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Parameter Estimations of Stochastic Volatility Model by Modified Adaptive Kalman Filter with QML

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Abstract

To determine the parameters of Stochastic Volatility Model (SVM), a modification to the Quasi Maximum Likelihood (QML) scheme has been proposed by employing (modified) Adaptive Kalman Filter (AKF). AKF allows optimization over lesser number of parameters as the variance (σ_v^2) of the noise in the volatility state equation is determined by the AKF. The adaptive method, instead of a constant σ_v^2, allows it to be time varying. Before applying the methodology on market data, the proposed method is characterized here by synthetic data through simulation investigations. Numerical experiments show that the performance of SVM based QML-KF and novel QML-AKF are comparable to that of more popular GARCH family based techniques.

Keywords : Adaptive Estimation, Noise Covariance Adaptation, Modified AKF, Stochastic Volatility Model, Quasi-Maximum Likelihood

I. Introduction

This work introduces the Stochastic Volatility Model (SVM), one of the existing methods, namely Quasi-Maximum Likelihood (QML) for parameter estimation [XX] of SVM, using ordinary Kalman filter (QML-KF) [I, VII, XII, XXI] and proposes an adaptive version of QML, generically called QML-AKF. The purpose is to investigate the efficacy of using Adaptive Kalman Filter (KF) [III, IV] in the Quasi-Maximum Likelihood (QML) framework for estimation of the parameters of SVM. This characterization of methods would be helpful in applying these techniques to model market data [III, IV, IX].

The state-space framework of the SVM assumes three parameters (namely α, δ and σ_v). Dynamic estimation of those parameters is a challenging task. In the QML-

KF the state estimation component is carried out using ordinary KF and the three parameters are selected for maximum likelihood. In this work, a systematic search method was used for maximizing the likelihood, instead of other automated optimizing methods in order to get more insight into the sensitivities of each parameter.

In the proposed QML-AKF method, the parameter σ_v is estimated by the adaptive KF methods (discussed in [III, IV]) and the other two parameters were selected based on systematic search in parameter space for maximum likelihood.

SVM, possibly first suggested in [XVI], is a distinctly different approach from EWMA and ARCH/GARCH models. The relative advantages of SVM approach as recently reviewed by [VIII, XV] include capability to provide one-step-ahead prediction and to better accommodate excess kurtosis and leverage effects compared to GARCH. [VI, XV, XVIII] are recent good textbooks in the domain of stochastic volatility literature. However, sections 3.9 to 3.11 of [V] and [II] reviewed the previous work on SVM in a systematic way presenting almost all possible varieties of the model and the estimation techniques. The disadvantages of SVM [ibid] include the requirement of simultaneous estimation of states and parameters, and consequent additional computation. Another limitation of the SVM is that it is non-linear though a logarithmic transformation may lead to its linearization but some problems still retained with the model due to its logarithmic scaling. Further, “ordinary” SVM can produce misleading estimates when there is significant “leverage effect” [XIII-XIV]. This work, however, concentrates on SVM only.

The rest of the paper is organized as follows. The next section II briefly revisits some previous work on SVM along with the theory of associated with it and the problems there in. Section III presents the proposed method SVM-QML-AKF and its modification. Section IV presents the results of simulation investigations with synthetic data using the proposed method. The paper is ended with a section on conclusion.

II. Theoretical Background

Discrete Time SVM

The discrete time model of the return r_t of a risky component given in [XIX] is a product process expressed as:

$$r_t = \sigma_t \varepsilon_t \tag{1}$$

where ε_t is zero mean unit variance Gaussian process and σ_t is some non-negative process, termed as the “latent stochastic volatility” at time t.

The state space SVM [VIII] with the latent stochastic volatility is possibly first characterized by [XVI]. The substitution assuming $\sigma_t = \exp\left(\frac{h_t}{2}\right)$ whereby $r_t = \varepsilon_t \exp\left(\frac{h_t}{2}\right)$ and a logarithmic transformation implies

$$\log r_t^2 = h_t + \log \varepsilon_t^2, \tag{2}$$

where $\log r_t^2$ is a measurable output.

The discrete time state evolution process, h_t with parameters δ (the state evolution gain must have a modulus less than unity for stability), α (a constant but unknown bias input), and σ_v (a positive constant giving the standard deviation of the process noise) can be expressed as

$$h_t = \alpha + \delta\, h_{t-1} + \sigma_v v_t, \text{ for } t = 1, 2, \ldots\ldots, \mathrm{T} \tag{3}$$

where v_t is a zero mean, unit variance Gaussian noise sequence i.e. $v_t \sim N(0,1)$ termed as process noise.

