

Exponentially backlogged shortage inventory model for deteriorating item with linear selling price of the product

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Abstract

This paper deals with an inventory model for deteriorating items with linear price and frequency of advertisement dependent demand and exponentially backlogged shortages. The deterioration rate follows three-parameter Weibull distribution. The corresponding non-linear problem have been formulated and solved. Numerical example has been considered to illustrate the model and the significant features of the result are discussed. Finally, we have performed the sensitivity analysis taking one or more parameters at a time.

Keywords: Inventory, Weibul distribution deterioration, linear price dependent demand, Partially backlogged shortage.

I. Introduction

Deterioration is the natural phenomenon in the reality. It cannot be ignored in the inventory analysis. It has huge impact on the business. Ghare and Schrader [XX] first proposed this concept in their research and developed an exponentially decaying inventory model. Then Emmons [XIX] introduced the variable deterioration concept in their work. This variable deterioration concept is namely as two-parameters or three parameters weibull distribution. Taking this type concept, several researchers introduced lot of research in the existing literature such as Covert and Philip [XVII], Giri et al. [XXII], Ghosh and Chaudhari [XXI]. On the other hand, Chakrabarty et al. [XVI], Giri et al. [XXIV], Sana et al. [XL], Sana and Chaudhari [XXXIX] and others developed inventory models for deteriorating items with there-parameter weibull distributed deterioration. Misra [XXXIV] developed an EOQ model with a weibull deterioration rate for perishable product without considering backlogging situation. These investigations were followed by several researchers like, Deb and Chaudhari [XVIII], Giri et. al.[XXIII], Goswami and Chaudhari [XXV], Mandal and Phaujdar [XXXI],

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 Padmanabhan and Vrat [XXXV], Pal et al.[XXXVII], Mandal and Maiti [XXXII], Goyal and Gunasekaran [XXVI], Sarkar et al.[XLI], Bhunia and Maiti [III],[II], Bhunia and Shaikh [VII],[V],[XII], Pal et al.[XXXVI], Bhunia et al. [XI], Bhunia and Shaikh [VI], Bhunia et al [XIII], etc., where a time-proportional deterioration rate was considered.

Advertisement of the product is one of the important issues in competitive business. Retailers always want put some advertisement of his product in the popular media such as Newspaper, Magazine, Radio, T. V., Cinema, etc. and also through the sales representatives have a motivational effect on the people to buy more. Kotler [XXVIII] incorporated marketing policies into inventory decisions and discussed the relationship between economic ordering quantity and pricing decision. Ladany and Sternleib [XXIX] studied the effect of price variation on selling and consequently on EOQ. However, they did not consider the effect of advertisement. Subramanyam and Kumaraswamy [XLIX], Urban [LII], Abad [I] and Luo [XXX] developed inventory models incorporating the effects of price variations and advertisement on demand rate of an item. In this connection we may refer some recent works such as Shaikh [XLV], Bhunia et al. [IX], [XIV], [XV], [IV], Bhunia and Shaikh [X], Shaikh et al. [XLVII], [XLIV], [XLVI], [XLVIII], Bhunia et al. [VIII], Tiwari et al. [LI], Mishra et al. [XXXIII], Shaikh [XLII], Shaikh et al. [XLIII], Panda et al. [XXXVIII], Khan et al. [XXVII], Taleizadeh et al. [L], etc.

In this paper, we have studied an inventory model for deteriorating items which follows three parameters weibull distribution. Shortages are allowed with the exponentially backlogging rate under the length of the waiting time of the customers. The corresponding non-linear problem have been formulated and solved. Numerical example has been considered to illustrate the model and the significant features of the result are discussed. Finally, we have performed the sensitivity analysis taking one or more parameters at a time.

II. Assumptions and Notations

We have developed the paper by considering the following assumption as notations:

Assumption:

- (i) Replenishments are instantaneous.

