ISSN (Online) : 2454-7190 ISSN 0973-8975
J.Mech.Cont.& Math. Sci., Vol.-11, No.-2, January (2017) Pages 8-22

SIMILARITY SOLUTION OF NATURAL CONVECTIVE BOUNDARY LAYER FLOW AROUND A VERTICAL SLENDER BODY WITH SUCTION AND BLOWING

By

[1]Md. Hasanuzzaman, [2]Akio Miyara

[1]Graduate School of Science and Engineering
[2]Department of Mechanical Engineering
Saga University, 1 Honjomachi, Saga-shi 840-8502, Japan

https://doi.org/10.26782/jmcms.2017.01.00002

Abstract:

In this paper, the similarity solution of natural convective laminar boundary layer flow around a vertical slender body with suction and blowing has been investigated. Firstly, the governing boundary layer partial differential equations have been made dimensionless and then simplified by using Boussinesq approximation. Secondly, similarity transformations are introduced on the basis of detailed analysis in order to transform the simplified coupled partial differential equations into a set a ordinary differential equations. The transformed complete similarity equations are solved numerically by using Fourth order Runge-Kutta method as well as MATLAB. Finally, the flow phenomenon has been characterized with the help of obtained flow controlling parameters such as suction parameter, buoyancy parameter, Prandtl number, body-radius parameter and other driving parameters. The effects of dimensionless parameters on the velocity and temperature distributions are presented graphically. It is found that a small suction or blowing can play a significant role on the patterns of flow and temperature fields.

Keywords: Similarity solution, natural convective; vertical slender body, suction or blowing.

I. Introduction

Natural convection is a mechanism, or type of heat transport, in which the fluid motion is not generated by any external source (like a pump, fan, suction device, etc.) but only by density differences in the fluid occurring due to temperature gradients. In natural convection, fluid surrounding a heat source receives heat, becomes less dense and rises. The surrounding, cooler fluid then moves to replace it. This cooler fluid is then heated and the process continues, forming convection current; this process transfers heat energy from the bottom of the convection cell to top. The driving force for natural convection is buoyancy, a result of differences in fluid density.

Natural convection has attracted a great deal of attention from researchers because of its presence both in nature and engineering applications. In nature convection cells formed from air raising above sunlight-warmed land or water are a major feature of all weather systems. Convection is also seen in the rising plume of hot air from fire, oceanic currents, and sea-wind formation. In engineering applications, convection is commonly visualized in the formation of microstructures during the cooling of molten metals, and fluid flows around shrouded heat-dissipation fins, and solar ponds. A very common industrial application of natural convection is free air cooling without the aid of fans: this can happen on small scales to large scale process equipment.

The problem of free, mixed and forced convection over a horizontal porous plate has been attracted the interest of many investigators (Viz. Clark and Riley [1], Schneider [2] and Merkin and Ingham [3] among several others) in view of its applications in many engineering and geophysical problems. Ramanaiah *et al*. [4] considered the problem of mixed convection over a horizontal plate subjected to a temperature or surface heat flux varying as a power of *x*. However, the problem of forced, free and mixed convection flows past a heated or cooled body with porous wall is of interest in relation to the boundary layer control on airfoil, lubrication of ceramic machine parts and food processing. Deswita *et al.* [5] obtained a similarity solution for the steady laminar free convection boundary layer flow on a horizontal plate with variable wall temperature. Hossain and Mojumder [6] presented the similarity solution for the steady laminar free convection boundary layer flow generated above a heated horizontal rectangular surface. Furthermore, they study of complete similarity solutions of the unsteady laminar natural convection boundary layer flow above a heated horizontal semi-infinite porous plate have been considered by Hossain et al [7]. Hossain *et al.* [8] obtained a complete similarity solution of the unsteady laminar combined free and forced convection boundary layer flow about a heated vertical porous plate in viscous incompressible fluid and investigated the effects of several involved parameters on the velocity and temperature fields and other flow parameters like skin friction, heat transfer coefficients across the boundary layer. Van Dyke [9] successfully analyzed a natural convection flow near a vertical thin needle for the case of a constant surface temperature. Kuiken [10] has studied the axi-symmetric free convective boundary layer along an isothermal vertical cylinder of constant thickness.

