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The Dynamics of SIR (Susceptible-Infected-Recovered) Epidemic Model in Greater Noakhali for Pneumonia and Dysentery

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Abstract

We study the SIR model for the mathematical modeling of diseases of greater Noakhali. This model describes the spread of infectious diseases in which an individual may move from susceptible to infected and to recover. We discussed the mathematics behind the model and various tools for judging effectiveness in a certain territory. We completed the paper with an example using the infectious diseases, Pneumonia and Dysentery, commonly the children are infected. The current results of this paper are greatly instructive for us to further understand the epidemic spreading and design some fruitful prevention and disposal strategies to fight the epidemics.

Keywords: SIR Model, Effective removal rate, Basic reproductive ratio, Effective reproductive ratio.

I. Introduction

The SIR (susceptible-infected-removed) model developed by Ronald Ross, William Hamer, and others in the early twentieth century consists of a system of three coupled non-linear ordinary differential equations, which does not possess an evident cue solution. The term of epidemiology is used to human diseases for a large time now. Epidemiology is concerned with disease spreading within populations. The word ‘epidemic’ or ‘endemic’ is a word which is currently related to tragic events of monumental proportions such as when the pneumonia or dysentery hit the world. Both the pneumonia and the dysentery cause the death of millions of people every year.

The dynamics of SIR (Susceptible-Infected-Recovered) epidemic model with a limited resource for treatment and immigrants are proposed and analyzed. It is assumed that the population is divided into three classes known as susceptible, infected and recovered. The existence, uniqueness and boundedness of the solution of the model have been investigated. The local stability analysis of SIR epidemic model without treatment and immigrants is discussed analytically. We have discussed elaborately about SIR epidemic model with assumptions, limiting behavior, basic reproductive ratio, effective reproductive number and found the actual number from the tables of "populations under 0-5 infected by Pneumonia in Noakhali district", "populations under 0-5 infected by dysentery in Noakhali district", "populations under 0-5 infected by Pneumonia in Lakhshmipur district", "populations under 0-5 infected by dysentery in Lakhshmipur district". We have discussed the future prediction of susceptible, infected and recovered population under the age 0-5 using SIR epidemic models.

II. Preliminaries

Population

In biology, a population is all the organisms of the same group or species, which live in a particular geographical area, and have the capability of interbreeding [VII]. The area that is used to define a sexual population is defined as the area where inter-breeding is potentially possible between any pair within the area, and where the probability of interbreeding is greater than the probability of cross-breeding with individuals from other areas [IV].

Susceptible

Individuals that are susceptible have in the case of the basic SIR model, who are not infected but could become infected that is they are able to catch the disease. Once they have it, they move into the infected axil. That is the state of being easily affected, influenced, or harmed by something.

Susceptible population is represented by S(t). The rate of change of S with respect to time t is given by

$$\frac{dS}{dt} = -rSI$$

where $r > 0$ is infectious rate and I is infected population.

Example of susceptibility in a sentence:

- The virus can infect susceptible individuals.

Infected

Infected individuals can spread the disease to susceptible individuals. The time they spend in the infected compartment is the infectious period, after which they enter the recovered compartment.

Infected population is represented by I(t). The rate of change of I with respect to time t is given by

$$\frac{dI}{dt} = rSI - aI$$

where a is removal rate and $a > 0$.

Example of infected in a sentence:

- If you are sick you should stay home to avoid infecting other people in office or any other place.
- The virus has infected many people.
- They are unable to prevent bacteria from infecting the wound.

Recovered

Individuals in the recovered compartment are assumed to be immune for life. Recovered population is represented by the letter R(t). The rate of change R with respect to t is

$$\frac{dR}{dt} = aI$$

where a is removal rate and $a > 0$.

Example of recovered in a sentence:

- He had a heart attack but is recovering well.
- She recovered consciousness in the hospital.