In the measurement equation (1), $\eta_t = \log \varepsilon_t^2$ is analogous to measurement noise. Though ε_t may be a zero mean, unit variance Gaussian sequence, $\eta_t = \log \varepsilon_t^2$ is not. However, from [XII], it may be verified that the mean and variance of $\log \varepsilon_t^2$ are -1.27 and $\frac{\pi^2}{2}$ respectively. The term $\log \varepsilon_t^2$ is often replaced by a zero mean noise η_t.

i.e. $$\log r_t^2 = -1.27 + h_t + \eta_t, \tag{4}$$

where $E(\eta_t) = 0$ and $\mathrm{var}(\eta_t) = \frac{\pi^2}{2}$.

So the summarized version of state-space SVM (log-normal) is given by

$$\log r_t^2 = -1.27 + h_t + \eta_t, \tag{4}$$

$$h_t = \alpha + \delta\, h_{t-1} + \sigma_v v_t \tag{3}$$

where $v_t \sim N(0,1)$, $E(\eta_t) = 0$, $\mathrm{var}(\eta_t) = \frac{\pi^2}{2}$, $\eta_t = \log \varepsilon_t^2$, $\varepsilon_t \sim N(0,1)$. α, δ and σ_v are parameters of the model. Here $\sigma_t = \exp\left(\frac{h_t}{2}\right)$ and r_t identifies the stochastic volatility and return (preferably Gaussian) processes respectively.

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The usage of the SVM for VaR computation [III, IV] calls for precisely the following:

(i) identification of the parameters of the model,
(ii) estimating the latent state variable h_t and
(iii) using the state equation for extrapolating h_{t+1} and thereby, the return r_{t+1} (or log return since return and log return have negligible differences).

SVM Parameter Estimation using QML-KF

The applicable numerical techniques for this simultaneous estimation of states and parameters as surveyed in [VIII, XV], include the moment matching method [XV], Quasi-Maximum Likelihood (QML) [XII, XI], and Monte Carlo simulation based method [X] etc. More elaborate discussions on the QML method may be obtained in [XII, I, X]. Estimation of SVM parameters using QML technique is usually based on Kalman Filter (KF). Kalman Smoothing for stochastic volatility estimation in a state space model for real and simulated situations are also discussed in [X].

The QML technique is one of the simplest but an approximate method for parameter estimation of the SVM. As the noise process η_t is non Gaussian, standard parameter estimation techniques cannot be used directly. However, the QML method makes the approximation by assuming that η_t is Gaussian so that KF may be used. Following the KF steps we can then numerically determine the point in the parameter space $\{\alpha, \delta, \sigma_v\}$ corresponding to the maximum likelihood of the measurement sequence $\{r_t\}$ or equivalently of $\{\log r_t\}$. The advantage of following QML approach is its speed and adaptability to diversified situations [VIII].

To initiate the KF steps, one must have initial distribution (mean and covariance) of the state variable h_t at t=0. It is customary to assume $h_0 \sim N\left(\frac{\alpha}{1-\delta}, \frac{\sigma^2{}_v}{1-\delta^2}\right)$, due to stationarity of the underlying time series [I].

Algorithm 1: QML-KF for SVM Parameter Estimation

Step 1: Initialization Step

Set t=0. Select suitable guess values of α, δ, σ_v. Initialise the filter state by

$h_{0|0} = \frac{\alpha}{1-\delta}$ and state error covariance $P_{0|0} = \frac{\sigma^2{}_v}{1-\delta^2}$. Also obtain the observation r_t.

Step 2: KF Prediction Step

$$h_{t|t-1} = \alpha + \delta\, h_{t-1|t-1}$$

and

$$P_{t|t-1} = \delta^2 P_{t-1|t-1} + \sigma_v^2 .$$

Step 3: KF Update Step

$$h_{t|t} = h_{t|t-1} + \frac{P_{t|t-1}}{\Omega_t}\left[\log(r_t^2) + 1.27 - h_{t|t-1}\right] \text{ and}$$

$$P_{t|t} = P_{t|t-1}\left[1 - \frac{P_{t|t-1}}{\Omega_t}\right] \text{ where } \Omega_t = P_{t|t-1} + \frac{\pi^2}{2} .$$

Step 4: Repeat Step2 and 3 for all epochs.

$$L(\alpha,\quad \delta,\quad \sigma_v) \propto -\tfrac{1}{2}\sum \log(\Omega_t) - \tfrac{1}{2}\sum \frac{v_t^2}{\Omega_t}$$

Step 5: Likelihood Calculation

where $v_t = \log(r_t^2) + 1.27 - h_{t|t-1}$ is the one step ahead prediction error.