The purpose of the present study is, therefore, to find a possible similarity solution of unsteady natural convection laminar boundary layer flow of viscous incompressible fluid caused by a heated (or cooled) axi-symmetric slender body of finite axial length immersed vertically in a viscous incompressible fluid. The thermal distributions on the outer surface of the body as well as the motion of the body itself are assumed to be unsteady. Furthermore, throughout the investigation, the effect of suction or blowing has been taken into consideration. We are attempted to investigate the effects of several involved parameters on the velocity and temperature fields and other flow parameters like skin friction and heat transfer coefficients across the boundary layer. The numerical results including the velocity and temperature fields are to be presented graphically for different selected values of the established dimensionless parameters. It is expected that the effects of suction and blowing play an important role on the velocity and temperature fields, so that their effects should be taken into account with other useful parameters associated.

II. Basic equations of the flow and mathematical analysis

An axi-symmetric heated (or cooled) slender body of finite axial length is immersed vertically in a viscous fluid of variable properties. The surface temperature ($=T_w$), the velocity and the temperature of the undisturbed fluid (u_e and T_e) close to the body surface but outside the boundary-layer are all general functions of x and t. r_w is the radial distance from the axis of symmetry to the surface of the body, x is the distance measured along the axis of symmetry of the body and t is the time. The physical configuration and the coordinate system of the problem are shown in Fig.1.

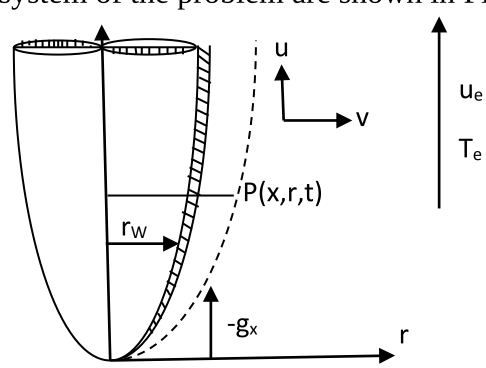


Fig: 1 physical configuration and coordinate system

The influence of body force generated by buoyancy effects on the flow field near the surface is significant if the Froude number of such a flow field is of order unity. That is the non dimensional form of the buoyancy force is

$$\frac{T_w - T_e}{T_e}\frac{g_x L_c}{U^2} \cong 0(1) \tag{1}$$

where g_x is the gravity component in the x-direction, L_c is a suitable characteristic length and U is a suitable characteristic velocity. The non-dimensional form of the equations expressing conservation of mass, momentum and energy for a Boussinesq fluid are follows:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv)=0 \tag{2}$$

$$\frac{Du}{Dt} = -g_x \beta_T \Delta T\theta + \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} + \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{3}$$

$$\frac{D\theta}{Dt} = -\left\{u\frac{\partial}{\partial x}(\log \Delta T) + \frac{\partial}{\partial t}(\log \Delta T)\right\}\theta + \frac{1}{P_r}\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) \tag{4}$$

where ρ is the density, μ dynamic coefficient of viscosity and ν kinematic coefficient of viscosity , T the temperature, k the thermal conductivity C_p the specific heat at constant pressure and $\beta_T = -\rho^{-1}\left(\frac{\partial \rho}{aT}\right)_P$ the coefficient of cubical expansion of the fluid and $\Pr = \frac{\mu C_p}{k}$ the Prandtl number . Here $T - T_0 = \Delta T_0$, $\Delta T = T_W - T_0$ and $T_e = T_0$ is treated here as constant temperature for the ambient fluid. Since the Boussinesq from of the state relations is $\rho = \rho(T)$, it follows that $\rho_e = \rho_0$ (constant). T and T_W in general depend on both x and t. A solution of the equations (2)-(4) is now sought, these equations being valid in the limit $Re \rightarrow \infty$ and $E \rightarrow 0$. Higher order effects are not discussed here, as the present investigation concerns the first order boundary- layer approximations only.

