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The Generalized Kudryashov Method: a Renewed Mechanism for Performing Exact Solitary Wave Solutions of Some NLEEs

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Abstract:

The present study deals with the applicability and effectiveness of the algorithm of generalized Kudryashov method (GKM), which is one of the most workable methods to constitute the exact traveling wave solutions of non-linear evolution equations (NLEEs) in physical and mathematical science. The recent paper, we enucleated this method for each of the following Couple Boiti-Leon-Pempinelli equations system, DSSH equation and fourth-order nonlinear Ablowitz-Kaup-Newell-Segur (AKNS) water wave dynamical equation. The prominent competence of this method is to naturalize the way of solving systems of NLEEs. Moreover, we can see that when the parameters are ascribed to the particular values, obtain solitary wave solution from the exact travelling wave solution. The obtained new solutions have a wide range of inflictions in the field of physics and other areas of applied science. To perceive the physical phenomena, we have plotted coupled with some 2D and 3D graphical patterns of analytic solutions obtained in this study by using computer programming wolfram Mathematica. The worked-out solutions ascertained that the suggested method is effectual, simple and direct and can be exerted to several types of nonlinear systems of partial differential equations.

Keywords: The generalized Kudryashov method; Couple Boiti-Leon-Pempinelli equations; DSSH equation; fourth-order nonlinear AKNS equation; traveling wave solution; exact solution.

I. Introduction

At the current time the swift evolvement of the experiment in nonlinear science, it will be manifest ever improving interest of mathematicians and researchers in recent techniques to search effective methods for attaining approximate solutions to NLEEs.

The exact solutions of NLEEs are the key stone to comprehend the manifold mathematical and physical phenomena that govern the physical world today. Therefore, the study of traveling wave solutions for NLEEs is of enormous importance because of their prospective implementations in various scientific and engineering fields, especially in fluid mechanics, water wave mechanics, plasma physics, meteorology, electromagnetic theory, solid state physics, chemical kinematics, optical fibers, physics, geochemistry, and nonlinear optics. With the aid of the computer symbolic systems like Mathematica, the task of inventing new computational methods to obtain exact solutions of NLEEs is very fascinating for many scholars. For this reason, in the past several decades many effective methods have been presented, such as the exp $(-\varphi(\xi))$-expansion method [I, XXI], the variational iteration method [XXXVII], the extended F-expansion method [XXIII], the improved F-expansion method [XVIII, XXX], the modified simple equation method [XX, V], the homogeneous balance method [XV], the Hirota's bilinear method [VIII], the Khater method [XXXV], the extended trial equation method [XL, IX], the exponential rational function method [IV, XXXI], the Bernoulli sub-equation function method [XVII], the Sine-Cosine method [VII, XXII], the (G'/G)-expansion method [VI, XXXIX], the tanh function method [XXXVIII], the extended tanh function method [XII], the He's exp-function method [III] and so on (see for example [XI, XXVI, XIII, XXIV, XXXIII, XIV, XXIX,]). Lately, Kudryashov [XXXII] has introduced a simple method to discuss the exact solutions for NLEE namely truncated expansion method. Kumar et al. [X] have developed a direct truncated expansion method for a nonlinear fractional differential equation, which called the improved of Kudryashov method. In recent decades, some researchers have improved the behavior of the current nonlinear differential equation, for example, Kabir et al. [XXVIII] proposed new approach method to acquire absolutely approximate analytic solutions which called the modified Kudryashov method. More recently, some scholars as like Mahmud et al. [XVI], koparan et al. [XXVII], Demiray et al. [XXXVI], Gepreel et al. [XIX] have the GKM to accomplish the exact and traveling wave solutions for nonlinear PDEs. In this current work, we have employed and made the best use of the novel GKM to obtain the exact and traveling wave solutions for the following nonlinear system of equations.

We first consider the following Couple Boiti-Leon-Pempinelli equations system is of the form [XXV]:

$$\begin{cases} u_{ty} = (u^2 - u_x)_{xy} + 2v_{xxx} \\ v_t = v_{xx} + 2uv_x \,. \end{cases}, \tag{1}$$

The second considerable problem DSSH equation which is widely used in mathematical physics in the form is given by [XXXIV]:

$$u_{xxxxxx} - 9u_x u_{xxxxx} - 18u_{xx}u_{xxx} + 18u_x^2 u_{xx} - \frac{1}{2}u_{tt} + \frac{1}{2}u_{xxxt} = 0. \tag{2}$$

Finally, we consider the well known dynamic problem which widely used in the field of applied physics is fourth-order nonlinear AKNS water wave dynamical equation with a perturbation parameter β in the form is given by [II]:

$$4u_{xt} + u_{xxxt} + 8u_x u_{xy} + 4u_{xx}u_y - \beta u_{xx} = 0. \tag{3}$$

The structure of this paper is arranged as the following portions: In section 2 "Basic ideas of the method", we proposed a novel GKM for more generality solitary solutions. Applications of the novel GKM to Couple Boiti-Leon-Pempinelli equations (1), DSSH equation (2) and fourth-order nonlinear AKNS water wave dynamical equation (3) is presented in "Implementations of the method" which is in section 3. In section 4 "Physical interpretation and graphical illustration", we provided different shapes of graphical patterns to obtained solutions. In section 5, we presented "Comparison" and in section 6, we have decorated our "Results and discussion". Lastly, we furnished "Conclusions" is in section 7.