Step 6: Repeat Step 1 to 5 to calculate $L(\alpha, \delta, \sigma_v)$ for a suitable range covering the expected range of values with appropriate increments.

Step 7: Determine α^*, δ^* and σ_v^* for which $L(\alpha^*, \delta^*, \sigma_v^*)$ is maximum and these α^*, δ^* and σ_v^* are the ML estimates of three parameters under consideration respectively.

The benefits of QMLE are its simplicity and consistency. Its drawbacks are that the estimates are inefficient, even asymptotically, and that its small-sample properties are suspecting. [XVII] surveyed and presented a number of alternative ways for improving the efficiency of the QML estimator.

Characterization of QML-KF with Synthetic Data

Before applying the QML-KF method to market data, it was considered worthwhile to test and characterize this method with synthetic data for a better understanding of the model and the algorithm. This was carried out with simulation experiment described below.

Data Generation: The simulation experiments, reported in this section, have been carried out with the synthetic data for characterizing the QML-KF. Constant values of parameters α, δ and σ_υ were first chosen for SVE. Such data are generated with the state-space SVM equations (3) and (4) with its assumptions. The noise variables v_t and ε_t have been generated using the Gaussian random number generator of Matlab-7 software. Time duration (or number of epochs) of the considered data sets are typically varies among 500, 1000 and 1500 for these simulation studies. The number of Monte Carlo runs for reporting standard errors of estimation has been restricted preferably to 1000 (rarely to 500) because of huge computational requirements of joint estimation of SVM parameters using QML, though number of Monte Carlo runs is much higher like 10000 or more for single parameter estimation.

Similar data sets have been used for the proposed QML-QAKF.

Joint Estimation of All Three SVM Parameters

The following table 1 presents the estimation performance of the QML-KF technique with synthetic data as stated above. The range being considered for the parameters α, δ and σ_v are [-0.999, 0.999], [-0.999, 0.999] and [0.001, 0.999] respectively while likelihood evaluation.

Table 1: Performance of QML-KF on joint estimation of all three SVM parameters

	α (s.e.*)	δ (s.e.*)	σ_υ (s.e.*)
Truth	-0.713	0.9	0.363
T=500	-0.6328 (0.1609)	0.9073 (0.0230)	0.3450 (0.1077)
T=1000	-0.5800 (0.1643)	0.9160 (0.0219)	0.3148 (0.0452)
T=1500	-0.6200 (0.1129)	0.9100 (0.0168)	0.3584 (0.0428)

*s.e.: standard error, T: Number of epochs, No. of MC run being 500.

A sample performance is shown in table 1. From the above table it may be observed that estimation performance remains almost same with the increase of time scale above 500 epochs, especially for δ and σ_v estimates. However, parameter α estimates are noticed to be less satisfactory compared to parameters δ and σ_v estimates in view of the deviation from truth and also the magnitude of standard error of the estimates over MC run.

It should be pointed out that after solving a large number of cases with various combinations of parameters while jointly estimating all three parameters it was found that such small estimation errors as shown in the table above is not possible for a large number of combinations of truth values of the parameters. By a systematic study it was observed that to have acceptable estimation errors:

(i) α should be between -0.5 to 0.5,
(ii) δ should be greater than 0.7 and
(iii) σ_v should be between 0.3 to 0.4.

