

Elastoplastic Deformation of Clay Brick Masonry under Biaxial Stresses and Mechanisms of its Performance

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Abstract

Plasticity properties of the masonry appear to be key requirements in order to predict seismic response from masonry construction. On the grounds of the experimental research results, the system of mechanisms of local damage in masonry elements (brick, mortar and contact nodes thereof) is formulated and justified. It is revealed that the state of interaction nodes of basic masonry materials under increasing stress is not irreversible: when the stress state of the node changes, the discrete transition from one state to another becomes possible. The proposed system of mechanisms of local damage and the tools to analyze the state of masonry elements serves grounds for elaboration of a structural model of clay brick masonry as piece-wise homogeneous multimodule composite environment. Based on the results of numerical studies of behavior of the clay brick masonry structural model with the proposed destruction mechanisms as well as on the grounds of the strength criteria system, an accurate prediction of the elastic and plastic deformation phases can be made to determine the plasticity characteristics of the masonry under biaxial stresses.

Keywords : Brick, Masonry Joint, Destruction Mechanisms, Strain-Stress State, Elastoplastic Deformation

I. Introduction

Clay brick masonry has been employed as a structural material for many centuries; many outstanding ancient monuments built of this material have survived up to the present with minimal damage. The core components of masonry are brick/stone (materials of a distinct geometric structure) and brickwork mortar (featuring good plasticity when being applied) allow creating a composite structure which, in fact, composes the structural material. At the initial stages of its implementation the mechanical properties of the clay brick masonry in general and core components in particular were not studied; still the requirements were introduced in order to observe

the masonry construction ratios which would ensure stability of the buildings being erected (The Ten Books on Architecture by Vitruvius).

Within a long-practiced use of masonry construction there were introduced and justified methods to assess rigidity and bearing capacity thereof. For the most concentrated stressing conditions (axial and eccentric compression) the methods were proposed to predict deformations. However, as for biaxial stress state condition, the strain forecast method to comprise the plastic deformation phase has not been developed to date. The lack of a forecast technique to predict the elastoplastic deformation stems from a very complex processes that generate plastic deformation in masonry construction. Within the framework of the solid deformable body theory, these processes represent mechanisms of local damage formation and accumulation which take place in clay brick masonry under increasing stressing.

The article focuses on studies of mechanisms of formation of local damage in masonry construction under biaxial stress state.

II. Analysis of the Stated Issue

One of the first comprehensive analysis of clay brick masonry as a homogeneous material was carried out by Onishchik (1939). Based on the research, destruction patterns for masonry construction were determined and empirical equation for the stress-strain state relation was introduced to illustrate the case of uniaxial stress state of clay brick masonry as a continuous medium:

$$\varepsilon = -\frac{R'}{E_0} \ln\left(1 - \frac{\sigma}{R'}\right) = -\frac{\mu R_{ch}}{E_0} \ln\left(1 - \frac{\sigma}{\mu R_{ch}}\right) \quad (1)$$

where R' – relative tensile rigidity of material at which tangential elastic modulus equals to zero;

R_{ch} – ultimate rigidity of clay brick masonry at uniaxial compression perpendicular to horizontal joints;

E_0 – initial elasticity modulus of clay brick masonry.

Equation (1) is still being used nowadays with certain refinement as required.

For the purposes of accounting for physically non-linear behavior of clay brick masonry under centric and eccentric compression, a range of models were introduced to provide evidence of the ‘generalized’ mechanical properties of masonry with due regard to behavioral specifics of basic materials. For instance, Kashevarova (2002, 2010) presents a model of how clay brick masonry performs under stress as a physically non-linear material. The model suggests an approach to describe mechanical behavior of clay brick masonry as a continuous material under complex stress state with provision for structural damage (types of damage under analysis are splitting and crumbling) and strain hardening. It is assumed that the material is initially linearly elastic and orthotropic (isotropic) and once damaged, it also remains orthotropic.

Gorshkov et al. (2014) has substantiated the impact of variable properties of basic materials on the ‘generalized’ (effective) properties of clay brick masonry being a

homogeneous continuum, which also allows for taking into consideration the most important aspects of physically non-linear performance of clay brick masonry centric and eccentric compression.