III. Similarity solutions

III.i Similarity transformations:

The complexity of the above governing differential equations makes the use of simplifying approximations desirable so that tractable solutions may be obtained. The method of similarity provides a convenient and accurate procedure for computing heat transfer, skin friction and other laminar boundary-layer characteristics, Guided by this idea, the independent variables (*x,r,t*) are changed to a new set of variables $(\xi, \phi\ , \tau)$ where the relations between the two sets are

$$\xi = x,\ \ \tau = t,\ \ \phi = \frac{r^2}{2\gamma(x,\, t)} \tag{5}$$

The continuity equation (2) is identically satisfied by introducing a stream function $\psi(x,\, r,\, t)$ defined by $ru = \frac{\partial \psi}{\partial r}\ , -rv = \frac{\partial \psi}{\partial x}$, we have , $u = 2\frac{\partial \psi}{\partial (r^2)} = \frac{\partial}{\partial \phi}\left\{\frac{\psi}{\gamma(x,t)}\right\}$. Using for the moment, a non-dimensional factor *U* (*x, t*) for the velocity component *u* is given by $\psi(\xi,\, \phi,\, \tau) = \gamma\, U\, F(\xi, \phi, \tau) + \psi(\xi, \phi_0,\, \tau)$

where $F(\xi, \phi,\, \tau) = \int_{\phi_0}^{\phi} \frac{u}{U} d\phi$ *and* ϕ_0 is the value of ϕ on the body. That is $\phi_0 = \frac{r_W^2}{2\gamma(x\ ,\ t))}$. The velocity components *u* and *v* are found to be *u*=*UL*, $-rv = (\gamma U\, F)_\xi - \phi\,\gamma_\xi\, U\, F_\phi - r_W v_W\,$, where $-r_W v_W = \psi_\xi(\xi, \phi_0, \tau)$, where v_W represents the non-zero wall velocity called suction or blowing velocity normal to the porous surface, so that fluid can either be sucked or blown throughout. Physically, $v_W < 0$ and $v_W > 0$ represent the suction and blowing velocity through the porous surface respectively. For uniform suction (or blowing) v_W = constant. However, $v_W \neq 0$ implies that the surface is impermeable to the fluid. In view of the above transformation, equations (2) to (4) become

$$2\nu\{(\phi+a_3)F_{\phi\phi}\}_{\phi}+\{a_0(\phi+a_3)+a_9\}F_{\phi\phi}+(a_1+a_2)F\,F_{\phi\phi}-a_2F_{\phi}^2-a_4F_{\phi}+a_5\vartheta+a_6=0 \quad (6)$$

$$\frac{2\nu}{P_r}\{(\phi+a_3)\vartheta_{\phi}\}_{\phi}+\{a_0(\phi+a_3)+a_9\}\vartheta_{\phi}+(a_1+a_2)F\vartheta_{\phi}-(a_7+a_8F_{\phi})\vartheta=0 \quad (7)$$

where

$$(i)\ \gamma_{\tau}=a_0,\ (ii)\ (\gamma UL)_{\xi}=\gamma_{\xi}UL+\gamma(UL)_{\xi}=a_1+a_2,(iii)\ \frac{r_W^2}{2\gamma}=a_3,\ (iv)\ \frac{\gamma(UL)_{\tau}}{UL}=a_4,(v)\ \gamma(UL)_{\xi}=a_2,$$

$$(vi)\ \gamma_{\xi}(UL)=a_1,(vii)\ -\frac{\nu}{UL}g_x\beta_T\Delta T=a_5,\ (viii)\ \frac{\nu}{UL}\{(u_e)_{\tau}+u_e(u_e)_{\xi}\}=a_6,\ (ix)\ \nu(\log\Delta T)_{\tau}=a_7,$$

$$(x)\ \nu UL\{\log\Delta T\}_{\xi}=a_8,\ (xi)-r_WV_W=a_9\ \text{ and }\ \phi=\frac{r^2-r_W^2}{2\gamma(x,t)} \quad (8)$$

The boundary conditions which are imposed in order to determine the solutions of the transformed boundary layer equations (6)-(7) are given by: $F(0)=F_{\phi}(0)=0$

$$F_{\phi}(\infty)=\vartheta(0)=1,\ \vartheta(\infty)=0 \quad (9)$$

The relations in equation (8) furnish us with the conditions under which similarity solutions are obtained provided that all $a's$ must be constants and thus the equations (6)-(7) will become non-linear ordinary differential equations. In view of the conditions (ii) and (i) stated in equation (8), we have $\gamma=a_0\tau+B(\xi)$ and $\gamma UL=(a_1+a_2)\xi+A(\tau)$

where $A(\tau)$ is either a function of τ or constant and $B(\xi)$ is a function of ξ or constant. From the above two relations we obtain $\frac{dA(\tau)}{d\tau}.\frac{dB(\xi)}{d\xi}=a_1(a_0+a_4)$ (10)

Since $\gamma(\xi,\tau)$ and $UL(\xi,\tau)$ depend wholly on the choice of $A(\tau)$ and $B(\xi)$, the equation (10) plays a significant role in determining the possible four cases of similarity solutions:

(A) $\frac{dA(\tau)}{d\tau}=0$ but $\frac{dB(\xi)}{d\xi}\neq 0$,

(B) Both $\frac{dA(\tau)}{d\tau}$ and $\frac{dB(\xi)}{d\xi}$ are zero.