II. Basic ideas of the method

In this section, we analyze the novel GKM for attaining traveling wave solutions to the NLEEs. Assume that a nonlinear equation for a function v, say in two independent variables x and t is in the form

$$T(v, v_t, v_x, v_{tt}, v_{xt}, v_{xx}, \dots \dots \dots \dots) = 0. \tag{4}$$

Or in case of three independent variables x, y and t in the form

$$T\left(v, v_t, v_x, v_y, v_{tt}, v_{xt}, v_{yt}, v_{xx}, v_{yy}, \dots \dots \dots\right) = 0. \tag{5}$$

where T is a polynomial of unknown function in $v = v(x,t)$ or $v = v(x,y,t)$ and its various partial derivatives and nonlinear terms are engaged. The main steps of the GKM are as follows

Step1: we consider the following traveling wave transformation

$$v(x,t) = V(\xi) \text{ or } v(x,y,t) = V(\xi), \quad \xi = x + y \pm ct, \tag{6}$$

here c is wave velocity.

The traveling wave transformation (6) consents us decreasing Eq. (4) and (5) to an ordinary differential equation (ODE) of the form for $V = v(\xi)$ is

$$H(V, V^{'}, V^{''}, V^{'''}, \dots \dots ..) = 0, \tag{7}$$

here prime indicates the derivative with respect to ξ. For obtaining the resulted expression becomes the simplest Eq. (7) can be integrated and letting the constant of integration to be zero or another character.

Step 2: The aforesaid technique suggests the exact solution of Eq. (7) can be expressed in the following rational form:

$$V(\xi) = \frac{\sum_{i=0}^{N} a_i Q^i(\xi)}{\sum_{j=0}^{M} b_j Q^j(\xi)}, \tag{8}$$

where $a_i (i = 0,1,2, \dots .., N)$, $b_j (j = 0,1,2, \dots \dots, M)$ are constants to be determined later. Again $a_N \neq 0$, $b_M \neq 0$ and we also note that $Q(\xi)$ is the solution of the auxiliary ODE

$$Q'(\xi) = Q(\xi)(Q(\xi) - 1). \tag{9}$$

It can be obvious that Eq. (9) has a solution of the form as follows

$$Q(\xi) = \frac{1}{1+Ae^{\xi}}, \tag{10}$$

herein A being an integrating constant.

Step 3: Evaluate the values of positive integers N and M in Eq. (8) by using homogeneous balance principle. As for instance, we balance the highest order derivative with the highest order nonlinear term appears in Eq. (7).

Step 4: Substituting Eq. (8) into Eq. (7) along with Eq. (9), we acquire a polynomial $R(Q(\xi))$ of $Q(\xi)$. Setting all of the coefficients of the like powers of $R\big(Q(\xi)\big)$ to zero, we attain the algebraic system of equations. For solving this system of equations, we use Mathematica software package program and obtaining the unknown parameters a_i, b_j and c and ultimately we furnish the exact solutions of the reduced Eq. (7).

III. Implementations of the method

In this section, the GKM has been assigned to investigate the new exact traveling wave solutions of the Couple Boiti-Leon-Pempinelli equations system, DSSH equation and fourth-order nonlinear AKNS water wave dynamical equation.

III.a Couple Boiti-Leon-Pempinelli equations system

In this part, we will be applied the GKM to constitute the exact traveling wave solutions of the Couple Boiti-Leon-Pempinelli equations system (1). First, we will consider the following wave transformation

$$u(x,y,t) = U(\xi)\,,\ v(x,y,t) = V(\xi), \xi = x + y - ct, \tag{11}$$

hither c is a velocity constant to be determined later. Using the above wave transformation and integrating the first equation of the system in Eq. (1) three times with respect to ξ (letting the constant of integration to be zero) then substituting it into the second equation of the system in Eq. (1), we attain the relation as follows:

$$U'' - 2U^3 - 3cU^2 - c^2U = 0. \tag{12}$$

Taking into account the homogeneous balance principle, balancing the highest order derivative U'' with non-linear term U^3 in Eq. (12), we get:

$$N = M + 1. \tag{13}$$

Choosing $M = 1$, we attain $N = 2$, so that the exact solution of Eq. (7) constitutes as follows:

$$U(\xi) = \frac{a_0 + a_1Q + a_2Q^2}{b_0 + b_1Q}, \tag{14}$$

where a_0, a_1, a_2, b_0 and b_1 are constants to be determined later and $Q = Q(\xi)$ satisfy Eq. (9). Inserting Eq. (14) into Eq. (12) and putting off the denominator, we obtain a polynomial in $Q(\xi)$. Equating all the coefficients like powers of $Q(\xi)^k$ to zero, we obtain the following system of algebraic equations