- (ii) The inventory planning horizon is infinite and the inventory system involves only one item.
- (iii) The deterioration rate follows three-parameter weibull distribution.
- (iv) The demand rate $D(A, p)$ is dependent on selling price (p) of an item and the frequency of advertisement (A). We assume it as follows:

$$D(A, p) = A^{\nu} a p^{-b}, a, b, \nu \geq 0.$$

Notations:

Notations	Units	Description
C_o	\$/order	Ordering cost
a	Constant	Demand parameter ($a > 0$)
b	Constant	Demand parameter ($b > 0$)
p	\$/unit	Selling price per unit
C_p	Units	Purchase cost per unit
C_l	\$/unit	Opportunity cost per unit
C_{ad}	\$/unit	Advertisement cost
C_b	\$/unit	Shortage cost per unit
C_h	\$/unit /unit time	Holding cost per unit
R	Units	Maximum backlogged units
S	Units	Maximum inventory level
Z	\$/year	Total profit per unit time.
Decision variables		
t_l	Years	Time at which the stock in RW reaches to zero
T	Years	The length of the replenishment cycle

III. Mathematical Model

In this model, we have allowed shortages partially with the exponential backlogging rate which is the length of the waiting time of the customers. By considering this situation, the rate is defined as $e^{-\delta(T-t)}$, $\delta > 0$.

Initially, a retailer purchased $(S+R)$. After fulfilling the shortages amount, the on-hand inventory level is S at $t=0$ and it declines continuously due to demand and deterioration up to the time $t=t_1$ and reaches the zero level. After the time $t=t_1$, shortage occurs and it accumulates at the rate $e^{-\delta(T-t)}$, $(\delta > 0)$ up to the time $t=T$ when the next lot arrives. At time $t=T$, the maximum shortage level is R . This entire cycle then repeats itself after the cycle length T .

Let $q(t)$ be the inventory level at any time $t \geq 0$. Then the inventory level $q(t)$ at any time t satisfies the differential equations

$$q'(t) + \theta(t)q(t) = -D(A, p), \quad 0 \leq t \leq t_1 \quad (1)$$

$$q'(t) = -D(A, p)e^{-\delta(T-t)}, \quad t_1 < t \leq T \quad (2)$$

with the boundary conditions

$$q(t) = S \text{ at } t=0, \quad q(t) = 0 \text{ at } t=t_1. \quad (3)$$

and

$$q(t) = -R \text{ at } t=T. \quad (4)$$

Also, $q(t)$ is continuous at $t=t_1$.

Using the conditions (3) and (4), the solutions of the differential equations (1)-(2) are given by

$$\begin{aligned} q(t) &= D(A, p)e^{-\alpha(t-\gamma)^\beta} \int_t^{t_1} e^{\alpha(t-\gamma)^\beta} dt, \quad 0 \leq t \leq t_1 \\ &= \frac{D(A, p)}{\delta} \left\{ 1 - e^{-\delta(T-t)} \right\} - R, \quad t_1 < t \leq T \end{aligned}$$

From (2), we have $q(t) = S$ at $t=0$.

Then,

$$S = D(A, p)e^{-\alpha(-\gamma)^\beta} \int_0^{t_1} e^{\alpha(t-\gamma)^\beta} dt \quad (5)$$

From the continuity condition, we have

$$R = \frac{D(A, p)}{\delta} \{1 - e^{-\delta(T-t_1)}\} \quad (6)$$

The total number of deteriorated units is given by

$$D' = \int_0^T \theta(t)q(t)dt$$

The total inventory cost per cycle of the system consists of the following components and which given below:

Ordering cost: C_o .

Purchasing cost: $C_p(S + R)$.

Inventory holding cost:

$$C_{hol} = C_h \int_0^{t_1} q(t) dt \quad (7)$$

$$\text{Shortage cost: } C_{sho} = c_b \left[\left(R - \frac{D(A, p)}{\delta} \right) (T - t_1) + \frac{D(A, p)}{\delta^2} \{1 - e^{-\delta(T-t_1)}\} \right]$$

$$\text{Lost sale cost: } CLS = c_l D(A, p) \left[(T - t_1) - \frac{1}{\delta} \{1 - e^{-\delta(T-t_1)}\} \right]$$

The total inventory cost (TC) of the system is given by

$$TC = \langle \text{ordering cost} \rangle + \langle \text{purchasing cost} \rangle + \langle \text{inventory holding cost} \rangle + \langle \text{advertisement cost} \rangle + \langle \text{transportation cost} \rangle + \langle \text{inventory shortage cost} \rangle + \langle \text{Opportunity cost} \rangle$$

$$= C_o + C_p(S + R) + C_h \int_0^{t_1} q(t) dt + C_{ad}A + C_{tran} + C_{sho} + CLS$$