(C) Both $\dfrac{dA(\tau)}{d\tau}$ and $\dfrac{dB(\xi)}{d\xi}$ are finite constants,

(D) $\dfrac{dA(\tau)}{d\tau} \neq 0$ but $\dfrac{dB(\xi)}{d\xi} = 0$

III.ii. Similarity case to be considered

Of these four similarity cases, only the Case (A) for which $\dfrac{dA(\tau)}{d\tau} = 0$ but $\dfrac{dB(\xi)}{d\xi} \neq 0$, has been studied here. Thus we have $\gamma = K_1 \{(a_1 + a_2)\xi + A\}^{\frac{a_1}{a_1+a_2}}$ and $UL = \dfrac{1}{K_1}\{(a_1 + a_2)\xi + A\}^{\frac{a_2}{a_1+a_2}}$. Substituting these in the conditions (i) to (ix) of the equation (8) yields the relations between the constants as follows: $a_0 = 0$, a_1, a_2 arbitrary, $a_3 = r_W^2 \Big/ 2K_1\{(a_1 + a_2)\xi + A\}^{\frac{a_1}{a_1+a_2}}$, $a_4 = 0$, $a_5 = -g_x \beta_T \Delta T K_1^2 \{(a_1 + a_2)\xi + A\}^{\frac{a_1-a_2}{a_1+a_2}}$, $a_6 = a_2$, $a_7 = 0$, $a_8 = a_2 - a_1$ and a_9 is arbitrary.

Here UL should be considered as the non-dimension characteristic velocity. Without loss of generality we may write $UL = u_e$. For purely natural convection flow we can put $U_F = u_e$ therefore $U_F^2\, \vartheta / u_e^2 = 1$ which yields $a_6 = 0$

Substituting the constants and choosing $F = \alpha_1 f$ and $\phi = \alpha_1 \eta$ the above equations (6) to (7) reduce to:

$$2(\eta + R_0) f_{\eta\eta\eta} + (2 + F_W + f) f_{\eta\eta} - \beta f_\eta^2 + \vartheta = 0 \tag{11}$$

$$2(\eta + R_0)\vartheta_{\eta\eta} + \mathrm{P_r}\left[\left(\frac{2}{\mathrm{P_r}} F_W + f\right)\vartheta_\eta + (1 - 2\beta) f_\eta \vartheta\right] = 0 \tag{12}$$

The boundary conditions are $f(0) = f_\eta(0) = 0$, $f_\eta(\infty) = 1$, $\vartheta(0) = 1$, $\vartheta(\infty) = 0$ (13)

where is also chosen $\alpha_1 = \alpha_2$, $\dfrac{a_1 + a_2}{\nu}\alpha_1 = 1$, $\dfrac{r_W^2\, u_e}{2\nu(\xi + \xi_0)} = R_0$, $\dfrac{a_2}{a_1 + a_2} = \beta$. For this situation (natural convection flows) similarity solutions exist only when the natures of ΔT and r_W are

(i) $\Delta T \propto x^{2\beta-1}$ (ii) $r_W \propto x^{\frac{1-\beta}{2}}$

The body shear stress and heat transfer rate are given by

$$\lambda_w = \sqrt{2R_0}\ \frac{\mu_r \nu}{(x+x_0)^2}\left[-\frac{g_x \beta_T \Delta T (x+x_0)^3}{\nu^2}\right]^{\frac{3}{4}} f_{\eta\eta}(0)$$

$$q_w = -\sqrt{2R_0}\ \frac{k_W \Delta T}{(x+x_0)}\left[-\frac{g_x \beta_T \Delta T (x+x_0)^3}{\nu^2}\right]^{\frac{1}{4}} \vartheta_\eta(0).$$