$Q^0: 2a_0^3 + 3ca_0^2b_0 + a_0c^2b_0^2 = 0,$

$Q^1: 6a_0^2a_1 + 6ca_0a_1b_0 - a_1b_0^2 + c^2a_1b_0^2 + 3ca_0^2b_1 + a_0b_0b_1 + 2c^2a_0b_0b_1 = 0,$

$Q^2: 6a_0a_1^2 + 6a_0^2a_2 + 3ca_1^2b_0 + 6ca_0a_2b_0 + 3a_1b_0^2 - 4a_2b_0^2 + c^2a_2b_0^2 + 6ca_0a_1b_1 - 3a_0b_0b_1 + a_1b_0b_1 + 2c^2a_1b_0b_1 - a_0b_1^2 + c^2a_0b_1^2 = 0,$

$Q^3: 2a_1^3 + 12a_0a_1a_2 + 6ca_1a_2b_0 - 2a_1b_0^2 + 10a_2b_0^2 + 3ca_1^2b_1 + 6ca_0a_2b_1 + 2a_0b_0b_1 - a_1b_0b_1 - 3a_2b_0b_1 + 2c^2a_2b_0b_1 + a_0b_1^2 + c^2a_1b_1^2 = 0,$

$Q^4: 6a_1^2a_2 + 6a_0a_2^2 + 3ca_2^2b_0 - 6a_2b_0^2 + 6ca_1a_2b_1 + 9a_2b_0b_1 - a_2b_1^2 + c^2a_2b_1^2 = 0,$

$Q^5: 6a_1a_2^2 + 3ca_2^2b_1 - 6a_2b_0b_1 + 3a_2b_1^2 = 0,$

$Q^6: 2a_2^3 - 2a_2b_1^2 = 0,$

Solving the above set of algebraic equations by using Mathematica software package program, we acquire different cases of solutions which are representing as follows

Case 1:

$$a_0 = -2b_0,\ a_1 = 4b_0,\ a_2 = -2b_0,\ b_1 = -2b_0 \text{ and } c = 2, \tag{15}$$

whereinto b_0 is arbitrary constant.

Plugging these results in Eq. (14) by using Eqs. (10) and (11), the following solution will be derived

$$u_{1,1}(x,y,t) = \frac{2A^2(Cosh(x+y-2t) + Sin\,h(x+y-2t))}{(1-A^2)\,Cosh(x+y-2t) - (1+A^2)\,Sin\,h(x+y-2t)}, \tag{16}$$

In Particular, if we set $A = 1$ then the solution turns into the form

$$u_{1,1}(x,y,t) = -\big(1 + Coth(x+y-2t)\big). \tag{17}$$

Case 2:

$$a_0 = b_0,\ a_1 = -(b_0 - b_1),\ a_2 = -b_1,\ c = -1 \text{ and } b_0,\ b_1 \text{ are arbitrary constant.} \tag{18}$$

Substituting these results into Eq. (14) by using Eqs. (10) and (11), we obtain the exact solution of Eq. (1) takes the form as follows

$$u_{1,2}(x,y,t) = \frac{A\left(Cosh\left(\frac{x+y+t}{2}\right) + Sin\,h\left(\frac{x+y+t}{2}\right)\right)}{(A+1)Cosh\left(\frac{x+y+t}{2}\right) + (A-1)Sin\,h\left(\frac{x+y+t}{2}\right)}. \tag{19}$$

For simplicity, if we choose $A = 1$ then,

$$u_{1,2}(x,y,t) = \frac{1}{2}\left(1 + tanh\left(\frac{x+y+t}{2}\right)\right). \tag{20}$$

Also if we choose $A = -1$ the solution attains

$$u_{1,2}(x, y, t) = \frac{1}{2}\left(1 + Coth\left(\frac{x+y+t}{2}\right)\right). \tag{21}$$

Case 3:

$$a_0 = 0, a_1 = b_1, a_2 = -b_1, b_0 = 0, c = -1, \tag{22}$$

Here b_1 is arbitrary constant.

These values coincide to the solution as analogous to the preceding solution. Due to congeniality, we did not issue it.

Csae 4:

$$a_0 = 0, a_1 = 0, a_2 = 2b_0, b_1 = -2b_0 \text{and } c = 2, \tag{23}$$

herein b_0 is arbitrary constant.

