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TWO DIMENSIONAL WAVE PROPAGATION IN A HIGHER ORDER VISCOELASTIC PLATE UNDER THE INFLUENCE OF INITIAL STRESS AND MAGNETIC FIELD

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Abstract

The aim of the present paper is to investigate the propagation of waves in a magneto-visco-elastic initially stressed electrically conducting plate of finite thickness involving time rate of strain and stress of higher order. The initial stress is assumed to be of the nature of hydrostatic tension or compression. The normal mode analysis is used to obtain the wave velocity equations for the waves propagated in the plate bounded by stress free plane boundaries. The wave velocity equations in different cases, obtained in this paper may be considered as more general in the sense that the results presented by other authors may be obtained as special cases in the absence of additional fields. Numerical computations are carried out and the effects of higher order viscoelasticity, magnetic field and initial stress on the phase velocity ratio are exhibited graphically.

সংক্ষিপ্তসার

উচ্চতর ক্রমের পীড়ণ ও বিকৃতির সময় - হারকে যুক্ত করে চৌম্বক -সান্দ্র-স্থিতিস্থাপক প্রারম্ভিক পীড়ণযুক্ত বিদ্যুৎ পরিবহণক্ষম সসীম পুরু প্লেটে তরঙ্গ প্রবাহকে অনুসন্ধান করা এই গবেষণা পত্রের উদ্দেশ্য। পীড়ণযুক্ত সমতলবেষ্টনীর দ্বারা আবদ্ধ প্লেটে প্রবাহিত তরঙ্গের তরঙ্গ গতিবেগ সমীকরণ নির্ণয় করার জন্য আদর্শ গণগরিষ্ঠমান বিশ্লেষণের সাহায্য নেওয়া হয়েছে। অন্যান্য লেখকেরা অতিরিক্ত ক্ষেত্রের অনুপস্থিতিতে বিশেষ ক্ষেত্র হিসাবে যে ফল প্রকাশ করেছেন তার থেকে আরও বেশী সাধারণ হিসাবে বিভিন্ন ক্ষেত্রের জন্য এই তরঙ্গ গতিবেগ সমীকরণটিকে নির্ণয় করা হয়েছে বলে বিবেচনা করাযেতে পারে। ইহাদের সাংখ্যমান গণনা করা হয়েছে এবং দশা গতিবেগ অনুপাতের উপর উচ্চতর ক্রমের সান্দ্র - স্থিতিস্থাপকতা, চৌম্বকক্ষেত্র এবং প্রারম্ভিক পীড়নের কার্যকারিতা লেখচিত্রের আকারে প্রকাশ করা হয়েছে।

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1. Introduction.

In the last few decades the interaction between strain and electromagnetic fields has been receiving greater attention from many investigators [1-7] owing to its applications to geophysical problems, certain topics in optics and acoustics and various branches of engineering sciences. Moreover, the earth is placed in its own magnetic field and the material medium of the Earth may be viscous in nature in some places. Again the earth is also an initially stressed layered structure where initial stresses exist due to variation of temperature, weight of the matter on it, overburden layer, slow process of creep, gravitation and largeness etc. It is generally supposed for simplicity that this initial equilibrium state of stress is approximately of hydrostatic nature.

The seismic signals propagating through the earth medium have to travel through such initially stressed magneto viscoelastic plates and hence it is doubtless to say that the properties of these layered materials do affect the propagation of the seismic waves. The subject of propagation of elastic waves through the plates is very important and researchers have investigated several problems [8-14] by assuming different models. Dilatational and rotational waves in a magneto elastic initially stressed conducting medium have been thoroughly studied by Yu and Tang [15]. Magneto-Elastic waves and the disturbances in initially stressed conducting media have been investigated by De and Sengupta [16]. Effect of viscosity of the material on the propagation of waves has been studied by Roy Choudhuri and Banerjee [17], Addy and Chakraborty [18], Song et al [19], Sharma and Othman [20], Yin-feng and Zhong-min [21], Rakshit and Mukhopadhyay [22]. However in all the previous papers so far published it is seen that the combined effect of magnetic field, initial stress and viscous nature of the material medium in the form of a plate involving strain rate and stress rate have not been considered in details. Keeping in view of such type of geophysical situation the authors investigate the combined effect of magnetic field, initial

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stress in the nature of hydrostatic tension or compression and the viscous nature of the material medium on the propagation of waves in an infinitely extended thick plate bounded by horizontal planes. Following Voigt [23] higher order viscoelastic model with stress rate and strain rate has been used in this problem. Starting from the field equations presented by Yu and Tang we have obtained the wave velocity equations to determine the phase velocity of waves propagated in the general higher order viscoelastic initially stressed plate in presence of a constant magnetic field parallel to x_1-axis. Two interesting special cases have been deduced. Numerical calculation and graphical representation have been made to highlight the effect of magnetic field, initial stress and order of viscoelasticity on the propagation of waves through a plate.

The governing equations

For electrically conducting charge free elastic solid under an initial stress permeated by an electromagnetic field, the equation of motion when the body is subjected to small perturbation, may be expressed as [15]

$$\begin{aligned} \rho\frac{\partial^2 u_i}{\partial t^2} &= -p_0\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \mu_e H_0\left(\frac{\partial \tilde{H}_i}{\partial x_1} - \frac{\partial \tilde{H}_1}{\partial x_i}\right) + \frac{\partial \tilde{\tau}_{ij}}{\partial x_j} \\ \tilde{H}_i &= H_0\left(\frac{\partial u_i}{\partial x_1} - \frac{\partial u_1}{\partial x_i}\right) \end{aligned} \tag{1}$$

where $\tilde{\tau}_{ij}$ are the components of incremental stress tensor, u_i are the components of displacement vector $\vec{u}$ with respect to the coordinates x_1, x_2, x_3 and time t, ρ is the mass density, μ_e is magnetic permeability, $p_0(<0)$ is the tension and $p_0(>0)$ denotes the compression. H_0 is the intensity of the initial constant magnetic field parallel to the x_1- axis and i, j=1, 2, 3.