IV. Numerical solutions and discussions

The set of differential equations (11)–(12) with the boundary conditions (13) are solved numerically by using MATLAB. Like Case A, here the velocity f_η and temperature ϑ are determined as a function of coordinate η. The skin friction coefficient $f_{\eta\eta}(0)$ and the heat transfer rate $-\vartheta_\eta(0)$ are also evaluated for this case and numerical results thus obtained in terms of the similarity variables are displayed in graphs and tables for several selected values of the established parameters F_w, β, R_0 and Pr below. To obtain the solution of differential equations (11) – (12) with the boundary conditions (13), we have adopted a numerical procedure based on MATLAB. The effects of suction parameter F_w, driving parameter β (the ratio between the changes of local boundary-layer thickness with regard to position and time), the body radius parameter R_0 and the Prandtl number Pr are plotted in the Figure 2 –9. To observe the effect of F_w, the other four parameters β, R_0 and Pr are taken constants. Similarly, we observe the effect of the parameters β, R_0 and Pr by taking the rest four parameters constant respectively.

The effects of F_w on the velocity and temperature fields are plotted in the Fig. 2 and Fig. 3 respectively. From Fig. 2 it is observed that, in all cases the velocity is starting at zero, and then velocity increases with the increase of η near the leading edge and finally moves towards 1.0 asymptotically but temperature is starting at 1.0, then it decreases asymptotically and finally leads to zero with the increase of η. From Fig. 2 we see that for the case of suction $(F_W > 0)$, the velocity increases with increasing F_W

but for blowing case $(F_W < 0)$, velocity decreases in the region $0.7 \le \eta \le 1.8$ and then the velocity increases with the increase of the magnitude of blowing. The usual stabilizing effect of the suction parameter on the boundary layer growth is also evident from this figure.

From Fig. 3 it is observed that for both the cases of suction and blowing, temperature decreases quickly close to the leading edge and away from it temperature decreases asymptotically and finally leads to zero with the increase of η. For the case of suction $(F_W > 0)$, temperature decreases with increasing suction. But for the case of blowing $(F_W < 0)$, temperature decreases more with the increase of the magnitude of blowing.

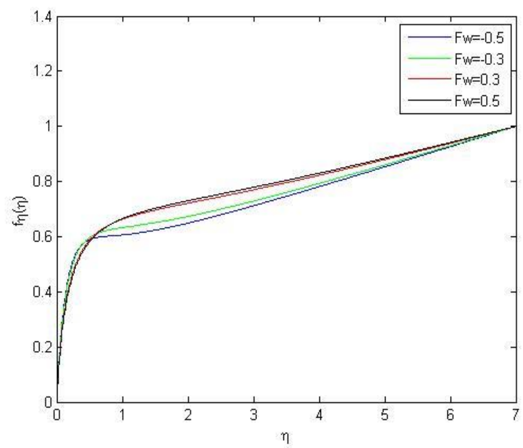


Fig. 2

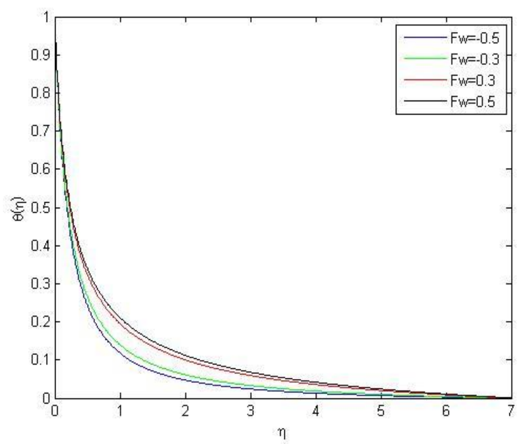


Fig. 3

Fig 2: Velocity profiles and Fig: 3 temperature profiles for different values of F_w (with fixed values of $\beta = 0.5,\ R_0 = 0.1$ and $\Pr = 0.71$).

The body radius parameter R_0 depends on the shape of the slender body. The velocity and temperature fields exhibit remarkable changes with the variation of R_0 as observed from Fig. 4 and Fig. 5. It is observed from Fig. 4 that in all cases the velocity is starting at zero and increasing asymptotically to 1.0. But with the increase in R_0 the rate of change of velocity increases primarily. Thus before being asymptotically 1.0 for higher values of R_0 velocity is higher within the boundary layer. It is observed from Fig. 5 that in all cases the temperature is starting at 1.0 and decreasing asymptotically to zero. But with the increase in R_0 the rate of change of temperature decreases primarily. Thus before being asymptotically zero for higher values of R_0 temperature is lower within the boundary layer.