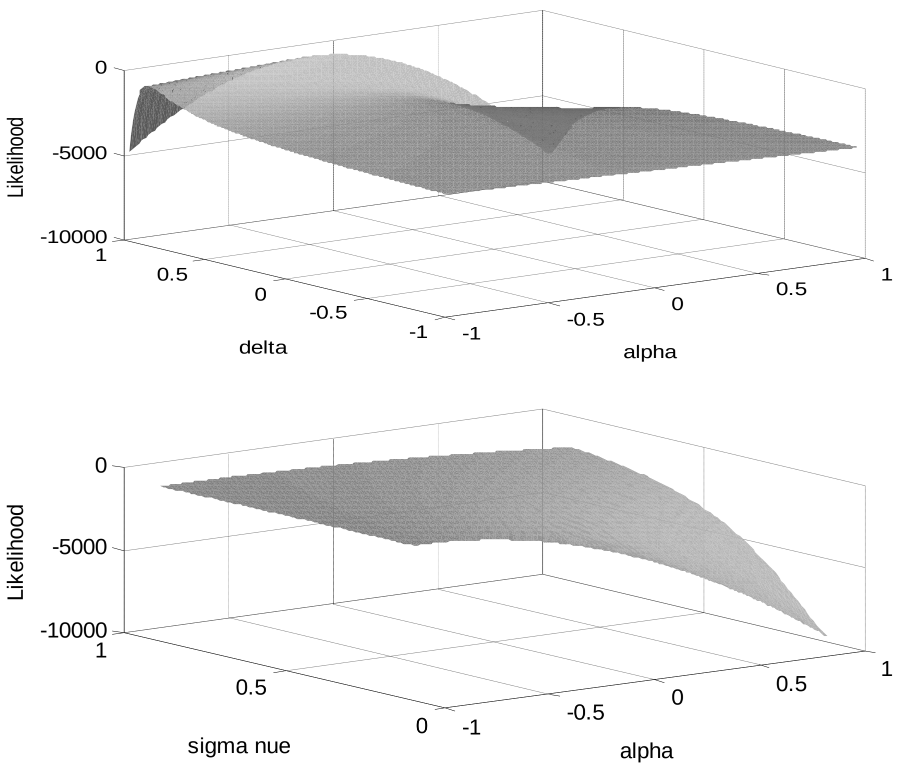
The above findings led to the following two types of queries:

(i) The reason for large estimation error outside the regions specified above and
(ii) Impact of the estimation errors on determining quantities such as VaR.

The first query would be addressed indirectly in a subsequent section. The second query would is left as future work with market data.

After a systematic analysis of the QML-KF method with synthetic data in this work, it is revealed that flat likelihood on certain domain of the parameter is the main reason of the estimation inaccuracies while jointly estimating the SVM parameters.

The following likelihood plots show the flat likelihood surface which is the underlying reason behind the poor or less satisfactory performance of the QML-KF estimator in joint estimation of all three SVM parameters. Figure 1 presents three likelihood surface plots taking two SVM parameters from the considered three SVM parameters. Considered domains of the three parameters for these likelihood plots are [-0.99, 0.99], [-0.99, 0.99] and [0, 0.9] for α, δ and σ_v respectively. The domains of the all three parameters are divided at discrete points with increments 0.01 to get the estimates accurate up to two decimal place starting at -0.99. (The third or missing values are taken from the truth value set α, δ and σ_v being -0.713, 0.9 and 0.363 respectively, as appropriate).



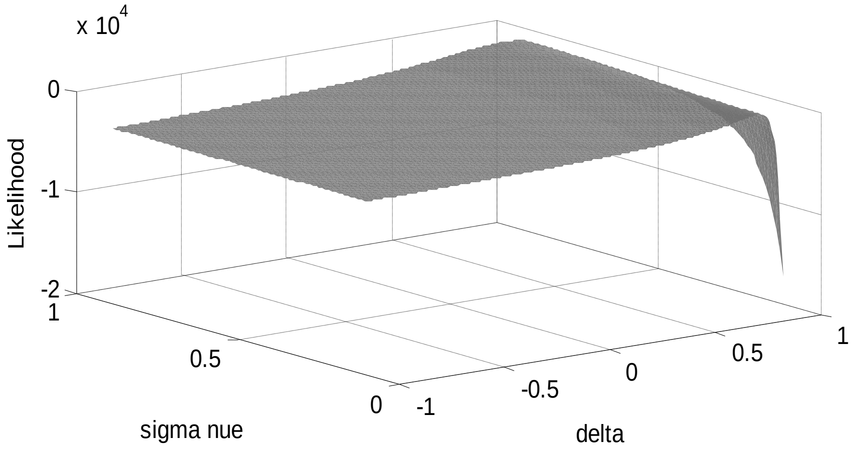


Fig. 1 Likelihood plots while all three parameters are unknown

Large, nearly flat surfaces could be seen in the likelihood surface plots, where due to small numerical error there could be large deviation in the maximum likelihood parameter estimate.

