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AN INVENTORY MODEL FOR DETERIORATING ITEM WITH ALLOWABLE DELAY IN PAYMENT

BY

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Abstract

In this paper, we have developed an inventory model for deteriorating item with permissible delay in payment. Demand function dependent on the selling price and frequency of advertisement cost. Partially backlogged shortages are allowed and backlogged rate dependent on the duration of waiting time up to arrival of next lot. The corresponding model have been formulated and solved. Three numerical examples have been considered to illustrate the model. Finally sensitivity analyses have been carried out taking one parameter at a time and other parameters as same.

Keywords : ***Inventory, deterioration, partially backlogged shortages, permissible delay in payment***

1. Introduction

Due to highly competition in marketing policy permissible delay in payment is one of the important factors to increase their business. As a result, wholesalers/suppliers offer different types of facilities to their retailers to promote their business. In that case, wholesalers/suppliers offer a certain credit period to their retailer. In this period, no interest is charged by the supplier to their retailer. However, after this period, a low rate of interest is charged by the supplier to the next credit period and after this time period, a high rate of interest is charged by the supplier under certain terms and conditions. This is known as inventory problem with permissible delay in payments. It is also known as trade credit financing inventory problem. This type of idea was first introduced by Haley and Higgins [1]. Thereafter, Goyal [2] formulated an EOQ model under the conditions of permissible delay in payments. Then, Aggarwal and Jaggi [3] extended the Goyal's model for deteriorating items. Shortages are not considered in their model. Jamal et al. [4] developed the general EOQ model, considering fully backlogged shortages.

After Jamal et al. [4], a number of works have been done by several researchers to their research. The detailed of some of the works have been shown in **Table 1A**.

Table 1A: Summary of related literature for single warehouse inventory model with permissible delay in payments

Author(s) and year	Warehouses	Deterioration	Demand Rate	Shortages	Level of permissible delay in payments	Inventory policies
Hwang and Shinn[5]	Single	Yes	Constant	No	Single	--
Chang, Ouyang and Teng[6]	Single	Yes	Constant	No	Single	--
Abad and Jaggi[7]	Single	No	Linearly time dependent	No	Single	--
Ouyang, Wu and Yang[8]	Single	Yes	Constant	No	Single	--
Huang [9]	Single	No	Linearly time dependent	No	Two level	---
Huang[10]	Single	No	Constant	No	Single	--
Huang[11]	Single	No	Constant	No	Two level	--
Sana and Chaudhuri [19]	Single	Yes	Selling price dependent	No	Single	--
Huang and Hsu Yang[20]	Single	No	Constant	No	Two level partial trade credit	--
Ho and Ouyang [21]	Single	No	Constant	No	Two level	--
Jaggi and Khanna [24]	Single	Yes	Inventory level dependent	Complete backlogging	Single	IFS
Jaggi and Kausar [25]	Single	No	Selling price dependent	Complete backlogging	Single partial trade credit	--
Jaggi and Mittal [29]	Single	Yes	Annual	Complete backlogging	Single	IFS
Shah, Patel and Lou[37]	Single	Yes	Inventory level dependent	No	Single	--
Present paper	Single	Yes	Selling price dependent demand	Partial Backlogging	Single	IFS

Table 1B: Summary of related literature for two warehouse inventory model

Author(s) and year	Warehouses	Deterio ration	Demand Rate	Shortages	Level of permissible delay in payments	Inventory policies
Das, Maity and Maiti[12]	Two	No	Inventory level dependent	No	Single	--
Niu and Xie[13]	Two	Yes	Constant	Complete backlogging	Single	IFS
Rong, Mahapatra and Maiti[14]	Two	Yes	Selling price dependent	Partial and complete backlogging	Single	IFS
Dey, Mondal and Maiti[15]	Two	No	Dynamic	No	Single	--
Hsieh, Dey and Ouyang[16]	Two	Yes	Constant	Complete backlogging	Single	IFS
Maiti[17]	Two	No	Inventory level dependent	No	Single	--
Jaggi and Verma [18]	Two	No	Selling price dependent	Complete backlogging	No	--
Lee and Hsu [22]	Two	Yes	Linearly time dependent			
Jaggi, Aggarwal and Verma[23]	Two	Yes	Selling price dependent	Partial backlogging	No	IFS
Bhunia and Shaikh[26]	Two	Yes	Price and time dependent	Partial Backlogging	No	IFS
Bhunia, Pal and Chattopsdhyay [27]	Two	Yes	Inventory level dependent	Partial backlogging	No	IFS
Jaggi, Khanna and Verma[28]	Two & Single	Yes	Linearly time dependent	Partial backlogging	No	IFS
Yang [30]	Two	Yes	Constant	Partial backlogging	No	IFS&SFI
Bhunia, Shaikh, Maiti and Maiti [31]	Two	Yes	Linearly time dependent	Partial Backlogging	No	IFS
Bhunia, Shaikh and Gupta[32]	Two	Yes	Linearly time dependent	Partial backlogging	No	IFS&SFI
Yang and Chang [33]	Two	Yes	Constant	Partial backlogging	Single	SFI
Chung and Huang [34]	Two	Yes	Constant	No	Single	--
Liang and Zhou [35]	Two	Yes	Constant	No	No	--
Bhunia, Jaggi, Sharma and Sharma [36]	Two	Yes	Constant	Partial backlogging	Alternative approach Single	IFS