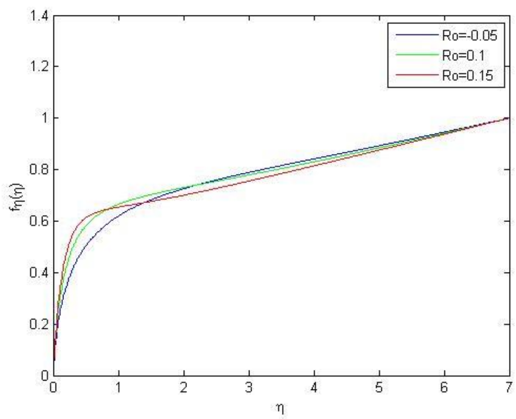


Fig. 4

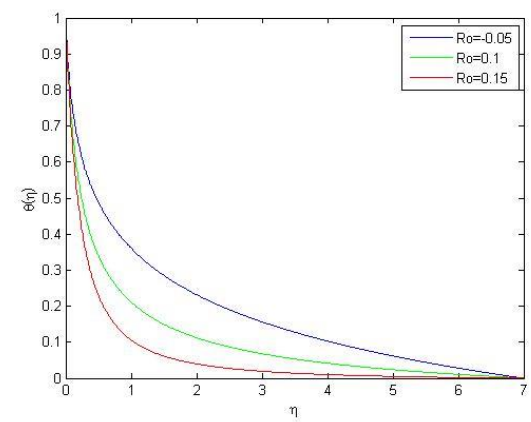


Fig. 5

Fig. 4 Velocity profiles and Fig 5 temperature profiles for different values of R_0 (with fixed values of $F_w = .0.5$, $\beta = 0.5$ and $\mathrm{Pr} = 0.71$)

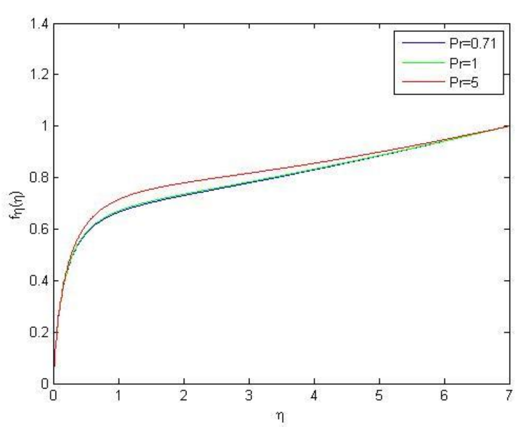


Fig. 6

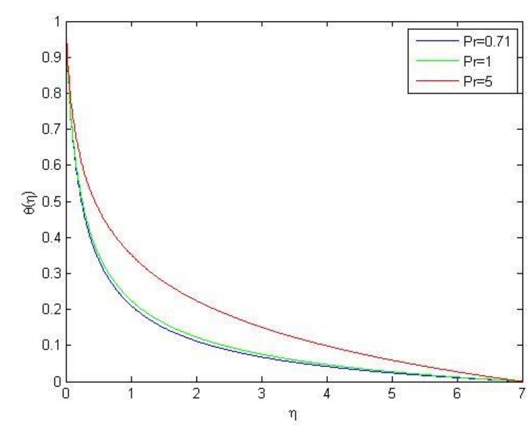
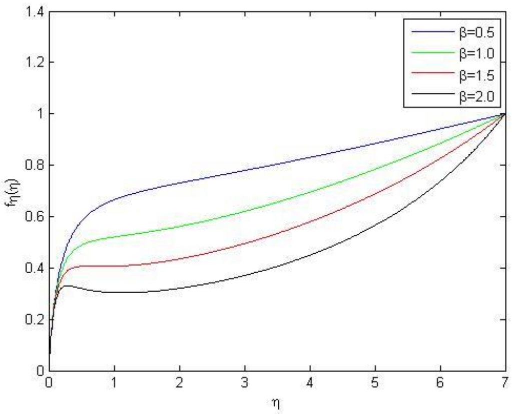


Fig. 7

Fig. 6 Velocity profiles and 7 temperature profiles for different values of Pr (with fixed values of $F_w = 0.5$ $\beta = 0.5$, and $R_0 = 0.1$)