Net population

Net population is the total population which contains susceptible, infected and removal individuals. That is the total population $N = S + I + R$ is constant because

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = -rSI + (rSI - aI) + aI = 0$$

Effective removal rate

Effective removal rate is the ratio of the rate at which individuals are removed from the infected category to the rate at which they are added to the same category.
It is denoted by ρ,

where $\rho = \frac{a}{r}$ (1)

Basic reproductive ratio

The basic reproduction number (sometimes called basic reproductive ratio, or basic reproductive rate and denoted by, B_r) of an infection can be thought of as the number of cases, one case generates on average over the course of its infectious period, otherwise uninfected population.

An important part of modeling diseases is the basic reproductive ratio [III]. The basic reproductive ratio is important since it tells us if a population is at risk from a disease. B_r is affected by the infection and removal rates, i.e., r and a, and is obtained by

$$B_r = \frac{r}{a} S_0 = \frac{S_0}{\rho} \quad (2)$$

When $B_r < 1$, the infection will die out in the long run.

When $B_r = 1$, the disease occurrence will be constant.
But if $B_r > 1$, the infection will be able to spread in a population.

Effective Reproductive Number

Effective reproductive number, denoted by E_r, is the average number of secondary cases generated by an infectious case during the epidemic. To calculate this number, we multiply the basic reproductive ratio by how many people are susceptible at time t, that is

$$E_r = B_r \frac{S_t}{N} \tag{3}$$

The effective reproductive number is important since it helps researchers and health officials determine how effective their policies are on controlling the disease. When $E_r < 1$, the policies concerning the containing of the disease are effective [III].

Assumptions of SIR models

The SIR model is used in epidemiology to compute the amount of susceptible, infected, and recovered people in a population. This model is an appropriate one to use under the following assumptions:

- The population is fixed.
- The only way a person can leave the susceptible group is to become infected. The only way a person can leave the infected group is to recover from the disease. Once a person has recovered, the person received immunity.
- Age, sex, social status, and race do not affect the probability of being infected.
- There is no inherited immunity.
- The member of the population mix homogeneously (have the same interactions with one another to the same degree).
- Removal of infective to the removed class is proportional to the number of infective, i.e., aI is constant, where $a > 0$.
- The incubation period is short enough to be negligible.

The model based on the above assumptions is

$$\frac{dS}{dt} = -rSI \tag{4}$$

$$\frac{dI}{dt} = rSI - aI \tag{5}$$

$$\frac{dR}{dt} = aI \tag{6}$$

III Development of SIR epidemic model

From (4) and (5), we have

$$\frac{dI}{dS} = \frac{rSI - aI}{-rSI} = -1 + \frac{aI}{rSI} = -1 + \frac{\frac{a}{r}}{s} = -1 + \frac{\rho}{S}$$

Where $\rho = \frac{a}{r}$ is called the effective removal rate, i.e., the ratio of the rate at which individuals are removed from the infected category to the rate at which they are added to the same category.

$$\Rightarrow \quad dI = \left(-1 + \frac{\rho}{S}\right) dS$$
$$\Rightarrow \quad \int dI = -\int dS + \rho \int \frac{1}{S} dS$$
$$\Rightarrow \quad I = -S + \rho \ln(S) + \text{Constant}$$
$$\therefore \quad I + S - \rho \ln(S) = \text{Constant} = I_0 + S_0 - \rho \ln(S_0) \tag{7}$$

Here, $R(0) = 0$, So $0 \leq S + I < N$

I(t) will be maximum if

$$\frac{dI}{dt} = 0$$

$$\Rightarrow \quad I(rS - a) = 0$$
$$\Rightarrow \quad rS - a = 0 \qquad [\text{Since } I \neq 0]$$
$$\Rightarrow \quad rS = a$$
$$\Rightarrow \quad S = \frac{a}{r}$$
$$\therefore S = \rho \tag{8}$$

Putting $S = \rho$ in (7), we get

$$I_{max} + \rho - \rho \ln \rho = I_0 + S_0 - \rho \ln S_0$$
$$\Rightarrow \quad I_{max} = \rho \ln \rho - \rho + I_0 + S_0 - \rho \ln S_0$$
$$\Rightarrow \quad I_{max} = I_0 + S_0 - \rho + \rho \ln\left(\frac{\rho}{S_0}\right)$$
$$\therefore I_{max} = N - \rho + \rho \ln\left(\frac{\rho}{S_0}\right) \qquad [\text{Since } N = I_0 + S_0] \tag{9}$$

If $I_0 > 0$ and $S_0 > \rho$, then the phase trajectory starts with $S > \rho$. Also in this case I increases from I_0 and hence an epidemic ensures. If $S_0 < \rho$, then I decreases from I_0 and as such no epidemic occurs.