In some cases (seismic effects, irregular base deformations, damage of certain structural elements of the load-bearing system, etc.) biaxial stress state is formed in masonry construction; its occurrence causes deformation and destruction of masonry construction and is characterized by a number of fundamental distinctions from the case of centric and eccentric compression.

A generalized approach to the evaluation of masonry construction under biaxial stress state is proposed and substantiated by Geniev (1979) and Tupin (1980). Masonry is regarded '... as an orthotropic material characterized by certain quantitative indices of rigidity in two directions: across and along horizontal joints'. The papers describe three mechanisms of clay brick masonry destruction under biaxial stresses:

1. Destruction from fragmentation which takes place under both uniaxial and biaxial non-linear or uniform compression.
2. Destruction from detachment which takes place under both uniaxial and biaxial non-linear or uniform compression.
3. Shear fracture which appears under mixed biaxial stress states of compression and tension.

Strength criteria by Geniev (1979) were developed with the help of a set of independent rigidity properties of clay brick masonry as a homogeneous continuous material, i.e.: strength limits of masonry to withstand uniaxial compression/tension perpendicular to horizontal (unbanded) and vertical (banded) joints; ultimate strength under biaxial uniform compression; strength of masonry against shear along unbanded (horizontal) and banded (vertical) sections. It should be noted that masonry is analyzed within the continuous model limits, i.e. interaction aspects of basic materials are beyond consideration.

Models of physically non-linear clay brick masonry performance are proposed by many foreign researchers, for example, in papers by Fattal and Jokel (1976), Papa (1996), Page (1979), Gabor et al. (2006), Lourenco (1996), etc. These studies also review conditions of in-plane stress state of masonry as well as non-linear type of deformation being determined both by physical non-linearity of mortar (fine grain concrete) and progressive disintegration of masonry joints followed by redistribution of forces. In the most in-depth studies, for example, by Page (1979), Lourenco (1996), masonry is regarded as medium consisting of dissimilar materials such as brick (stone) and mortar and which is in line with the concept of a composite piecewise homogeneous material. Nonetheless, conditions of contact interaction of basic materials are not considered in those models.

Suggestions on how to account for interaction of basic masonry materials in masonry joints are formulated by Lemos (2006). However, mechanical properties of the contact elements are defined on the grounds of mechanical properties of basic

masonry materials and that (as will be shown below) is not in compliance with the actual operation conditions of the contact node.

The model of contact interaction between brick and mortar is proposed by Sousa et al. (2013). Nonetheless, the contact interface of mortar and bricks is modeled pursuant to the pattern of absolute contact of materials. Friction models in the contact zone (e.g., Mohr-Coulomb theory and alike) are not taken into account on the author's assumption in order to introduce some simplification into numerical studies.

The models of clay brick masonry as proposed help elaborate quite an accurate prediction of load-bearing capacity of masonry construction both for axial (eccentric) loading and in-plane stress conditions. However, the strain prediction for masonry construction under biaxial stress state including elastic and plastic phases cannot be elaborated on the grounds of the presented models.

It should be highlighted that specific mode of operation of masonry construction, for instance, to respond to seismic effects, requires crucial parameters of operation of masonry construction being the parameters of strain including, quintessentially, plastic phase. Under conditions of seismic effects the regulation in force (EN 1998-1. Eurocode 8: Design of structures for earthquake resistance) permits certain degree of plastic strain for load-bearing structures and which, indeed, reduces seismic response of the load-bearing system and, as a result, reduces value of the estimated seismic loading.

Significance of obtaining parameters of elastoplastic deformation of masonry construction under biaxial stresses is emphasized by contemporary researchers: Förster (2017), Butenweg et al. (2016). Therein it is stated that theoretical grounds for elastoplastic parameters of masonry is a very complex task to solve and this leads to the necessity to recourse to simplified techniques.

In principle, the plastic phase of deformation of any structural material and its characteristics are determined by the degree of local damage accumulated within the structure when being exposed to stresses. Therefore, in order to theoretically substantiate the elastoplastic response of masonry, it is needed to have a model able to correctly respond not only to mechanical properties of basic materials, but also to processes of formation and accumulation of local damage in masonry construction under increasing stresses generating biaxial stress state.