The other controlling parameter is the Prandtl number $\Pr\left(=\dfrac{\mu C_p}{k}\right)$ which depends on the properties of the medium. The velocity and temperature fields exhibit considerable changes with the variation of Pr as observed from Fig. 6 and Fig. 7. It is observed from Fig. 6 that in all cases the velocity is starting at zero and increasing asymptotically to 1.0. But with the increase in Pr the rate of change of velocity increases primarily. Thus before being asymptotically 1.0 for higher values of Pr velocity is higher within the boundary layer. It is observed from Fig. 7 that in all cases the temperature is starting at 1.0 and decreasing asymptotically to zero. But with the decrease in Pr the rate of change of temperature decreases primarily. Thus before being asymptotically zero for lower values of Pr temperature is lower within the boundary layer.

Fig. 8 and Fig. 9 exhibit the effects of the driving parameter β on the velocity and temperature fields, respectively. The velocity and temperature fields exhibit remarkable changes with the variation of β as observed from Fig. 8 and Fig. 9. It is observed from Fig. 8 that for all cases the velocity is starting at zero and increasing asymptotically to 1.0. But with the increase in β the rate of change of velocity increases primarily. After $\eta > 0.3$ the velocity increase for $\beta = 0.5$ and finally leads to 1.0 asymptotically. Then increase β the velocity decrease and after certain stage the velocity increase leads to 1.0 asymptotically. Like before, temperature decreases faster with the increasing β and finally leads to zero asymptotically as is seen in Fig. 9.



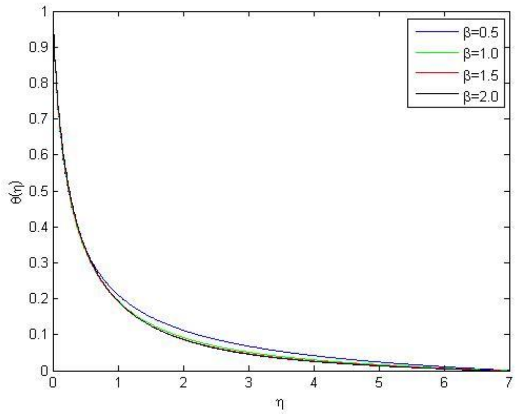


Fig. 8 Fig. 9

Fig. 8: Velocity profiles and Fig. 9: temperature profiles for different values of β (with fixed values of $F_w = 0.5$, $\Pr = 0.71$, and $R_0 = 0.1$)

The values proportional to the coefficients of skin friction $f^{//}(0)$ and heat transfer –$\vartheta^{/}(0)$ are tabulated in Table (1) – (4).

Table 1: Values proportional to the coefficients of skin-friction ($f^{//}(0)$) and heat transfer ($-\vartheta^{/}(0)$) with the variation of suction parameter F_w (for fixed $R_0 = 0.1$, $\beta = 0.5$ and $\Pr = 0.71$)

F_w	$f^{//}(0)$	$-\vartheta^{/}(0)$
0.50	0.769883	-0.68053
0.30	0.745995	-0.67369
-0.30	0.672158	-0.65275
-0.50	0.647313	-0.64597

Table 2: Values proportional to the coefficients of skin-friction ($f^{//}(0)$) and heat transfer ($-\vartheta^{/}(0)$) with the variation of body radius parameter R_0 (for fixed $F_w = 0.5$, $\beta = 0.5$ and $\Pr = 0.71$)

R_0	$f^{//}(0)$	$-\vartheta^{/}(0)$
0.05	0.789601	-0.63751
0.10	0.769883	-0.68053
0.15	0.730051	-0.66007

Table 3: Values proportional to the coefficients of skin-friction ($f^{//}(0)$) and heat transfer ($-\vartheta^{/}(0)$)with the variation of Prandtl number $\Pr$ (for fixed $F_w = 0.5$, $\beta = 0.5$ and $R_0 = 0.1$)

Pr	$f^{//}(0)$	$-\vartheta^{/}(0)$
0.71	0.769883	-0.68053
1.00	0.606899	-0.55108
7.00	0.139052	-0.21924

Table 4: Values proportional to the coefficients of skin-friction ($f^{//}(0)$) and heat transfer ($-\vartheta^{/}(0)$) with the variation of driving parameter β (for fixed $F_w = 0.5$, $\Pr = 0.71$ and $R_0 = 0.1$)