Again from (4) & (6), we get

$$\frac{dS}{dR} = -\frac{r}{a} S = -\frac{S}{\rho} \qquad \left(\text{where } \rho = \frac{a}{r}\right) \tag{10}$$

Integrating (10), we obtain

$$\int \frac{dS}{S} = -\frac{1}{\rho} \int dR$$

$\Rightarrow \ln S = -\frac{R}{\rho}$

$\Rightarrow \ln S - \ln S_0 = -\frac{R}{\rho}$

$\Rightarrow \ln \frac{S}{S_0} = -\frac{R}{\rho}$

$\Rightarrow \frac{S}{S_0} = e^{\left(-\frac{R}{\rho}\right)}$

$$\therefore S = S_0 e^{\left(-\frac{R}{\rho}\right)} \tag{11}$$

By applying Euler's method of systems, we can solve the differential equations (4), (5) & (6). The solutions to the differential equations are:

$$S_{n+1} = S_n - rS_n I_n (t_{n+1} - t_n) \tag{12}$$

$$I_{n+1} = I_n \{1 + (rS_n - a)(t_{n+1} - t_n)\} \tag{13}$$

$$R_{n+1} = R_n + aI_n (t_{n+1} - t_n) \tag{14}$$

Where S_{n+1} ,I_{n+1} and R_{n+1} are the number of susceptible, infected and recovered people at time t_{n+1}. Time difference $(t_{n+1} - t_n)$ is a small change of time and will be equal for all. It is important to note that researchers and health officials first collect data at a given period of time. The equations (12) to (14) are primarily used to calculate r and a.

Populations under 0-5 year infected by pneumonia in Noakhali district

Table 1: Children under 0-5 years old threatened by pneumonia last 5 years in Noakhali district

Year	Net Population N(t)	Susceptible person S(t)	Infectious person I(t)	Recovered person R(t)
2012	471440	464373	3624	3443
2013	479737	469230	5388	5119
2014	487797	472811	7685	7301
2015	496160	484840	5805	5515
2016	504667	485961	9593	9113
2017	513320	488114	12926	12280

Source: ''Civil Surgeon Office Noakhali and Population & Housing Census 2011, Zila Report: Noakhali", Bangladesh Statistical Bureau, Bangladesh [I].

From equation (12) and (14), we can write

$$r = \frac{S_n - S_{n+1}}{S_n I_n (t_{n+1} - t_n)} \text{ and } a = \frac{R_{n+1} - R_n}{I_n (t_{n+1} - t_n)} \tag{15}$$

Using this equation (15), we can get the values of r and a for each period. These r and a are shown in the following table:

Table 2: Infection and Removal rate of pneumonia during last 5 years

Year	Susceptible person S(t)	Infection rate (r)	Removal rate (a)	Effective removal rate (ρ)	If $(S > \rho)$ No epidemic occurs	If $(S < \rho)$ Epidemic occurs
2012	464373	-	-	-	-	-
2013	469230	$2.86 * 10^{-6}$	0.46	161703	No epidemic occurred	-
2014	472811	$1.42 * 10^{-6}$	0.41	288732	No epidemic occurred	-
2015	484840	$3.31 * 10^{-6}$	0.23	69486	No epidemic occurred	-
2016	485961	$3.98 * 10^{-7}$	0.62	1557788	-	Epidemic occurred
2017	488114	$4.62 * 10^{-7}$	0.33	714285	-	Epidemic occurred

Here the negative sign of r and a should be neglected since r and a are always greater than zero.