Inculcating these values in Eq. (14) with Eqs. (10) and (11), then we come up the closed form solution of Eq. (1) is

$$u_{1,3}(x, y, t) = \frac{2\big(Cosh(x+y-2t) - Sin\,h(x+y-2t)\big)}{(A^2-1)Cosh(x+y-2t) + (A^2+1)Sin\,h(x+y-2t)}. \tag{24}$$

Also, if we put $A = 1$ in the above equation yields,

$$u_{1,3}(x, y, t) = Coth(x + y - 2t) - 1 \tag{25}$$

III.b DSSH equation

In this part, the GKM will be used to constitute the exact traveling wave solution of the DSSH equation (2). First we will consider the following transformation

$$u(x, t) = U(\xi), \xi = x - ct, \tag{26}$$

here c is a celerity certain to be determined later. Inserting this transformation into Eq. (2), this converted into the following nonlinear ODE

$$U^{(6)} - 9U'U^{(4)} - 18U''U''' + 18(U')^2U'' - \frac{1}{2}c^2U'' - \frac{1}{2}cU^{(4)} = 0. \tag{27}$$

Integrating one time the ODE of Eq. (27) in terms of ξ, will give us

$$U^{(5)} - 9U'U''' - \frac{9}{2}(U'')^2 + 6(U')^3 - \frac{1}{2}c^2U' - \frac{1}{2}cU''' - g = 0. \tag{28}$$

where g is an integrating constant that can be determined later. By considering the homogeneous balance principle, balancing the highest order derivative $U^{(5)}$ with the nonlinear term $(U')^3$ in Eq. (28) and then the following relation is obtained

$$N = M + 1. \tag{29}$$

Now, by treating GKM, we attain the exact solution of Eq. (28). That solution is the similar solution of Eq. (12) and is in the form Eq. (14). Transforming Eq. (14) in Eq. (28) and removing the denominator, we get a system of algebraic equations by solving it, we obtain several results as follows

Case 1:

$$a_1 = -2a_0,\, a_2 = -4b_0,\, b_1 = -2b_0,\, c = 4 \text{ and } g = 0, \tag{30}$$

hither a_0 and b_0 are arbitrary constant.

Inserting these values in Eq. (14) with Eqs. (10) and (26), the following solution will be performed

$$u_{2,1}(x,t) = 1 + \frac{4\left(Cosh(x-4t) - Sinh(x-4t)\right)}{(1-A^2)Cosh(x-4t) - (1+A^2)Sinh(x-4t)}. \tag{31}$$

Taking hyperbolic function identities and assuming $A = 1$ Eq. (31) becomes

$$u_{2,1}(x,t) = 3 - 2Coth(x-4t). \tag{32}$$

Case 2:

$$a_1 = 2(a_0 + \sqrt{2}a_0 + b_0),\;\; a_2 = 4(b_0 + \sqrt{2}b_0),\;\; b_1 = 2(b_0 + \sqrt{2}b_0),\;\; c = 1 \text{ and } g = 0 \tag{33}$$

Herein a_0 and b_0 are arbitrary constant.

Consequently, the exact traveling wave solution of Eq. (2) will be in the form as follows

$$u_{2,2}(x,t) = 1 + \frac{2\left(Cosh\left(\frac{x-t}{2}\right) - Sinh\left(\frac{x-t}{2}\right)\right)}{(A+1)Cosh\left(\frac{x-t}{2}\right) + (A-1)Sinh\left(\frac{x-t}{2}\right)}. \tag{34}$$

Setting $A = 1$ then Eq. (34) can be expressed as

$$u_{2,2}(x,t) = 2 - tanh\left(\frac{x-t}{2}\right). \tag{35}$$

Again if we choose $A = -1$ Eq. (34) yields

$$u_{2,2}(x,t) = 2 - Coth\left(\frac{x-t}{2}\right). \tag{36}$$

Case 3:

$$a_1 = -2(a_0 - b_0),\;\; a_2 = -4b_0,\;\; b_1 = -2b_0,\, c = 1 \text{ and } g = 0, \tag{37}$$

where a_0, b_0 are arbitrary constant.

These values coincide to the solution as like as the preceding solution. Due to congeniality, we did not express it.

Case 4:

$$a_1 = \frac{2}{21}(5a_0 + 21b_0),\ a_2 = \frac{20\ b_0}{21},\ b_1 = \frac{10\ b_0}{21},\ c = 1,\ g = 0, \tag{38}$$

Where a_0 and $b_0$are arbitrary constant.

$$u_{2,4}(x,t) = \frac{2}{3} + \frac{2\left(Cosh\left(\frac{x-t}{2}\right) - Sin\,h\left(\frac{x-t}{2}\right)\right)}{(1+A)Cosh\left(\frac{x-t}{2}\right) - (1-A)Sin\,h\left(\frac{x-t}{2}\right)}. \tag{39}$$