β	$f^{//}(0)$	$-\vartheta^{/}(0)$
0.5	0.769883	-0.68053
1.0	0.749632	-0.67015
1.5	0.730051	-0.66007
2.0	0.711108	-0.65028

V. Conclusions

Similarity solution of natural convective laminar boundary layer flow around a vertical slender body with suction or blowing with the similarity case $\frac{dA(\tau)}{d\tau} = 0$ but $\frac{dB(\xi)}{d\xi} \neq 0$ has been studied in this paper. It is observed that this case is very case sensitive relative to the values of controlling parameters and no symmetric relationships of controlling parameters on the flow variables are observed here. On the basis of the findings the following conclusions can be drawn:

(a) For the case of suction $(F_W > 0)$, the velocity increases with increasing F_W but for blowing case $(F_W < 0)$, velocity decreases in the region $0.7 \leq \eta \leq 1.8$ and then the velocity increases with the increase of the magnitude of blowing. For the case of suction $(F_W > 0)$, temperature decreases with increasing suction. But for the case of blowing $\left(F_W < 0\right)$, temperature decreases more with the increase of the magnitude of blowing.

(b) With the increase in R_0 the rate of change of velocity increases primarily. Thus before being asymptotically 1.0 for higher values of R_0 velocity is higher within the boundary layer. But with the increase in R_0 the rate of change of temperature decreases primarily. Thus before being asymptotically zero for higher values of R_0 temperature is lower within the boundary layer.

(c) With the increase in $\Pr$ the rate of change of velocity increases primarily. Thus before being asymptotically 1.0 for higher values of $\Pr$ velocity is higher within the boundary layer. It is observed from. But with the decrease in $\Pr$ the rate of change of temperature decreases primarily. Thus before being asymptotically zero for lower values of $\Pr$ temperature is lower within the boundary layer.

(d) But with the increase in β the rate of change of velocity increases primarily. After $\eta > 0.3$ the velocity increase for $\beta = 0.5$ and finally leads to 1.0 asymptotically. Then increase β the velocity decrease and after certain stage the velocity increase leads to 1.0 asymptotically. Like before, temperature decreases faster with the increasing β and finally leads to zero asymptotically.

References

1) Clarke, J. F. Riley, N. : Natural convection induced in a gas by the presence of a hot porous horizontal surface, Q. J. Mech. Appl. Math., 28, 373–396, 1975.

2) Schneider, W. : A similarity solution for combined forced and free convection flow over a horizontal plate, Int. J. Heat Mass Transfer, 22, 1401–1406, 1979.

3) Merkin, J. H. Ingham, D. B.: Mixed convection similarity solutions on a horizontal surface, ZAMP, 38, 102–116, 1987.

4) Ramanaiah, G. Malarvizhi, G. and Merkin, J. H.: A unified treatment of mixed convection on a permeable horizontal plate, Wäfirme-und Stoffübertragung, 26, 187–192, 1991.

5) Deswita, L. Nazar, R. Ahmad R. Ishak, A and Pop, I.: Similarity Solutions of Free Convection Boundary Layer Flow on a Horizontal Plate with Variable Wall Temperature, European Journal of Scientific Research, 27(2), 188–198, 2009.

6) Hossain, M. M. T. and Mojumder, R. : Similarity solution for the steady natural convection boundary layer flow and heat transfer above a heated horizontal surface with transpiration, Int. J. Appl. Math. Mech., 6(4), 1–16, 2010.

7) Hossain, M. M. T. Mojumder, R. and Hossain, M. A. : Solution of natural convection boundary layer flow above a semi-infinite porous horizontal plate under similarity transformations with suction and blowing, Daffodil International University Journal of Science and Technology, 6(1), 43–51, 2011.

8) Hossain, M. M. T. Mandal, B. and Hoossain, M. A. : Similarity Solution of Unsteady Combined Free and Force Convective Laminar Boundary Layer Flow about a Vertical Porous Surface with Suction and Blowing, Procedia Engineering, 56,134–140, 2013.

9) Van Dyke, M. : Free convection from a vertical needle, Problems of Hydrodynamics and Continuum Mechanics (SIAM Publication, Philadelphia, Penn.), 1969.

10) Kuiken, H. K. : The thick free-convective boundary layer along a semi-infinite isothermal vertical cylinder, ZAMP, 25, 497, 1974.