Single Parameter Estimation (while Other Two are Known)

While noting the complexity and somewhat unsatisfactory performance of estimating the three parameters simultaneously, it is interesting to find that the performance of QML-KF is very high when estimating one single parameter of these three parameters assuming other two parameters as known. The following section presents the estimation performance of QML-KF while two of these three parameters are known.

The following table 2 illustrates the performance of the QML-KF for estimation of a single SVM parameter assuming the other two as known. The second row of the table gives the true values of the parameters where as third, fourth and fifth row present the estimates of those parameters with corresponding standard errors of estimation by 1000 Monte Carlo run in brackets where the considered time duration of the estimation consists of 500, 1000 and 1500 epochs. Indicated (in second row) true values of the other two parameters are assumed to be known while estimating a single parameter.

From the table 2, it is observed that the performance of QML-KF is satisfactory for all three individual SVM parameters estimation. δ estimation performance is much more satisfactory than other two as evident from the s.e. results.

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Table 2: Performance of QML-KF on individual estimation of the SVM parameters

	α (s.e.*)	δ (s.e.*)	σ_v (s.e.*)
Truth	-0.713	0.9	0.363
T=500	-0.7110 (0.0187)	0.9009 (0.0025)	0.3546 (0.0747)
T=1000	-0.7123 (0.0133)	0.9009 (0.0018)	0.3589 (0.0484)
T=1500	-0.7092 (0.0107)	0.9010 (0.0015)	0.3603 (0.0393)

*s.e.: standard error, T: Number of epochs, No. of MC run=1000.

Joint α and δ Estimation while σ_v is Known

This section presents the estimation performance of QML-KF when parameter σ_v is assumed to be known and joint estimation of α and δ is of concern. The following table 3 presents the performance. The second row of the table gives the true values of the parameters where as third, fourth and fifth row presents the estimates of those parameters with corresponding standard errors of estimation by 1000 Monte Carlo run in brackets where the considered time duration of the estimation consists of 500, 1000 and 1500 epochs. The known value of σ_v being 0.363 while jointly estimating α and δ using QML-KF.

Table 3: Performance of QML-KF on joint estimation of the α and δ

	α (s.e.*)	δ (s.e.*)
Truth	-0.713	0.9
T=500	-0.7540 (0.1575)	0.9030 (0.0213)
T=1000	-0.7555 (0.1384)	0.9025 (0.0194)
T=1500	-0.7359 (0.1290)	0.9054 (0.0180)

*s.e.: standard error, T: Number of epochs, No. of MC run=1000.

From the above table 3, it is observed that estimation performance is comparable and similar to that has been reported in table 2. Similar performance has been noted when true value of α and δ are within the bounds specified earlier. However performance degrades elsewhere in the parameter space. The following table 4 presents the joint estimation performance of α and δ at some arbitrary true values spanned on their domain where known value of σ_v is 0.363 with time duration consisting of 500 epochs.

From the table 4, it is observed that the joint estimation performance QML-KF is highly unsatisfactory compared to its previous cases.

To identify the reasons behind this unsatisfactory performance numerical simulation and surface plotting experiments were carried out (as in the previous section). It was confirmed that flat likelihood surfaces are the primary reason for such performance.

Table 4: Performance of QML-KF on joint estimation of the α and δ

True		Estimates	
α	δ	$\hat{\alpha}$ (s.e.)	$\hat{\delta}$ (s.e.)
-0.2	0.9	-0.2415 (0.1078)	0.8861 (0.0504)
-0.713	0.5	-0.7499 (0.2459)	0.4738 (0.1769)
-0.2	0.5	-0.2563 (0.1516)	0.3416 (0.3323)
0.2	0.1	0.1859 (0.1281)	0.2488 (0.3482)
0.5	-0.3	0.3258 (0.1503)	0.1890 (0.3279)
0.8	-0.9	0.4129 (0.1587)	0.0200 (0.2633)

*s.e.: standard error, No. of MC run = 1000.