In this paper, we have developed an inventory model for deteriorating item with permissible delay in payment. Demand function dependent on the selling price and frequency of advertisement cost. Partially backlogged shortages are allowed and backlogged rate dependent on the duration of waiting time up to arrival of next lot. The corresponding model have been formulated and solved. Three numerical examples have been considered to illustrate the model. Finally sensitivity analyses have been carried out taking one parameter at a time and other parameters as same.

2. Assumptions and Notations

The following notations and assumptions have been used in this paper.

Assumptions:

(i) The entire lot size is delivered in one batch.

(ii) Replenishment rate is infinite and lead time is constant.

(iii) The demand rate is dependent on price i.e., $D = a - bp$, where $a, b \geq 0$.

(iv) Deterioration is considered only after the quantities are stored in the warehouse. There is neither repair nor replacement of the deteriorated units during the inventory cycle.

(v) Shortages, if any, are allowed and unsatisfied demands are partially backlogged. During the stock-out period, the backlogging rate is dependent on the length of the waiting time up to the arrival of next lot. In this situation, the rate is defined as $\left[1+\delta\left(t_s - t\right)\right]^{-1}, \delta > 0$.

(vi) The inventory planning horizon is infinite and the inventory system involves only one item.

Notations:

$I(t)$	Inventory levels, at time t
S	Highest stock level at the beginning of stock-in period
R	Highest shortage level
θ	Deterioration rates in OW and RW, respectively($0 < \theta << 1$)
C_o	Replenishment cost per order

δ	Backlogging parameter
c_p	Purchasing cost per unit
p	Selling price per unit of item and is denoted by $p = mC_p$
D	Price dependent demand
C_h	Holding cost per unit per unit time
C_b	Shortage cost per unit per unit time
C_{ls}	Unit opportunity cost due to lost sale
t_1	Time at which the stock level reaches to zero
T	Time at which the highest shortage level reaches to the lowest point
M	Credit period offered by the supplier
I_e	Interest earned by the retailer
N	Next credit period offered by the supplier
I_p	Interest charged by the suppliers to the retailers
$Z(.)$	The average cost

3. Mathematical formulation

Initially, at time $t = 0$ an enterprise stock the good *S* unit. Inventory level depleting due to demand of customer and deterioration during the tie period $0 < t \le t_1$. At time $t = t_1$ inventory level reaches to zero. There after shortages occurs and its accumulates at the rate $\frac{1}{[1 + \delta(T - t)]}, (\delta > 0)$ up to the time $t = T$ when the next lot arrives. At time $t = T$, the maximum shortage level is *R*.

Let *I(t)* be the inventory level at any time $t \ge 0$. The inventory level *I(t)* satisfies the following differential equation at any time *t* as follows

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \quad 0 < t \le t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = \frac{-D}{1 + \delta(T - t)}, \quad t_1 < t \le T \tag{2}$$

with the conditions

$$I(t)=0 \text{ at } t=t_1 \tag{3}$$

$$I(t)=S \text{ at } t=0 \tag{4}$$

and

$$I(t)=-R \text{ at } t=T \tag{5}$$

from (1) and using condition (3) we have

$$I(t)=\frac{De^{\theta(t_1-t)}}{\theta}-\frac{D}{\theta} \tag{6}$$

using condition (4) from equation (6) we have

$$S=\frac{D(e^{\theta t_1}-1)}{\theta}$$

Again from equation (2) and using condition (5) we have

$$I(t)=\frac{D}{\delta}\log\left|1+\delta(T-t)\right|-R \tag{7}$$

using condition (3) from equation (7) we get

$$R=\frac{D}{\delta}\log\left|1+\delta(T-t_1)\right|$$

Now holding cost is given by

$$C_{hol}=c_h\int_0^{t_1} I(t)dt \tag{8}$$

Shortage cost is given by

$$C_{sho}=c_b\int_{t_1}^{T}\left[-I(t)\right]dt \tag{9}$$

cost of lost sale is given by

$$OCLS=cls\int_{t_1}^{T}\left\{1-\frac{1}{1+\delta(T-t)}\right\}Ddt \tag{10}$$

Interest earned and interest charged depends upon the length of the cycle and allowable credits time M and N. The flowing three cases arise:

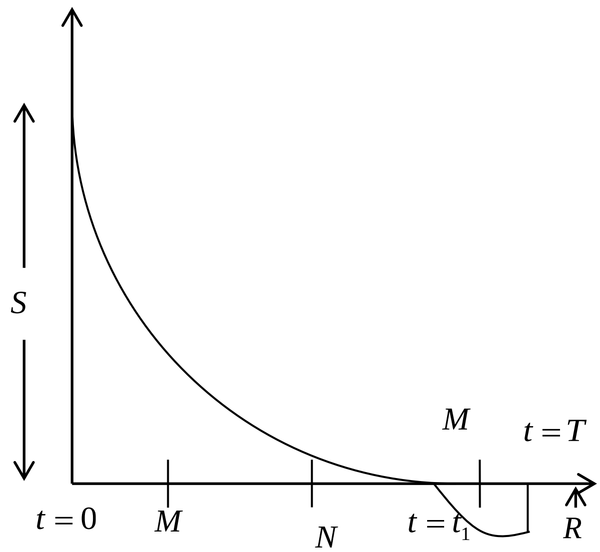

Fig: Inventory model for different credit period

Case 1: $t_1 < M$

Case 2: $M < t_1 < N$

Case 3: $M < N < t_1$

Case 1: In this case there will be no interest charge i.e., $IC1 = 0$, interest earn in this period is given by

$$IE1 = pI_e \left[\int_0^{t_1} tDdt + (M - t_1) \int_0^{t_1} Ddt \right] \tag{11}$$

In this case retailer total cost is given by

Tc=<ordering cost>+<holding cost>+<shortage cost>+<cost of lost sale>+<purchase cost>-<interest earn>.

Therefore

$$X = Co + C_{hol} + C_{sho} + Ocls + cp(S + R) - IE1$$

Therefore, the cost function is given by

$$Z_1(t_1, T) = \frac{X}{T} \tag{12}$$

Case 2: In this case there will be interest charge during the period $[M, t_1]$ i.e., $IC2 = cpI_{p1}\int_{M}^{t_1} I(t)dt$, interest earn in this period $[0, M]$ is given by

$$IE2 = pI_e\left[\int_0^M tDdt\right] \tag{13}$$

In this case retailer total cost is given by

Tc=<ordering cost>+<holding cost>+<shortage cost>+<cost of lost sale>+<purchase cost>+<interest charge>-<interest earn>.

Therefore

$X = Co + C_{hol} + C_{sho} + Ocls + cp(S+R) + IC2 - IE2$

Therefore, the cost function is given by

$$Z_2(t_1, T) = \frac{X}{T} \tag{14}$$

Case 3: In this case there will be interest charge during the period $[M, N]$ with the rate I_{p1} and during the period $[N, t_1]$ with the rate I_{p2} i.e., $IC3 = cpI_{p1}\int_{M}^{N} I(t)dt + cpI_{p2}\int_{N}^{t_1} I(t)dt$, interest earn in this period $[0, M]$ is given by

$$IE3 = pI_e\left[\int_0^M tDdt\right] \tag{15}$$

In this case retailer total cost is given by

Tc=<ordering cost>+<holding cost>+<shortage cost>+<cost of lost sale>+<purchase cost>+<interest charge>-<interest earn>.

Therefore

$$X = Co + C_{hol} + C_{sho} + Ocls + cp(S+R) + IC3 - IE3 \tag{16}$$

Therefore, the cost function is given by

$$Z_3(t_1, T) = \frac{X}{T} \tag{17}$$

4. Numerical Example:

For numerical illustration of the proposed inventory model, we have considered four numerical examples with the values of different parameters shown in **Table 1**.

Table 1: Values of parameters of different examples

Parameters	Example-1	Example-2
C_o	\$100.00	\$150.00
C_h	\$1.00	\$2.00
C_b	\$8.00	\$10.00
C_p	\$10.00	\$14.00
a	200.00	220.00
b	0.6	0.4
θ	0.06	0.06
I_e	\$0.12	\$0.10
I_p	\$0.15	\$0.13
δ	1.50	1.50
M	90/365	90/365
C_{ls}	\$14.00	\$16.00
N	120/365	120/365

The values of the model parameters considered in these numerical examples are not selected from any real life case study, but these values considered here are realistic. For solving the problems of different scenarios corresponding to each example, we have used GRG method. However the optimality cannot be tested analytically. The computational results have been shown in **Table 2**.