The profit function

The net profit (X) for the entire system is the difference between the sale revenue per cycle and the total cost of the system i.e.,

$$X = pA^V(a - bp)T - TC \quad (8)$$

Therefore, the profit function $Z(A, t_1, T)$ (average profit per unit time for the entire cycle) of the inventory system is given by

$$Z(A, t_1, T) = \frac{X}{T}$$

i.e.,

$$Z(A, t_1, T) = \left[pA^v(a-bp) - \left\{ C_4 + C_3(S+R) + C_1 \int_0^{t_1} q(t) dt + C_5 A + C_{sho} + CLS \right\} \right] / T \quad (9)$$

Here the profit function is a function of three continuous variables t_1 , T and one integer variable A .

The optimal solution of the above problem can be obtained with the help by using GRG method.

IV. Numerical Example

To validate the proposed model, we have considered one numerical example. We cannot consider any case study but the data are seeming realistic.

$C_h = \$0.5$ per unit per unit time, $C_b = \$8$ per unit per unit time, $p = \$14$, $C_p = \$10$ per unit, $C_o = \$250$ per order, $C_{ad} = \$50$ per advertisement, $\alpha = 0.05$, $\beta = 2$, $\gamma = 2$, $a = 80$, $b = 0.4$, $\delta = 0.5$, $\nu = 0.2$, $c_l = 10$.

The optimal solution has been obtained with the help of GRG method for different values of m . The optimum values of A , t_1 , T , S and R along with maximum average profit which are given below:

$$Z(A, t_1, T) = 303.8765, T = 3.339817, t_1 = 3.260368, A = 6.000000, S = 299.4886, R = 8.292700$$

V. Sensitivity Analysis

From the above example, we have performed a sensitivity analysis in order to measure the sensitivity of the proposed model. This analysis has been carried out by changing (increasing and decreasing) the parameters from -20% to $+20\%$, taken one or more parameters at a time making the other parameters at their more parameters at a time and making the other parameters at their original values.

The results of this analysis are shown in **Tables 1**.

Parameter	% changes of parameters	changes in Z^*	A^*	change in			
				R^*	S^*	t_1^*	T^*
C_h	-20	324.9088	7	7.755087	325.9304	3.433728	3.505636
	-10	314.1167	7	8.502019	319.2518	3.366362	3.445334
	10	294.1795	6	8.966261	293.6154	3.197887	3.283930
	20	284.7640	5	9.622377	288.0135	3.137854	3.230341
a	-20	205.5035	5	7.983312	236.5617	3.397710	3.499326
	-10	253.7506	6	8.615188	274.4180	3.344602	3.437392
	10	355.5845	7	8.883530	338.5175	3.227274	3.301694
	20	408.5999	8	9.461510	378.0665	3.199375	3.269647
b	-20	310.9346	6	8.245367	302.9614	3.249548	3.327340
	-10	307.4044	6	8.269096	301.2261	3.254930	3.333545
	10	300.3509	6	8.316177	297.7490	3.265860	3.346156
	20	296.8277	6	8.339526	296.0072	3.271410	3.352563
p	-20	35.03772	2	5.154487	230.9285	3.088194	3.148513
	-10	162.4326	4	6.739171	270.3523	3.169901	3.239223
	10	455.7324	9	10.62984	335.2107	3.386038	3.481024
	20	615.3786	12	12.75606	364.8579	3.500153	3.608960
C_p	-20	500.2007	10	13.30548	352.0472	3.452560	3.568706
	-10	399.7008	8	10.84071	326.7549	3.355127	3.453646
	10	213.3376	5	6.635872	284.2362	3.210441	3.276153
	20	128.4635	3	4.113128	249.7495	3.125730	3.170608

Table 1: Sensitivity analysis with respect to different parameters

VI. Concluding Remarks

In this work, we have investigated an inventory model for deteriorating items with linear price dependent demand along with the frequency of advertisement. Shortages are considered with the exponentially backlogging rate. In this model, the demand rate is taken as $D(A, p) = A^V(a - bp)$. It is well known that $D(A, p) \propto (a - bp)$ for fixed A . The demand of items increases with the increase of frequency of advertisement and is directly proportional to the number of advertisements. Hence, we take $D(A, p) \propto A^V$ for fixed p .

The problem can be further modified by taking different types of trade credit, advance payment, non-linear holding cost, inflation etc.

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