It has been observed that QML-KF is efficient in estimating a single parameter out of three SVM parameters assuming other two are known. QML-KF is also found quite satisfactory while estimating all three or any two SVM parameters jointly while true combination of α, δ and σ_v are within a "specified region" in parameter space. Problem may arise when two or all three SVM parameters are not known and estimated by QML where the values of the parameters are outside the "specified region". The computational efforts necessary for the joint estimation of the SVM parameters is also noticed to be very high. It is also observed that standard error of estimation reduces as time scale is enhanced. Performance of QML-KF is noticed to be not satisfactory in several situations. Investigations revealed that joint likelihood surface contains extensive flat regions. In such flat regions small numerical errors or changes in one parameter can cause large change in the ML value of other parameters.

III. Proposed SVM Parameter Estimation Technique QML-QAKF

SVM parameter σ_v is the determinant of process noise uncertainties which can be time varying. If such time evolving assumption is valid (and seems appealing here) then this parameter can be estimated by adaptive techniques. An algorithm called QML-QAKF is proposed in this work for such adaptively estimation of time varying σ_v.

While exploring the QML-KF method with synthetic data, the difficulties associated with simultaneous estimation of all the three parameters were noted. The AKF method characterized and presented in [II], allows continuous adaptation of the process noise (

σ_v in the SVM-QML perspective). Implementing AKF for estimation of σ_v in the SVM-QML indicates that one less parameter would have to be estimated by the ML method. On the other hand, the simultaneous adaptation of σ_v does not ensure that its value would remain constant over time, which is the implicit assumption in SVM. In general, one has to admit the concept of an evolving σ_v in the SVM, which is, in effect an extension of the SVM concept.

A novel family of algorithms, called the QAKF method is proposed here by embedding the AKF in the QML framework. QAKF is applicable here since observation noise is constant and known in this case (hence the nomenclature). The QML-QAKF algorithm is described next. In this method, σ_v would be determined by QAKF and only α and δ need be iterated.

Algorithm 2: QML-QAKF for SVM Parameter Estimation

Step 1: Initialization Step

Set t=0. Select suitable guess values of α , δ and σ_{vt} . Initialise the filter state by $h_{0|0} = \dfrac{\alpha}{1-\delta}$, state error covariance $P_{0|0} = \dfrac{\sigma^2{}_{vt}}{1-\delta^2}$ and m.

Step 2: KF Prediction Step

$h_{t|t-1} = \alpha + \delta\ h_{t-1|t-1}$ and

$P_{t|t-1} = \delta^2 P_{t-1|t-1} + \sigma_{vt}^2$.

Step 3: KF Update Step

$$h_{t/t} = h_{t/t-1} + \frac{P_{t|t-1}}{\Omega_t}\left[\log(r_t^2) + 1.27 - h_{t|t-1}\right] \text{ and}$$

$$P_{t|t} = P_{t|t-1}\left[1 - \frac{P_{t|t-1}}{\Omega_t}\right] \text{ where } \Omega_t = P_{t|t-1} + \frac{\pi^2}{2} .$$

Step 4: Check the value of t.

Step 5: If t> m then execute step 7 otherwise repeat step 2 to 4 with $\sigma_{vt}^2 = \sigma_{vt-1}^2$.

Step 6: Update state noise covariance: $\sigma_{vt}^2 = \sigma_{vt-1}^2 \sqrt{\lambda}$ where $\lambda = \dfrac{\frac{1}{m}\sum_{i=0}^{m-1} v_{t-i} v'_{t-i} - \frac{\pi^2}{2}}{P_{t|t-1}}$.

Step 7: Repeat steps 2 to 6 for all epochs.

Step 8: Repeat Step 1 to 7 to calculate log likelihood $L(\alpha, \delta)$ for a suitable range (covering the tentative range) of values with appropriate increments. Likelihood calculation formula is given by $L(\alpha, \delta) \propto -\frac{1}{2}\sum \log(\Omega_t) - \frac{1}{2}\sum \frac{v_t^2}{\Omega_t}$ where $v_t = \log(r_t^2) + 1.27 - h_{t|t-1}$ is the one step ahead prediction error.

Step 9: Determine α^* and δ^* for which $L(\alpha^*, \delta^*)$ is maximum and these α^* and δ^* are the ML estimates of three parameters under consideration respectively.

