomical body has a rest mass$m = Constant, K$ and the speed of light is a Universal constant$C = Universal\ Constant, K_U$

$$(\frac{K}{m_r})^2 = 1 - \frac{V^2}{K_U{}^2} \quad (4)$$

$$(\frac{K}{m_r})^2 + (\frac{V}{K_U})^2 = 1 \quad (5)$$

Let $m_r = Variable\ x$ and $V = Variable\ y$

$$(\frac{K}{x})^2 + (\frac{y}{K_U})^2 = 1 \quad (6)$$

Let $K = Constant\ a$ and $K_U = Constant\ b$

$$\frac{a^2}{x^2} + \frac{y^2}{b^2} = 1 \quad (7)$$

Thus, relativistic mass and velocity of the same particle have non-linear relationship. And they are related with the equation 7. A sufficiently large rest mass is assumed (like 100000 Kg) to plot the following graphs.

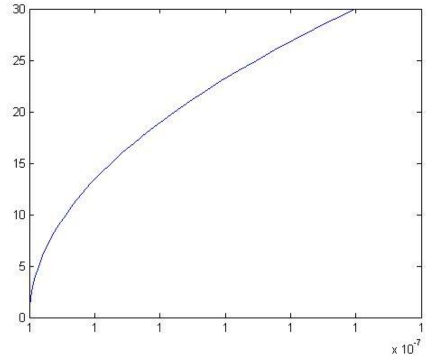

Fig 1.Plot between relativistic mass m_r of a particle and its velocity is when 30 km/sec

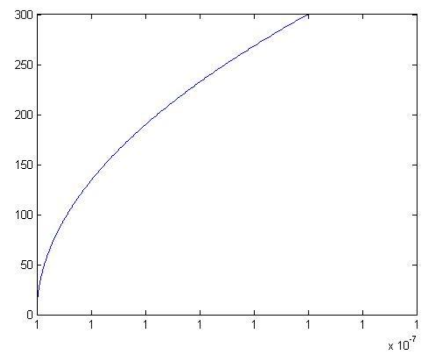

Fig 2.Plot between relativistic mass m_r of a particle and its velocity is when 300 km/sec

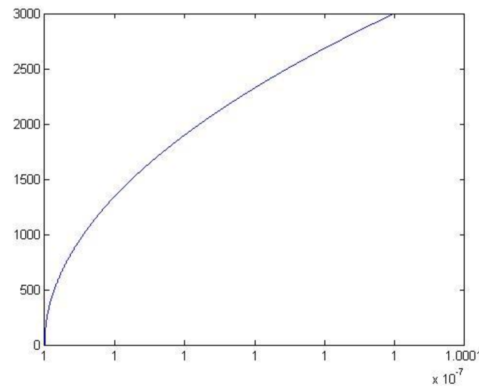

Fig 3.Plot between relativistic mass m_r of a particle and its velocity is when 3000 km/sec

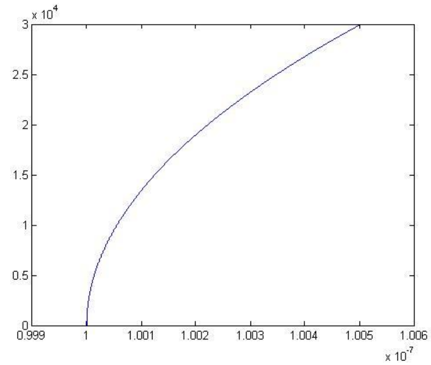

Fig 4. Plot between relativistic mass m_r of a particle and its velocity is when 30000 km/sec

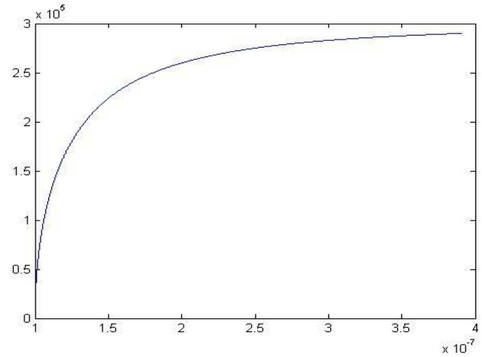

Fig 5. Plot between relativistic mass m_r of a particle and its velocity is when 290000 km/sec