For the purposes of studying elastoplastic characteristics of masonry under conditions of biaxial stress state it is proposed to use the model of a piece-wise homogeneous multimodule composite with expansion by way of interaction of basic materials at the contact points. Needless to say, interaction conditions can change during formation and accumulation of local damage at the contact nodes of basic materials given modification in the stresses borne by the structure and which need to be accounted for in the model simulating actual mechanisms of local and major fractures.

Studies conducted by Polyakov (1959), Polyakov and Safargaliev (1991) proved that mechanical properties of basic materials of clay brick masonry (brick and mortar) have the slightest (if any) impact on both the value of load-bearing capacity of masonry construction under biaxial stress state and the level of realization of plastic

phase of deformation. The key factor is determined by Polyakov (1959), Polyakov and Safargaliev (1991) and it defines the level of load-bearing capacity of masonry construction under biaxial strain state, i.e. the value of tensile strength of the brick-mortar contact node in horizontal masonry joint. Studies by S. Polyakov, S. Safargaliev prove this factor independent of mechanical properties of basic materials and defined by the implied moisture conservation in the mortar of horizontal masonry joint. Research carried out by Tonkikh et al. (2012) provides rationalization that tensile strength of the brick-mortar contact node in horizontal masonry joint can be determined and is equal to adhesive rigidity of contact interaction of brick and mortar (R_{adh}). It is important to point out that field surveys of the structures in situ determine that the level of adhesive rigidity of contact interaction of brick and mortar in existing masonry structures of buildings is within the range as follows: $0,02 \text{ MPa} \leq R_{adh} \leq 0,1 \text{ MPa}$. Given special additives are used to boost adhesion, it is likely to reach the value of $R_{adh} = 0,6 \text{ MPa}$ and above.

Our Research and Discussion

As indicated earlier, the process of plastic deformation of structural material represents the process of degradation (i.e. formation of local damage/destruction) of this material under increasing stresses. Owing to the fact that clay brick masonry is composite material with expressive piece-wise homogeneous structure, local damage can be formed both in masonry basic materials and in nodes of contact (interaction) thereof.

Thus, in order to proceed with detailed analysis of elastoplastic deformation process and destruction of clay brick masonry triggered by biaxial stress state it is mandatory to employ the set of experimentally proven mechanisms of masonry destruction and rigidity mechanisms corresponding to those of clay brick masonry destruction.

Research on mechanisms of formation and accumulation of local damage resulting in destruction of clay brick masonry under biaxial stress-strain state is carried out within the framework of the paper by Kabantsev (2013), Kabantsev and Tamrazyan (2014). All kinds of local damage and major destruction can be shown by the following grouping (ref. Fig. 1-4).

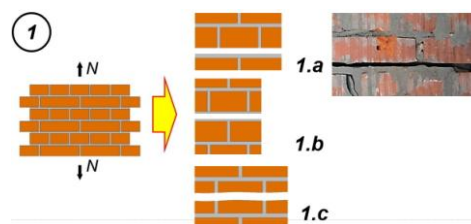


Fig. 1. Clay brick masonry destruction pattern under tensile strains acting perpendicular to horizontal masonry joint

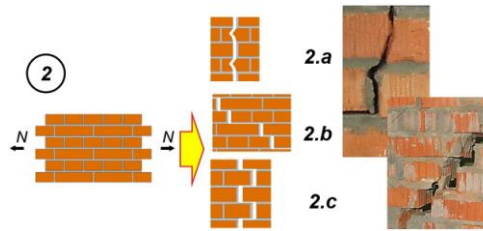


Fig. 2. Clay brick masonry destruction pattern under tensile strains acting parallel to horizontal masonry joint

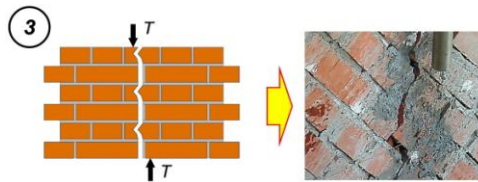


Fig. 3. Clay brick masonry destruction pattern under shear loads (acting perpendicular to horizontal masonry joint)