Basic reproductive ratio for pneumonia in Noakhali district

The basic reproductive number is 2.87 for 2013, since $S_0 = 464373, \rho = 161703$

$$\therefore B_r = \frac{S_0}{\rho} = \frac{464373}{161703} = 2.87$$

Similarly, we will find the remaining basic reproductive number for 2014, 2015, 2016 and 2017 from the following table:

Table 3: Basic reproductive ratio for pneumonia in Noakhali district

Year	Susceptible Person S(t)	Effective removal rate (ρ)	Basic reproductive ratio (B_r)	Comment
2012	464373	-	-	-
2013	469230	161703	2.87	Infections had spread in the population.
2014	472811	288732	1.63	Infections had spread in the population.
2015	484840	69486	6.80	Infections had spread in the population.
2016	485961	155778	3.11	Infections had spread in the population.
2017	488114	714285	0.68	Infection has died out in the long run.

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Effective reproductive number for pneumonia in Noakhali district

Table 4: Effective reproductive number for pneumonia in Noakhali district

Year	Net Population N(t)	Susceptible person S(t)	Basic reproductive ratio(B_r)	Effective Reproductive Number (E_r)	Comment
2013	479737	469230	2.87	2.81	Infection spread through the population and infected large number of people
2014	487797	472811	1.63	1.58	Infection spread through the population and infected large number of people
2015	496160	484840	6.80	6.64	Infection spread through the population and infected large number of people
2016	504667	485961	3.11	2.99	Infection spread through the population and infected large number of people
2017	513320	488114	0.68	0.65	Infection would be decreased in case of eliminating the disease

Populations under 0-5 year infected by dysentery in Noakhali district

Table 5: Children under 0-5 years old threatened by dysentery last 5 years in Noakhali district

Year	Net Population N(t)	Susceptible person S(t)	Infectious person I(t)	Recovered person R(t)
2012	471440	462313	4804	4323
2013	479737	455524	12744	11469
2014	487797	452201	18735	16861
2015	496160	465753	16004	14403
2016	504667	464794	20986	18887
2017	513320	474135	20624	18561

Source: ''Civil Surgeon Office Noakhali and Population & Housing Census 2011, Zila Report: Noakhali", Bangladesh Statistical Bureau, Bangladesh [I].

Using equation (15) we can get the values of r and a for each period. These r and a are shown in the following table:

Table 6: Infection and Removal rate of dysentery during last 5 years

Year	Susceptible person S(t)	Infection rate (r)	Removal rate (a)	Effective removal rate (ρ)	If ($S > \rho$) No epidemic occurs	If ($S < \rho$) Epidemic occurs
2012	462313	-	-	-	-	-
2013	455524	$3.06 * 10^{-6}$	1.49	486928	-	Epidemic occurred
2014	452201	$0.57 * 10^{-6}$	0.42	736842	-	Epidemic occurred
2015	465753	$1.60 * 10^{-6}$	0.13	81250	No epidemic occurred	-
2016	464794	$0.14 * 10^{-6}$	0.28	2153846	-	Epidemic occurred
2017	474135	$0.96 * 10^{-6}$	0.02	20833	No epidemic occurred	-

Basic reproductive ratio for dysentery in Noakhali district

The basic reproductive number for 2013 is 0.95, since $S_0 = 462313$, $\rho = 486928$

$$\therefore B_r = \frac{S_0}{\rho} = \frac{462313}{486928} = 0.95$$

Similarly we will find the remaining basic reproductive number for 2014, 2015, 2016 and 2017 from the following table:

Table 7: Basic reproductive ratio for dysentery in Noakhali district

Year	Susceptible Person S(t)	Effective removal rate (ρ)	Basic reproductive ratio (B_r)	Comment
2012	462313	-	-	-
2013	455524	486928	0.95	Infection died out in the long run
2014	452201	736842	0.62	Infection died out in the long run
2015	465753	81250	5.57	Infections had spread in the population
2016	464794	2153846	0.22	Infection died out in the long run.
2017	474135	20833	22.31	Infections had spread in the population

We see that in 2015 and 2017, infection has spread in the population since $B_r > 1$. But in 2013, 2014 and 2016, infection has died out in the long run.