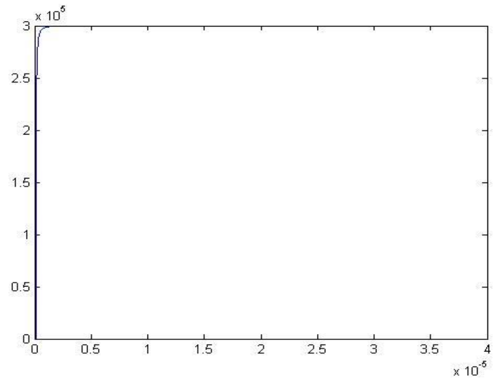

Fig 6. Plot between relativistic mass m_r of a particle and its velocity is when 300000 km/sec

The variation in the relativistic mass of a big enough astronomical body is shown with variation of its speed. The plot is non-linear, shaping like a growing exponential signal. Thus, mass and velocity have non-linear relationship can be approximated as growing exponential signal.

II. Doppler Effect is a Property of Space – Time

We know that, velocity of a body is ratio of distance to time. [V, VI]

$$Velocity(V) = \frac{Distance\ (d)}{Time\ (t)} \tag{8}$$

Doppler Effect is that when a source of light or sound moving towards relative to the observer, it attains higher and higher frequency and when a source of light or sound moving away relative to the observer, it loses frequency [VII]. It cannot be anabsolute truthbut it is an apparent truth [VIII]. The gaining and losing frequency is relative to the observer only. It is only that what an observer feels about the sound or light when moving in or moving away of the source relative to the observer is basically the Doppler Effect. The Doppler Effect can be extended to prove the constancy of the speed of light which, in turn, proves that Doppler Effect is a property of space-time and not the property of any medium.

We know, for a periodic signal (all wave propagations are periodic signals, like, sound wave, light wave)

$$Frequency(f) = \frac{1}{Time\ \ Period\ (T)} \tag{9}$$

For light, the velocity is C , $a\ Constant$. Then equation 8 becomes, [IX]

$$C = \frac{Distance\ \ (d)}{Time\ (t)} \tag{10}$$

For a periodic signal like all wave propagations, time t can be written asKT , in words, given time is integer multiple of fundamental time period (for periodic signal)

. $K = \pm 1, \pm 2, \pm 3 \dots \dots \dots$

$$t = KT \tag{11}$$

$$t = \frac{K}{f} \tag{12}$$

If source of light coming towards relative to observer, distance (d) will decreases, because the speed of light is always constant, time (t) must decrease too. It means KT decreases too. So, the time period (T) gets decreased, thus, frequency increases. The observer feels higher frequency of light when source is moving towards relative to observer. If source of light moving away relative to observer, distance (d) will increases, because the speed of light is always constant, time (t) must increase too. It means KT increases too. So, the time period(T) gets increased, thus, frequency decreases. The observer feels shorter time period(T) and higher frequency(f) of light when source is moving towards relative to observer. The increment or decrement of distance incorporates the increment or decrement of time in such a way that the ratio of distance to time is always constant for light. That is the reason why the speed of light is always constant disregard of moving in or away of source relative to observer. It is the observer's measurement gets changed with the movement of source of light relative to observer. That is the reason why there is always constancy in the speed of light. Because Doppler Effect is valid in space too, the Doppler Effect is a property of space-time.

III. Conclusion

The relativistic mass of a body and its velocity have a non-linear relationship, can be approximated as growing exponential signal. The Doppler Effect can be used to prove the constancy of the speed of light disregard of any movement in or away of the source relative to the observer. Doppler Effect is that when a source of light or sound moving towards relative to the observer, it attains higher and higher frequency and when a source of light or sound moving away relative to the observer, it loses frequency. Gaining and losing frequency is not absolute truth, but an apparent truth only. Gaining and losing frequency is an observer dependent realism only. Because Doppler Effect can be used in light and light can travel through free space, Doppler Effect is a property of space-time and not of any medium.

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