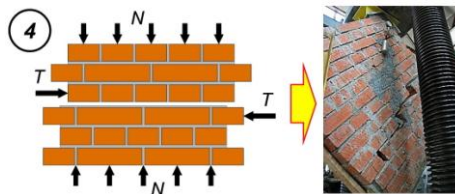


Fig. 4. Clay brick masonry destruction pattern under shear loads in-plane parallel to horizontal joint

Analysis of experimental research on mechanisms of local damage and major destruction enables formulating the conclusions as below:

Prevalent kind of destruction is the destruction along the brick-mortar contact node. Types of destruction 1.b and 1.c when fracture is formed on along basic material (brick or mortar) are not recorded in the course of research; in studies carried out by other authors such fractures are mentioned as of single occurrence. Prevalent kind of destruction (of type 1.a) is characterized by significantly lower level of adhesive interaction rigidity (R_{adh}) in relation to the strength values of basic materials.

The outcome of the completed studies proves that the brick and mortar interaction in vertical masonry joints is not formed in general. This is determined by the processes of mortar shrinkage in the joint during hardening and strength generation. Thus, the adhesion mechanism in a vertical masonry joint should not be accounted for.

In order to consider the process of destruction of basic masonry materials (brick and mortar) it is mandatory to employ relevant strength criteria.

For the purposes of assessment of stress state in basic materials of clay brick mortar it is feasible to adopt the strength criteria for ceramics and concrete as elaborated by Küpfer (1973) and which are widely and successfully used by other

researches, e.g., Sousa et al. (2013). Biaxial stress state requires the K pfer strength criterion as defined by the equation below:

$$1 + \frac{\sigma_1}{R_c} - \frac{\sigma_3}{R_t} \geq 0 \quad (2)$$

where R_c and R_t are rigidity limits of the material (brick, mortar) under uniaxial compression and tension respectively.

Analysis of the basic materials contact nodes stress state needs to be carried out with regard to experimentally defined patterns of formation of local damage and major destruction. In addition, it is advised to consider possible (and fit for implementation) mechanism of change in the state of contact node when generating one or another type of local damage.

Interaction between brick and mortar in vertical masonry joints is determined by absence of adhesion between these materials. Fig. 5 illustrates possible scenarios for the state of the node, where F is a generalized strength parameter of interaction.

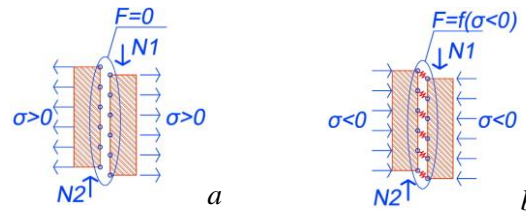


Fig. 5. Vertical masonry joint patterns for different states

Under biaxial stress state the stresses as listed below are formed:

- σ – tensile/compressive stresses, normal planes of the joint;
- T – shear stresses, parallel planes of the joint (stresses are determined by shear forces N_i).

A. Given tensile stresses normal to joint axis ($\sigma > 0$), interaction between mortar and brick is null owing to absence of adhesion (Fig. 5,a).

B1. Given compressive stresses normal to joint axis ($\sigma < 0$), performance of the joint is determined by shear stress value. At shear stresses being $T = N_1 - N_2 \leq [T]$ and not exceeding restraining stresses $[T] = K/\sigma$, (defined by significant compressive stress ($\sigma < 0$) and static friction coefficient K), vertical joint mortar interacts with adjacent bricks in the static friction mode both in accordance with stresses associated with normal joint axis and in accordance with shear stresses (Fig. 5,b).

B2. With shear stresses being $T = N_1 - N_2 > [T]$ and exceeding restraining stresses $[T] = K/\sigma$, the interaction of vertical joint mortar with bricks goes along the joint plane in dry friction mode by force $F = k/\sigma$ (k is sliding friction coefficient) in contrast with the direction perpendicular to the joint plane given the stresses are normal to joint axis.