Effective reproductive number for dysentery in Noakhali district

Table 8: Effective reproductive number for dysentery in Noakhali district

Year	Net population N(t)	Susceptible person S(t)	Basic reproductive ratio (B_r)	Effective reproductive number (E_r)	Comment
2013	479737	455524	0.95	0.90	Infection would decrease in case of eliminating the disease
2014	487797	452201	0.62	0.57	Infection would decrease in case of eliminating the disease
2015	496160	465753	5.57	5.23	Infection spread through the population and infected larger number of people
2016	504667	464794	0.22	0.20	Infection would decrease in case of eliminating the disease
2017	513320	474135	22.31	20.61	Infection spread through the population and infected larger number of people

Populations under 0-5 year infected by pneumonia in Lakhshmipur district

Table 9: Children under 0-5 years old threatened by pneumonia last 5 years in Lakhshmipur

Year	Net Population N(t)	Susceptible person S(t)	Infected person I(t)	Recovered person R(t)
2012	260138	236133	12124	11881
2013	264070	243259	10511	10300
2014	268061	245309	11491	11261
2015	272112	252009	10153	9950
2016	276225	252457	12004	11764
2017	280399	261268	9662	9469

Source: ''Civil Surgeon Office Lakhshmipur and Population & Housing Census 2011, Zila Report: Lakhshmipur", Bangladesh Statistical Bureau, Bangladesh [II].

Using equation (15), we can get the values of r and a for each period. These r and a are shown in the following table:

Table 10: Infection and Removal rate of pneumonia during last 5 years

Year	Susceptible person S(t)	Infection rate (r)	Removal rate (a)	Effective removal rate (ρ)	If $(S > \rho)$ No epidemic occurs	If $(S < \rho)$ Epidemic occurs
2013	243259	$2.49 * 10^{-6}$	0.13	52208	No epidemic occurred	-
2014	245309	$0.80 * 10^{-6}$	0.09	112219	No epidemic occurred	-
2015	252009	$2.38 * 10^{-6}$	0.11	46218	No epidemic occurred	-
2016	252457	$0.18 * 10^{-6}$	0.18	1000000	-	Epidemic occurred
2017	261268	$2.91 * 10^{-6}$	0.19	65292	No epidemic occurred	

Basic reproductive ratio for pneumonia in Lakhshmipur district

The basic reproductive number for 2013 is 4.52, since $S_0 = 236133$, $\rho = 52208$

$$\therefore B_r = \frac{S_0}{\rho} = \frac{236133}{52208} = 4.52$$

Similarly we will find the remaining basic reproductive number for 2014, 2015, 2016 and 2017 from the following table:

Table 11: Basic reproductive ratio for pneumonia in Lakhshmipur district

Year	Susceptible person S(t)	Effective removal rate (ρ)	Basic reproductive ratio, (B_r)	Comment
2012	236133	-	-	-
2013	243259	52208	4.52	Infections had spread in the population.
2014	245309	112219	2.17	Infections had spread in the population.
2015	252009	46218	5.31	Infections had spread in the population.
2016	252457	1000000	0.25	Infection died out in the long run.
2017	261268	65292	3.87	Infections had spread in the population.

In this case from 2013-15 and 2017, infection has spread in the population since $B_r > 1$. But in 2016, infection has died out in the long run since $B_r < 1$.

Effective reproductive number for pneumonia in Lakhshmipur district

Table 12: Effective reproductive number for pneumonia in Lakhshmipur district

Year	Net Population N(t)	Susceptible person S(t)	Basic reproductive ratio (B_r)	Effective reproductive number (E_r)	Comment
2012	260138	236133	-	-	-
2013	264070	243259	4.52	4.16	Infection spread through the population and infected large number of people
2014	268061	245309	2.17	1.99	Infection spread through the population and infected large number of people
2015	272112	252009	5.31	4.92	Infection spread through the population and infected large number of people
2016	276225	252457	0.25	0.23	Infection would decrease in case of eliminating the disease
2017	280399	261268	3.87	3.61	Infection spread through the population and infected large number of people

Populations under 0-5 year infected by dysentery in Lakhshmipur district

Table 13: Children under 0-5 years old threatened by dysentery last 5 years in Lakhshmipur

Year	Net Population N(t)	Susceptible person S(t)	Infected person I(t)	Recovered person R(t)	Infected person (%)	Recovered person (%)
2012	260138	246968	6754	6416	2.60	2.45
2013	264070	252622	5871	5577	2.22	2.11
2014	268061	252299	8083	7679	3.02	2.86
2015	272112	256206	8157	7749	2.99	2.85
2016	276225	270806	2779	2640	1.01	0.96
2017	280399	274602	2973	2824	1.06	1.01

Source: ''Civil Surgeon Office Lakhshmipur and Population & Housing Census 2011, Zilla Report: Lakhshmipur'', Bangladesh Statistical Bureau, Bangladesh [II].