For instance, we input $A = -1$ the solution takes the form

$$u_{2,4}(x,t) = \frac{5}{3} - Coth\left(\frac{x-t}{2}\right). \tag{40}$$

If we set $A = 1$we to come up the solution as follows

$$u_{2,4}(x,t) = \frac{5}{3} - tanh\left(\frac{x-t}{2}\right). \tag{41}$$

III.c Fourth-order nonlinear AKNS water wave dynamical equation

In this part, the GKM will be employed to establish the exact traveling wave solution of the fourth-order nonlinear AKNS water wave dynamical equation (3). We first take into consideration the following traveling waves transformation,

$$u(x,y,t) = U(\xi),\ \xi = x + y + \omega t, \tag{42}$$

here ω is wave speed to evaluated later. Plugging this transformation into Eq. (3), this transforms into the following nonlinear ODE and integrated with respect to ξ (letting the constant of integration to be zero), we obtain:

$$(4\omega - \beta)U' + 6(U')^2 + \omega U''' = 0. \tag{43}$$

In Eq. (8) the relation between N and M is conferred by homogeneous balance principle, balancing highest order derivative U''' with highest order nonlinear term $(U')^2$ in Eq. (43), we attain the relation as follows:

$$N = M + 1. \tag{44}$$

The solution of Eq. (43) is similar to Eq. (12), which form is in Eq. (14). Substituting Eq. (14) into Eq. (43) and cleaning the denominator, we acquire a system of algebraic equations by solving it, we attain

Case 1:

$$a_1 = -2a_0,\ a_2 = \frac{\beta b_0}{4},\ b_1 = -2b_0,\ \omega = \frac{\beta}{8}, \tag{45}$$

wherein a_0, b_0 are arbitrary constant.

Inserting these conditions of Eq. (45) into Eq. (14), hence the following solution will be furnished

$$u_{3,1}(x,y,t) = 1 + \frac{\beta}{4}\frac{\left(Cosh\left(x+y+\frac{\beta}{8}t\right)-Sin\,h\left(x+y+\frac{\beta}{8}t\right)\right)}{(A^2-1)Cosh\left(x+y+\frac{\beta}{8}t\right)+(A^2+1)Sin\,h\left(x+y+\frac{\beta}{8}t\right)}. \tag{46}$$

If we set $A = 1$ then Eq. (46) turns into the form

$$u_{3,1}(x,y,t) = 1 + \frac{\beta}{8}\left(Coth\left(x+y+\frac{\beta}{8}t\right)-1\right). \tag{47}$$

Similar result will be found if we set $A = -1$.

Case 2:

$$a_1 = \frac{-\beta b_0^2+5a_0b_1}{5b_0},\ a_2 = -\frac{\beta b_1}{5},\ \omega = \frac{\beta}{5}, \tag{48}$$

therein a_0, b_0, b_1 are arbitrary constant.

Accordingly, the new exact solution of Eq. (3) can be explored as follows

$$u_{3,2}(x,y,t) = 1 - \frac{\beta}{5}\frac{\left(Cosh\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)-Sin\,h\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)\right)}{(A+1)Cosh\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)+(A-1)Sin\,h\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)}. \tag{49}$$

Using hyperbolic function identities and choose $A = 1$ Eq. (49) yields,

$$u_{3,2}(x,y,t) = 1 + \frac{\beta}{10}\left(tanh\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)-1\right). \tag{50}$$

Also, if we set $A = -1$ Eq. (49) can be written as

$$u_{3,2}(x,y,t) = 1 + \frac{\beta}{10}\left(Coth\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)-1\right). \tag{51}$$

Case 3:

$a_0 = 0,\ a_2 = -\frac{\beta b_1}{5},\ ,\ b_0 = 0,\ \omega = \frac{\beta}{5}$ and b_1 is arbitrary constant. (52)

So that, the new exact traveling wave solution of Eq. (3) can be described the following form

$$u_{3,3}(x,y,t) = \frac{1}{2} - \frac{\beta}{5}\frac{\left(Cosh\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)-Sin\,h\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)\right)}{(A-1)Sin\,h\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)+(A+1)Cosh\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)}. \tag{53}$$

Choosing $A = -1$ and simplifying we attain

$$u_{3,3}(x,y,t) = \frac{\beta}{10}\left(Coth\left(\frac{x+y+\frac{\beta}{5}t}{2}\right)-1\right)+\frac{1}{2}. \tag{54}$$

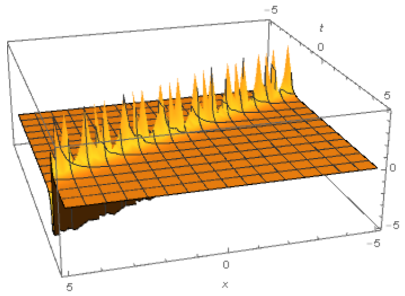
And if $A = 1$ then we deduce the solution, which converts into

$$u_{3,3}(x,y,t) = \frac{\beta}{10}\left(tanh\left(\frac{x+y+\frac{\beta}{5}t}{2}\right) - 1\right) + \frac{1}{2}. \quad (55)$$

Remark: All of the solutions in this treatise have been corroborated by substituting back into the original equation via the symbolic computer program Mathematica and found them new.