IV. Experimental Results

Experiments with QML-QAKF using Synthetic Data

QAKF characterized in [XX] is applied with QML framework by QML-QAKF algorithm on the state-space model given by equations (3) and (4). Here the objective is to estimate the process noise covariance identified by σ_v^2 assuming it as time varying and unknown since QAKF has the potential of estimating such time varying process noise covariance parameter. It has been observed that the same problem of singularity occurred as in case of [XX], i.e. estimated state noise covariance became negative. As a consequence MQAKF has become the next choice and it has been used in place of QAKF. The only change required in the algorithm 1 is to set the λ calculation formulae in step 6 as $\lambda = \left| \frac{\frac{1}{m}\sum_{i=0}^{m-1} e_{t-i} e_{t-i}^{'} - \frac{\pi^2}{2}}{P_{t|t-1}} \right|$.

Results of SVM Parameter Estimation using QML-MQAKF

The following table 5 presents the performance of QML-MQAKF for individual estimates of α and δ along with the standard error of estimation of those parameters while σ_v is assumed to be time varying starting at 0.363 with synthetic data. While estimating either α or δ, the other parameter is assumed to be known as stated in the truth. The chosen value of m is 100 for the following experimental results with QML-MQAKF.

Table 5: Performance of QML-MQAKF on individual estimation of the SVM parameters

	α (s.e.*)	δ (s.e.*)
Truth	-0.713	0.9
T=500	-0.6993 (0.0259)	0.9097 (0.0037)
T=1000	-0.7134 (0.0175)	0.9107 (0.0017)
T=1500	-0.7122 (0.0148)	0.9109 (0.0010)

*s.e.: standard error, T: Number of epochs, No. of MC run= 1000.

From above table 5, it is observed that the performance of QML-MQAKF is comparable to that of QML-KF for individual and joint estimation of α and δ as presented in table 1, 2 and 3 while the values of α and δ are near by -0.713 and 0.9 respectively though the parameter σ_v is treated as time varying and estimated by MQAKF. Estimation accuracies increase with the increase of time duration similar to the above cases as evident from the standard errors.

Joint α and δ estimation while σ_v is Unknown

The following table 6 presents the performance of QML-MQAKF for joint estimation of α and δ where true values of α and δ are -0.713 and 0.9 respectively together with the standard error of estimation using 1000 MC runs. The true value and starting value of σ_v is 0.363 for this reporting. The estimation is carried out for three time duration consisting of 500, 1000 and 1500 epochs.

Table 6: Performance of QML-MQAKF on joint estimation of the α and δ

	α (s.e.*)	δ (s.e.*)
Truth	-0.713	0.9
T=500	-0.955 (0.0173)	0.875 (0.0058)
T=1000	-0.953 (0.0236)	0.884 (0.0068)
T=1500	-0.960 (0.0346)	0.873 (0.0058)

*s.e.: standard error, T: Number of epochs=500, No. of MC run= 1000.

Performance of QML-MQAKF is noted above in table 6 which shows the performance is not as satisfactory as it was observed in case of either of α and δ assumed to be known as it is presented in table 5 as well as in QML-KF cases. In particular, the estimate of alpha showed considerable offset with respect to the truth.

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The investigations were then repeated for joint estimation of those parameters over wider range. The performances of QML-MQAKF on these investigations are presented in table 7 below. The true value as well as starting value of σ_v while estimating the other two parameters is 0.363.

Table 7: Performance of QML-MQAKF on joint estimation of α and δ in wider range

True		Estimates	
α	δ	$\hat{\alpha}$ (s.e.)	$\hat{\delta}$ (s.e.)
-0.2	0.9	-0.2340 (0.0989)	0.8960 (0.0313)
-0.7	0.5	-0.9260 (0.1207)	0.3260 (0.1092)
-0.2	0.5	-0.3200 (0.0424)	0.0850 (0.1392)
0.2	0.1	0.1650 (0.1038)	0.3950 (0.3841)
0.5	-0.3	0.3875 (0.1723)	-0.0150 (0.4011)
0.8	-0.9	0.7900 (0.1364)	-0.8025 (0.0785)

*s.e.: standard error, T: Number of epochs=500, No. of MC run= 1000.

From the above table 7, it is observed that the necessary condition to satisfactory performance with QML-MQAKF is $|\delta| >= 0.9$.

Volatility Estimation by QML-KF and QML-MQAKF

Present section concentrate on the checking and comparing the performance of QML-KF and QML-MQAKF on estimation of volatility using the synthetic dataset generated for the time duration consisting of 500 epochs using the model given by equation (3) and (4). Volatility is determined by the formulae $\sigma_t = \exp\left(\frac{h_t}{2}\right)$ from the estimated state variable given by h_t. The following figure 2 presents and qualitatively compares the volatility estimation performance of QML-KF and QML-MQAKF with its true value. The true value of α, δ and σ_v are -0.713, 0.9 and 0.363 respectively for the estimation presented below. The starting value of σ_v is 0.363 while estimating.