Using equation (15), we can get the values of r and a for each period. These r and a are shown in the following table:

Table 14: Infection and Removal rate of dysentery during last 5 years

Year	Susceptible person S(t)	Infection rate (r)	Removal rate (a)	Effective removal rate (ρ)	If $(S > \rho)$ No epidemic occurs	If $(S < \rho)$ Epidemic occurs
2013	252622	$3.39 * 10^{-6}$	0.12	35398	No epidemic occurred	-
2014	252299	$0.22 * 10^{-6}$	0.36	1636363	-	Epidemic occurred
2015	256206	$1.92 * 10^{-6}$	0.01	5208	No epidemic occurred	-
2016	270806	$6.99 * 10^{-6}$	0.63	90128	No epidemic occurred	-
2017	274602	$5.04 * 10^{-6}$	0.07	13888	No epidemic occurred	-

Basic reproductive ratio for dysentery in Lakhshmipur district

The basic reproductive number for 2013 is 6.98, since $S_0 = 246968$, $\rho = 35398$

$$\therefore B_r = \frac{S_0}{\rho} = \frac{246968}{35398} = 6.98$$

Similarly, we will find the remaining basic reproductive number for 2014, 2015, 2016 and 2017 from the following table:

Table 15: Basic reproductive ratio for dysentery in Lakhshmipur district

Year	Susceptible S(t)	Effective removal rate (ρ)	Basic reproductive ratio, (B_r)	Comment
2012	246968	-	-	-
2013	252622	35398	6.98	Infections had spread in the population
2014	252299	1636363	0.15	Infection died out in the long run
2015	256206	5208	48.44	Infections had spread in the population
2016	270806	90128	2.84	Infections had spread in the population
2017	274602	13888	19.50	Infections had spread in the population

We see that in 2013 and 2015-17, infection has spread in the population since $B_r > 1$. But in 2014, infection has died out in the long run $B_r < 1$.

Effective reproductive number for dysentery in Lakhshmipur district

Table 16: Effective reproductive number for dysentery in Lakhshmipur district

Year	Net Population N(t)	Susceptible person S(t)	Basic reproductive ratio (B_r)	Effective reproductive number (E_r)	Comment
2012	260138	246968	-	-	-
2013	264070	252622	6.98	6.68	Infection spread through the population and infected large number of people
2014	268061	252299	0.15	0.14	Infection would decrease in case of eliminating the disease
2015	272112	256206	48.44	45.61	Infection spread through the population and infected large number of people
2016	276225	270806	2.84	2.78	Infection spread through the population and infected large number of people
2017	280399	274602	19.50	19.10	Infection spread through the population and infected large number of people

IV. Future Prediction of SIR Epidemic Model

Populations under 0-5 year infected by pneumonia in Noakhali district

Using equation (7) & (12) we will construct the following table for the future prediction of the pneumonia in Noakhali district.

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Table 17: Future prediction of population under 0-5 which are threatened by pneumonia in Noakhali

Year	Net Population N(t)	Susceptible person S(t)	Infectious person I(t)	Recovered person R(t)
2018	521626	499079	11563	10984
2019	530570	490720	20436	19414
2020	539667	500991	19834	18842
2021	548919	491652	29368	27899
2022	558331	501312	29241	27778

Using equation (15) we get the values of r and a for each period. These r and a are shown in the following table:

Table 18: Infection and Removal rate of pneumonia for next 5 years

Year	Susceptible person S(t)	Infection rate (r)	Removal rate (a)	Effective removal rate (ρ)	If $(S > \rho)$ No epidemic occur	If $(S < \rho)$ Epidemic occur
2018	499079	$1.74 * 10^{-6}$	0.10	57471	No epidemic will occur	-
2019	490720	$1.45 * 10^{-6}$	0.73	503448	-	Epidemic will occur
2020	500991	$1.02 * 10^{-6}$	0.03	29411	No epidemic will occur	-
2021	491652	$9.40 * 10^{-7}$	0.46	489361	No epidemic will occur	-
2022	501312	$6.69 * 10^{-7}$	0.01	14947	No epidemic will occur	-

Comment: We have seen that in 2019 epidemic will occur and in 2021 epidemic may occur since ρ is near about S.