IV. Physical interpretation and graphical illustration

In this section, we have exemplified physical explanation and graphical representation of the attained solution of the Couple Boiti-Leon-Pempinelli, DSSH and fourth-order nonlinear AKNS equations. Let us now examine Figs. 1-6 as it illustrates 3D and 2D plots of some of our obtained solutions, which will provide different types of kink, soliton, singular kink etc. solutions. To this goal, we choose various special values of the constants obtained, such as Fig.1 shows the profile of Eq. (17) that behaves like the soliton solution for $y = 0$ and $A = 1$ within the interval $-5 \leq x \leq 5$ and $-5 \leq t \leq 5$. Fig. 2 represent the profile of kink solution of $3D$ and corresponding $2D$ shape of Eq. (20) for $y = 0$, $A = 1$ within the interval $-5 \leq x \leq 5$ and $-5 \leq t \leq 5$. Fig. 3 which represent the shape of singular soliton solution of Eq. (32) for $a_0 = 1$, $b_0 = 1$ and $A = 1$ within the interval $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$. Fig. 4 depicts the $3D$ and the corresponding $2D$ plot of Eq. (35) for $a_0 = 1$, $b_0 = 1$ and $A = 1$ within the interval $-5 \leq x \leq 5$ and $-5 \leq t \leq 5$. The kink solution of Eq. (50) for $a_0 = 1$, $b_0 = 1$, $y = 0.5$, $\beta = 15$ and $A = 1$ within the interval $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$ which is in Fig. 5. Fig. 6 shows the singular kink solution of Eq. (51) for $a_0 = 1$, $b_0 = 1$, $y = 0$, $\beta = -5$ and $A = -1$ within the interval $-5 \leq x \leq 5$ and $-5 \leq t \leq 5$.



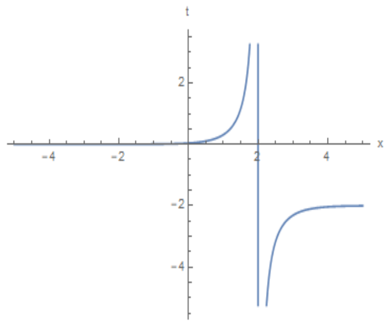


Fig. 1: Shape of the exact solution of Eq. (17) and its projection at $t = 1$ when $y = 0$.

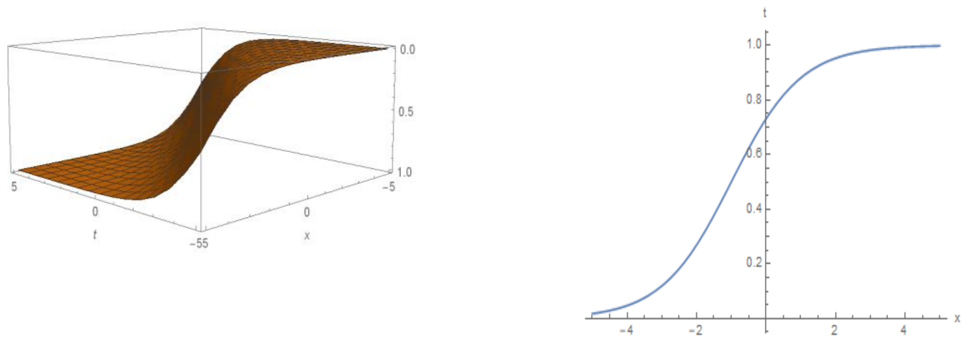


Fig. 2: Shape of the exact solution of Eq. (20) and its projection at $t = 0$ when $y = 0$.

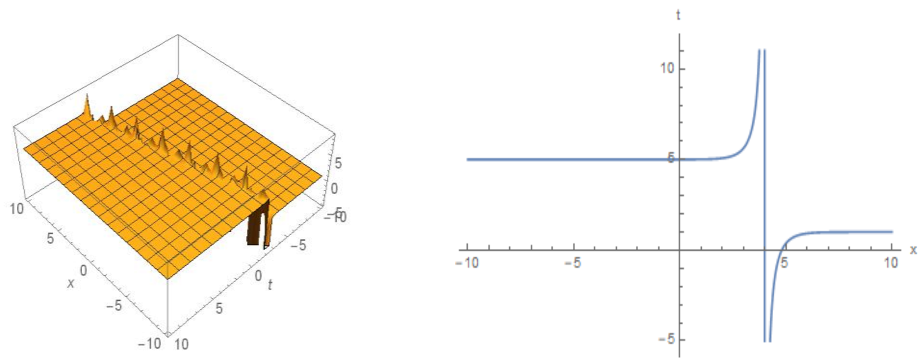


Fig. 3: Shape of the exact solution of Eq. (32) and its projection at $t = 1$.