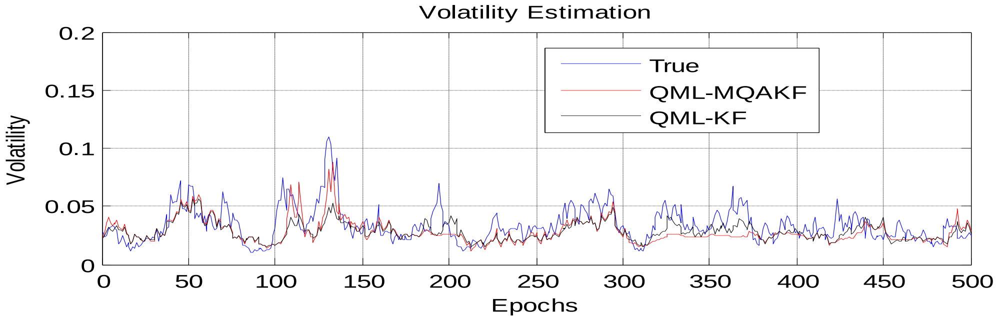

Fig. 2 Comparison of estimated volatilities using QML-KF and QML-MQAKF with synthetic data where true values of α and δ are -0713 and 0.9 respectively

From the above figure 2, and the RMSE values over time of Monte Carlo runs, it is observed that performance of QML-MQAKF is better than QML-KF though σ_v is assumed to be unknown in case of QML-MQAKF with this set of parameters.

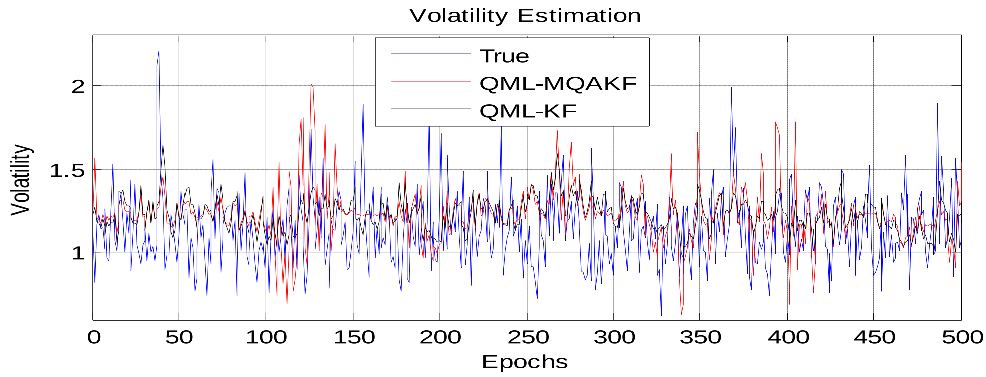

Fig. 3 Comparison of estimated volatilities using QML-KF and QML-MQAKF with synthetic data where true values of α and δ are 0.2 and 0.1 respectively

On the other hand from the figure 3 and RMSE over time of Monte Carlo runs, it is observed that performance of QML-MQAKF is less satisfactory compared to QML-KF when the values of the SVM parameters are different from the previous set.

V. Conclusion

Experiments with synthetic data revealed that QML-QAKF failed with the standard log normal SVM due to negativity occurrence and hence QML-QAKF is modified to QML-MQAKF and applied for the purpose. In specified regions in parameter space, the estimation performance of QML-MQAKF is found to be comparable to that of standard QML-KF with known σ_v even though the parameter σ_v is assumed to be unknown and evolving with respect to time in case of estimation by MQAKF method. The performance of QML-MQAKF is significantly good when the parameters σ_v and any one other parameter are to be estimated. It may be noted that σ_v is obtained from the AKF and only one parameter is determined by MLE. In this "single parameter MLE" case QML-MQAKF provides good estimate over wider regions. It may be conjectured that if the parameter α is estimated by any auxiliary method and only δ is to be estimated by QML-MQAKF method, good and convenient estimation would result. This is left for future work. In QML-MQAKF, the parameter σ_v is adaptively estimated and varies with time, even though the truth model assumes a constant value.

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