The following equations summarize the generalized operation conditions of vertical masonry joint:

- for scenario **A**:

$$\sigma > 0 \rightarrow F = 0 \quad (3)$$

- for scenario **B1**:

$$\begin{aligned} &\sigma < 0 \\ &T = N_1 - N_2 < K |\sigma| \rightarrow F = K |\sigma| \end{aligned} \quad (4)$$

- for scenario **B2**:

$$\begin{aligned} &\sigma < 0 \\ &T = N_1 - N_2 > K |\sigma| \rightarrow F = k |\sigma| \end{aligned} \quad (5)$$

The state of brick and mortar interaction node in vertical joint is not irreversible: at some point of stressing tensile stresses ($\sigma > 0$) are generated in masonry joint and this leads to disintegration of basic materials (brick and mortar), i. e. masonry joint 'opens up'. The joint will remain open till intensity of stress imposed onto masonry construction remains sufficient enough for the vertical joint to produce compressive stresses ($\sigma < 0$). A similar process may occur in the vertical joint at the shear mode under compressive stresses: under active shear stresses exceeding restraining stresses a shear takes place in contact node; shear may cease with increasing restraining stresses.

In overall, it is possible to record changes in the node both following the pattern of 'state **A**' – 'state **B1**' – 'state **B2**' as well as any other sequence of states pattern, for instance, 'state **B1**' – 'state **B2**' – 'state **B1**'.

Transition of the brick-mortar interaction node in vertical joint from state **B1** to **B2** occurs discretely (in discrete steps) and requires discrete changes of rigidity parameter only in terms of shear without any changes of rigidity parameter in terms of compression, and this should be taken into account while creating a model of clay brick masonry corresponding to the concept of piece-wise homogeneous multimode composite.

Interaction between brick and mortar in horizontal masonry joint is determined by a complex stress state of the node. Still the key part of this node performance is governed by the presence of adhesive interaction between brick and mortar with the rigidity value R_{adh} .

Given biaxial stress state of masonry construction, the stresses as listed below are generated in masonry construction:

- σ – tensile/compressive stresses, normal planes of joint;
- T – shear stresses, parallel planes of the joint (stresses are determined by shear forces N_i).

Horizontal masonry joint in masonry construction under biaxial stress state may be described by the states as follows:

C. In presence of *tensile* stresses in horizontal masonry joint normal to the joint axis ($\sigma > 0$), the following scenarios are possible (Fig. 6.):

C1) $0 > |\sigma| > R_{adh}$: in this scenario adhesion is destroyed which results in departure from normal interaction between brick and mortar of horizontal joint in view of all types of stress (Fig. 6.a).

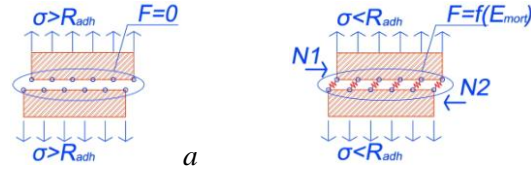


Fig. 6. Horizontal masonry joint state patterns under tensile stresses ($\sigma > 0$)

C2) $0 > |\sigma| \leq R_{adh}$: in this scenario, under increasing shear stresses not exceeding the value of restraining stresses $T=f(N_i) \leq [T]=f(R_{adh})$, interaction between brick and mortar of horizontal joint is achieved as formonolithic formation in view of types of stresses normal to joint axis and with regard to shear stresses (Fig. 6.b).

C3) $0 > |\sigma| > R_{adh}$: given the value of shear stress exceeding the value of restraining stresses $T=f(N_i) > [T]=f(R_{adh})$, adhesion is destroyed which results in departure from normal interaction between brick and mortar of horizontal joint in view of all types of stress (Fig. 6.a).

D. In presence of **compressive** stresses in horizontal masonry joint normal to joint axis ($\sigma < 0$), the following scenarios are possible (Fig. 7):

D1) when $|\sigma| < 0$ under shear stresses not exceeding the value of restraining stresses $T=f(N_i) \leq [T]$ (determined by adhesive strength, compressive stress value (σ) and internal friction value), interaction between brick and horizontal joint mortar is achieved as for monolithic formation in view of types of stresses normal to the joint axis and with regard to shear stresses (Fig. 7a).

D2) when $|\sigma| < 0$ under shear stresses exceeding the value of restraining stresses $T=f(N_i) > [T]$, adhesion is destroyed and leads to shear in dry friction mode along the joint plane; at that interaction between brick and mortar of horizontal joint is achieved in view of types of stresses normal to the joint axis (Fig. 7b).