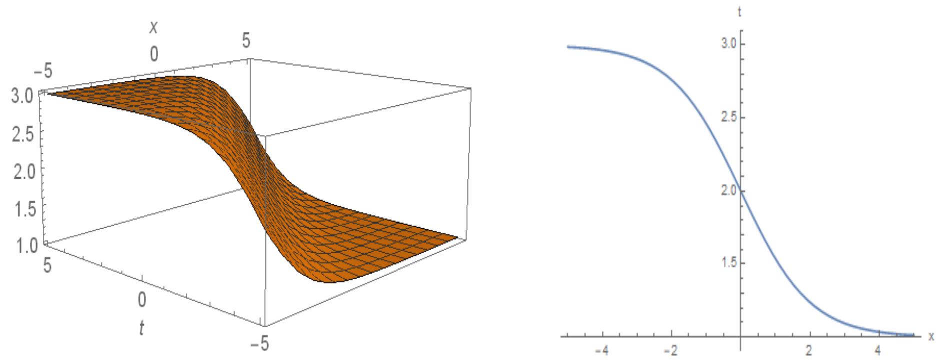


Fig. 4: Shape of the exact solution of Eq. (35) and its projection at $t = 0$.

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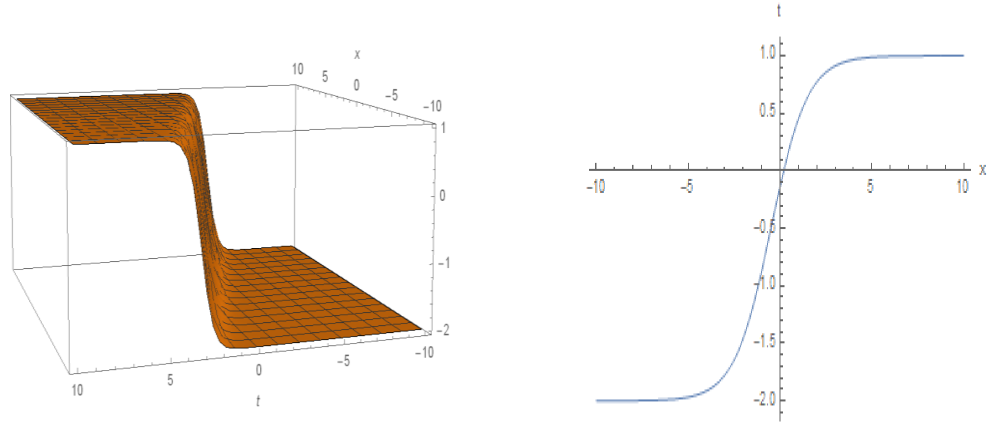

Fig. 5: Shape of the exact solution of Eq. (50) and its projection at $t = 0$.

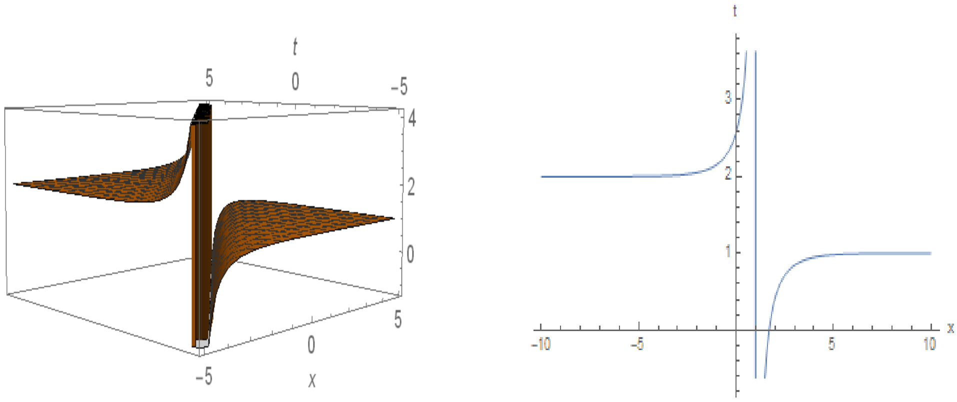

Fig. 6: Shape of the exact solution of Eq. (51) and its projection at $t = 1$

V. Comparison

In this portion, we compare our performed solutions with Khater et al. [XXV], Feng and Zheng [XXXIV], Ali et al. [II], results for Eqns. (1), (2), (3) respectively. Wherein Khater et al. [XXV] solved Couple Boiti-Leon-Pempinelli equations system by using Khater method, Feng and Zheng [XXXIV] solved the DSSH equation by implementing $\left(\frac{G'}{G}\right)$- expansion method and Ali et al. [II] solved fourth-order nonlinear AKNS water wave dynamical equation by applying Simple equation method. The comparison is ascertained as follows

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Couple Boiti-Leon-Pempinelli equations system:

Khater et al. [XXV] solution	Our solution
For $\alpha = \frac{-(\beta^2-c_1^2)}{4\sigma}$, $a_0 = \frac{\beta-c_1}{2}$ and $a_1 = \sigma$ then the solitary wave solution $u(x,y,t) =$ $-\frac{\beta+c_1}{2} - \sqrt{(\beta^2-\alpha\sigma)}\, tanh\left(\frac{\sqrt{(\beta^2-\alpha\sigma)}}{2}(x+y-c_1t)\right).$ and $u(x,y,t) =$ $-\frac{\beta+c_1}{2} - \sqrt{(\beta^2-\alpha\sigma)}\, Coth\left(\frac{\sqrt{(\beta^2-\alpha\sigma)}}{2}(x+y-c_1t)\right).$	Since $a_0 = b_0$, $a_1 = -(b_0-b_1)$, $a_2 = -b_1$ and $c = -1$ then exact solution $u_{1,2}(x,y,t) =$ $\frac{1}{2}\left(1 + tanh\left(\frac{x+y+t}{2}\right)\right).$ and $u_{1,2}(x,y,t) =$ $\frac{1}{2}\left(1 + Coth\left(\frac{x+y+t}{2}\right)\right).$

DSSH equation:

Feng and Zheng [XXXIV]	Our solution
When $a_1 = -2$, $a_0 = a_0$, $g = 0$, $c = \lambda^2 - 4\mu$, then for particular values of $c_1 = 1$, $c_2 = 0$, $\lambda = 2$ and $\mu = 0$ the hyperbolic solution comes up $u(x,t) = 2 - 2\tanh(x-4t) + a_0.$	For $a_1 = \frac{2}{21}(5a_0 + 21b_0)$, $a_2 = \frac{20\, b_0}{21}$, $b_1 = \frac{10\, b_0}{21}$, $c = 1$, $g = 0$ the exact solution is $u_{2,4}(x,t) = \frac{5}{3} - Coth\left(\frac{x-t}{2}\right).$ and $u_{2,4}(x,t) = \frac{5}{3} - tanh\left(\frac{x-t}{2}\right).$

Fourth-order nonlinear AKNS water wave dynamical equation:

Ali et al. [II]	Our solution
For $k = \frac{\beta}{b_1^2-4b_0b_2+4}$, $A_1 = \frac{\beta b_2}{-b_1^2+4b_0b_2-4}$, $A_{-1} = 0$, $A_{-2} = 0$, $A_2 = 0$ then the solution yields $v_2(x,y,t) =$ $A_0 +$ $\frac{\beta\left(b_1 - \sqrt{4b_0b_2-b_1^2}\, tan\left(\frac{\sqrt{4b_0b_2-b_1^2}}{2}(\xi+\xi_0)\right)\right)}{(2b_1^2-8b_0b_2+8)}$, $4b_0b_2 > b_1^2$, where $\xi = x + y + \frac{\beta}{b_1^2-4b_0b_2+4}t.$	$a_1 = \frac{-\beta b_0^2+5a_0b_1}{5b_0}$, $a_2 = -\frac{\beta b_1}{5}$, $\omega = \frac{\beta}{5}$ then we obtained the exact travelling wave solution is $u_{3,3}(x,y,t) = \frac{\beta}{10}\left(Coth\left(\frac{x+y+\frac{\beta}{5}t}{2}\right) - 1\right) +$ $\frac{1}{2}.$ and $u_{3,3}(x,y,t) = \frac{\beta}{10}\left(tanh\left(\frac{x+y+\frac{\beta}{5}t}{2}\right) - 1\right) +$ $\frac{1}{2}.$

VI. Results and Discussion

The fundamental key stone of the GKM is to punctuate the new exact solutions of the above mentioned Equations (1-3). These equations have been solved by several scholars (Khater et al. [XXV]; Feng and Zheng [XXXIV]; Ali et al. [II]). Also, utilizing different methods they have found different types of the exact solution such as Khater et al. [XXV] obtained solitary wave solutions by using Khater method and these results expressed tan, cot, tanh, coth form. Besides Feng and Zheng [XXXIV] and Ali et al. [II] has gotten some new traveling wave solutions which represented by sin, cos, sinh, cosh and sinh, cosh, tanh form. In our term, the first time we envisage the novel GKM and used a different technique for obtaining several types of new exact traveling wave solution of the aforecited equations (1-3). All the solutions in this article represented by hyperbolic form and these are new.

VII. Conclusion

The main purpose of this work has been effectively utilized novel GKM to attain exact traveling wave solutions of Couple Boiti-Leon-Pempinelli, DSSH and fourth-order nonlinear AKNS equations. The obtained solutions are discerned by putting them back into the said physical model and are accomplished to be very commodious over various existing methods. The obtained solutions may be congenial to perceive the mechanism of the tangled non-linear physical phenomena in wave collaboration. These solutions have an extraordinary specialty that keeps its uniformity upon interacting with others. The results out solutions emphasize that the GKM provides a powerful mathematical tool that some new exact solutions are frequently extracted. This method is very legitimate, modest, efficient, simple and effective for NLEEs in mathematical and pragmatical point of view. Finally, we consider that the method can easily be applicable to a large category of physical problems in a wide range of applied nonlinear science.

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