Table 2: Computational result for different values of *m*

Cases	Different values of *m*	Z	S	R	t_1	T
Case 1	$m=1.25$	2188.98	109.79	20.39	0.56	0.68
	$m=1.27$	2187.71	109.75	20.38	0.56	0.67
	$m=1.30$	2185.79	109.69	20.38	0.56	0.67
	$m=1.32$	2184.52	109.65	20.37	0.56	0.67
	$m=1.35$	2182.61	109.59	20.36	0.56	0.67
Case 2	$m=1.25$	2190.28	109.15	20.50	0.55	0.67
	$m=1.27$	2189.01	109.11	20.49	0.55	0.67
	$m=1.30$	2187.10	109.05	20.49	0.55	0.67
	$m=1.32$	2185.82	109.02	20.48	0.55	0.67
	$m=1.35$	2183.92	108.95	20.47	0.55	0.67
Case 3	$m=1.25$	2245.31	47.81	25.19	0.24	0.39
	$m=1.27$	2243.71	47.78	25.16	0.24	0.39
	$m=1.30$	2241.31	47.74	25.11	0.24	0.39
	$m=1.32$	2239.72	47.71	25.08	0.24	0.39
	$m=1.35$	2237.39	47.66	25.03	0.24	0.39

From **Table 1**, it is observed that the average cost of the system is minimum for Case 1.

5. Sensitivity Analysis

Sensitivity analyses have been performed for the this model to study the effects of under or over estimation of different system parameters on the optimal policies corresponding to the best found value of the average cost of the system (which is basically the minimum value of the average cost, but the property of minimization cannot be established theoretically). Here the percentage changes are taken as measures of sensitivity. These analyses have been carried out by changing (increasing and decreasing) the parameters by – 20% to + 20%. The computational results have been obtained by changing one parameter at a time and keeping the other parameters at their original values (cf. **Table 3**).

Table 3: Sensitivity analysis with respect to different parameters

Parameter	% changes of parameters	% changes in Average cost	t_1^*	T^*	R^*	S^*
a	-20	-19.51	10.71	10.29	-12.21	-8.74
	-10	-9.74	3.57	4.41	-5.97	-4.17
	10	9.70	-3.57	-4.41	5.76	3.98
	20	19.37	-7.14	-8.82	11.31	7.76
b	-20	0.79	-1.79	-1.47	0.48	0.34
	-10	2.02	-41.07	-32.35	-41.32	14.98
	10	-0.39	0.00	0.00	-0.23	-0.15
	20	-0.79	0.00	0.00	-0.47	-0.29
C_o	-20	-1.42	-8.93	-10.29	-9.22	-12.62
	-10	-0.69	-5.36	-5.88	-4.49	-6.14
	10	0.66	3.57	4.41	4.29	5.99
	20	1.30	8.93	8.82	8.39	11.74
C_h	-20	-0.36	3.57	1.47	3.41	-3.19
	-10	-0.15	0.00	0.00	1.37	-1.33
	10	0.26	-3.57	-2.94	-2.44	2.36
	20	0.46	-5.36	-4.41	-4.21	4.17
C_b	-20	-0.08	-1.79	0.00	-1.49	13.36
	-10	-0.01	-1.79	0.00	-1.01	6.58
	10	0.12	0.00	-1.47	-0.19	-4.81
	20	0.17	0.00	-2.94	0.16	-9.63
C_p	-20	-17.24	8.93	1.47	8.88	-20.68
	-10	-8.56	3.57	0.00	4.29	-11.20
	10	8.43	-3.57	-1.47	-4.07	13.70
	20	16.79	-7.14	-1.47	-8.71	31.58
θ	-20	-0.26	1.79	0.00	9.48	-2.31
	-10	-0.13	0.00	0.00	1.08	-1.13
	10	0.13	-1.79	-1.47	-1.04	1.18
	20	0.25	-3.57	-1.47	-2.06	2.31
δ	-20	-0.08	0.00	0.00	-0.48	7.51
	-10	-0.04	0.00	0.00	-0.24	3.63
	10	0.03	0.00	-1.47	0.23	-3.34
	20	0.07	0.00	-1.47	0.44	-6.48

Form **Table 3**, the observations are self expletory.

6. Concluding Remark

In this paper, we have developed a considering trade credit financing under different situation. This model can be applied in many practical situations.

Highly competitive business situation in marketing policy, suppliers are forced to offer trade credit financing for their retailers to increase the turn over of their business. Sometimes, they allow partial payment instead of full payment at the end of trade credit period. As a result, a retailer is motivated to procure a large quantity of goods at a time. For further research, one can extend the proposed model in several ways. This model can be extended for different types of variable demand dependent on displayed stock-level, time and others. On the other hand, it can also be generalized by considering two level credit policy. The model can be extended in fuzzy and interval environments also.

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