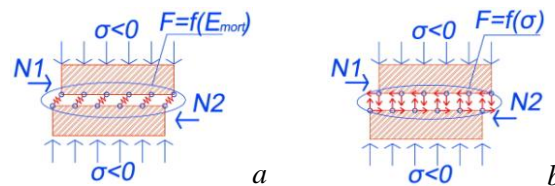


Fig. 7. Horizontal masonry joint state patterns under compressive stresses ($\sigma < 0$)

Research paper by Capozucca (2011) touches upon experimental studies which enable to define dry friction value for horizontal joint in clay brick masonry. It is found out that in masonry made of solid ceramic bricks dry friction coefficient equals to $k=0,74$ which is significantly higher than the value of 0,4 recommended by Eurocode (European Committee for Standardization, EC6, Design of masonry structures.). In paper by Polyakov and Safargaliev (1991) the value of internal friction

coefficient is experimentally determined. As revealed, for masonry made of solid ceramic bricks internal friction coefficient value should be equal to $k=1,18$.

The following equations summarize the generalized operation conditions of horizontal masonry joint:

- for scenario **C1**:

$$\sigma > R_{adh} \rightarrow F = 0 \quad (6)$$

- for scenario **C2**:

$$\sigma \leq R_{adh} \rightarrow F = f(E_{mort}) \quad (7)$$

$$T = N_1 - N_2 < K |\sigma|$$

- for scenario **C3**:

$$\sigma \leq R_{adh} \rightarrow F = 0 \quad (8)$$

$$T = N_1 - N_2 > K |\sigma|$$

- for scenario **D1**:

$$\sigma < 0 \rightarrow F = f(E_{mort}) \quad (9)$$

$$T = N_1 - N_2 < K |\sigma|$$

- for scenario **D2**:

$$\sigma < 0 \rightarrow F = k |\sigma| \quad (10)$$

$$T = N_1 - N_2 > K |\sigma|$$

Thus, the brick and mortar interaction node in horizontal joint may be available in five different states which shall be taken into account while analyzing interaction conditions for basic materials in masonry joints under any types of stresses experienced by contact nodes.

It is compulsory to consider that the state of brick and mortar interaction node of in vertical joint is not irreversible: at some stage of loading tensile stresses ($\sigma > 0$) appear in masonry joint and this leads to disintegration of basic materials (brick and mortar), i. e. masonry joint 'opens up'. The joint will remain open till intensity of loading imposed onto masonry construction remains sufficient enough for the horizontal joint to produce compressive stresses ($\sigma < 0$). A similar process may occur in the horizontal joint at the shear mode under compressive stresses: under active shear stresses exceeding restraining stresses a shear takes place in the contact node; the shear may cease with increasing restraining stresses.

Transition of the brick-mortar interaction node in horizontal joint from state **D1** to **D2** occurs discretely (in discrete steps) and requires discrete changes of rigidity parameter only in terms of shear without any changes of rigidity parameter in terms of compression.

IV. Conclusions

The experimentally proven system of mechanisms of formation and evolution of local damage in clay brick masonry under biaxial stresses as well the criteria of rigidity corresponding to these mechanisms allow assessing of stress state for any element or node of clay brick masonry.

Nonetheless, availability of tools to perform the state analysis for any element or node serves grounds for elaboration of structural model of clay brick masonry in compliance with masonry concept as a piece-wise homogenous multimode composite. Elements of such model are grouped together as per the material characteristics or in accordance with the node type; every group of elements possesses unique state characteristics defined by rigidity properties.

Rigidity analysis of elements of masonry structural model inclusive of the basic materials nodes of interaction in masonry joints may be carried out in accordance with any system of criteria regardless of any other components of system of criteria. Based on the outcome of the state analysis for elements of masonry structural model, it is possible to observe changes of this state, i.e. contact node in horizontal joint may shift from scenario **D1** into the state of scenario **D2**, or contact node in vertical joint may shift from scenario **B2** into the state of scenario **B1**; in view of the mentioned above design analysis of masonry model can be carried on.

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