Populations under 0-5 year infected by pneumonia in Lakhshmipur district

Using equation (7) & (12) we will construct the following table for the future prediction of the pneumonia in Lakhshmipur district.

Table 19: Future prediction of population under 0-5 which are threatened by pneumonia in Lakhshmipur

Year	Net Population N(t)	Susceptible person S(t)	Infectious person I(t)	Recovered person R(t)
2018	284636	255104	15145	14387
2019	288938	256147	16816	15975
2020	293305	261864	16124	15317
2021	297738	256619	21087	20032
2022	302237	260115	21601	20521

Using equation (15) we get the values of r and a for each period. These r and a are shown in the following table:

Table 20: Infection and Removal rate of pneumonia during next 5 years

Year	Susceptible person S(t)	Infection rate (r)	Removal rate (a)	Effective removal rate (ρ)	If $(S > \rho)$ No epidemic occur	If $(S < \rho)$ Epidemic occur
2018	255104	$2.44 * 10^{-6}$	0.51	209016	No epidemic will occur	-
2019	256147	$2.70 * 10^{-7}$	0.11	407469	-	Epidemic will occur
2020	261864	$1.33 * 10^{-6}$	0.04	30075	No epidemic will occur	-
2021	256619	$1.24 * 10^{-6}$	0.29	233870	No epidemic will occur	-
2022	260115	$6.46 * 10^{-7}$	0.02	30959	No epidemic will occur	-

Comment: We have seen that in 2019 epidemic will occur and in 2021 epidemic may occur since ρ is near about S.

V. Discussion

In tables 1 & 9 for pneumonia and 5 & 13 for dysentery represent the net populations in Noakhali and Lakhshmipur which contain three categories of individuals such as susceptible, infected and removal individuals for analysis of SIR epidemic model in last five years. From these tables we have got the infection and removal rate r and a by equation (15) using SIR epidemic model, which are represented in table 2. In that table we have found effective removal rate (ρ). With the help of ρ and S(t), we conclude that whether epidemic occurred or not. In that case we have seen that no epidemic occurred in 2013, 2014 and 2015 while epidemic occurred in 2016 & 2017. Now in table 3, by using equation (2) we have found the value of basic reproductive ratio which is used for whether infection would spread or not. In that case we have seen from table 3 that during 2013-2016 infection has spread in the population whereas in 2017 infection has died out in the population. In table 4, we have discussed about effective reproductive ratio (E_r) for pneumonia in noakhali district which is very important to test how many people have been infected in a single period of time. We estimate the value of E_r from equation (3). In this table we have seen that from 2013-2016, infection spread through the population and infected a large number of people and in 2017, if the disease had been eliminated then infection would decrease and the similar process would occur for Lakhshmipur district for pneumonia and dysentery.

VI. Conclusion

Pneumonia and dysentery are the most common diseases in our country. Every year a number of people die by these diseases, especially children who are under five years. In this paper we have discussed about the dynamics of SIR model developed with the ordinary differential equation. We have tried to test whether epidemic diseases of pneumonia and dysentery occurred or not during last five years (2013-2017) in Noakhali and Lakhshmipur district and whether epidemic will occur or not during next five years (2018-2022). We hope that this research work will come handy for ministry of health. Studying this paper government can be aware of future effect of pneumonia and dysentery. Conscious citizens, officers of civil surgeon office and any other health NGOs in Noakhali, and Lakhshmipur district can study our prediction of dynamics of SIR model for pneumonia and dysentery and they will be able to take necessary steps to prevent the epidemic against these diseases. We hope it will help to take effective measure for those who are going to be infected in future.

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