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Let us consider a homogeneous, isotropic general higher order viscoelastic infinitely extended thick flat plate of finite thickness 2H occupying the region Ω given by

$$\Omega = \{(x_1, x_2, x_3) | -\infty < x_1 < \infty, -\infty < x_2 < \infty, -H \le x_3 \le H\}$$

with the middle plane surface of the plate coinciding with the plane $x_3 = 0$. The plate which is under an initial hydrostatic state of stress is permeated by a constant magnetic field parallel to the x_1-axis. The material of the plate will not continue to remain homogeneous and isotropic when it is subject to initial stress and magnetic field. Here, in the present investigation, we ignore such variations. Introduce a set of orthogonal Cartesian coordinate axes $ox_1x_2x_3$, the origin o being any point on the middle plane and x_3-axis being a line drawn vertically downwards. We consider only a two dimensional problem. We also assume that all causes producing the wave propagation are independent of variable x_2 and waves are propagated only in the x_1x_3-plane. Thus all functions appearing in the field equations are independent of variable x_2 and the displacement vector has components $(u_1(x_1, x_3, t), 0, u_3(x_1, x_3, t))$ (plane strain problem).

From the nature of the problem the non-zero displacement components u_1 and u_3 at any point may be expressed as

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1} \tag{2}$$

where ϕ and ψ are displacement potentials which are functions of the co-ordinates x_1, x_3 and t and

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$$\nabla^2\phi = \Delta\,,\ \nabla^2\psi = \frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3}\,,\ \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}\,,\ \Delta = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \tag{3}$$

For a homogeneous isotropic higher order visco-elastic solid plate including both strain rate and stress rate, the stress-strain relations may be presented as [23, 24]

$$D_\eta \bar{\tau}_{ij} = D_\lambda \Delta \delta_{ij} + 2D_\mu e_{ij} \tag{4}$$

where

$$D_\eta = \sum_{k=0}^{n} \eta_k \frac{\partial^k}{\partial t^k},\ D_\lambda = \sum_{k=0}^{n} \lambda_k \frac{\partial^k}{\partial t^k},\ D_\mu = \sum_{k=0}^{n} \mu_k \frac{\partial^k}{\partial t^k}$$

in which η_0, λ_0 and μ_0 are the elastic parameters and η_k, λ_k and μ_k (k=1,2,......n) are parameters associated with k-th order viscoelasticity, $e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \delta_{ij}$ and Δ are the strain tensor, Kronecker delta and dilatation respectively.

Using (4) in (1) one obtains the displacement equations of motion in the generalized visco-elastic medium involving stress rate and strain rate under the hydrostatic tension or compression in presence of a constant initial magnetic field as

$$\begin{aligned} \rho D_\eta \frac{\partial^2 u_1}{\partial t^2} &= \left(D_\lambda + D_\mu\right)\frac{\partial \Delta}{\partial x_1} + D_\mu \nabla^2 u_1 - D_\eta p_0 \nabla^2 u_1 \\ \rho D_\eta \frac{\partial^2 u_3}{\partial t^2} &= \left(D_\lambda + D_\mu\right)\frac{\partial \Delta}{\partial x_3} + D_\mu \nabla^2 u_3 - p_0 D_\eta \nabla^2 u_3 + \mu_e H_0^2 D_\eta \left(\frac{\partial^2 u_3}{\partial x_1^2} - \frac{\partial^2 u_1}{\partial x_1 \partial x_3}\right) \end{aligned} \tag{5}$$

The equations of motion (5) in view of (2) and (3) yield the following wave equations

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$$\frac{\partial^2\phi}{\partial t^2}=\left(D_T-\frac{p_0}{\rho}\right)\nabla^2\phi+c_A^2\frac{\partial^2\psi}{\partial x_1\partial x_3}$$
$$\frac{\partial^2\psi}{\partial t^2}=\left(D_S-\frac{p_0}{\rho}\right)\nabla^2\psi+c_A^2\frac{\partial^2\psi}{\partial x_1^2} \tag{6}$$

in which

$$D_T=\frac{D_\lambda+2D_\mu}{\rho D_\eta},\ D_S=\frac{D_\mu}{\rho D_\eta},\ c_A=\sqrt{\frac{\mu_e H_0^2}{\rho}}=\text{Alfven velocity}.$$

Boundary conditions.

Since the plane $x_3=\pm H$ are assumed to be free of stresses, the boundary conditions of the problem are

$$\tilde{\tau}_{13}=\tilde{\tau}_{33}=0 \quad \text{at } x_3=\pm H, \tag{7}$$

where $\tilde{\tau}_{13}$ and $\tilde{\tau}_{33}$ are given by

$$\tilde{\tau}_{13}=\rho D_S\left(2\frac{\partial^2\phi}{\partial x_1\partial x_3}-\frac{\partial^2\psi}{\partial x_3^2}+\frac{\partial^2\psi}{\partial x_1^2}\right)$$
$$\tilde{\tau}_{33}=\rho D_T\nabla^2\phi+2\rho D_S\left[\frac{\partial^2\psi}{\partial x_1\partial x_3}-\frac{\partial^2\phi}{\partial x_1^2}\right] \tag{8}$$

Normal mode analysis.

Normal mode analysis is, in fact, to look for the solution in the Fourier transform domain. We assume that all the relations are sufficiently smooth on the real line such that normal mode analysis of these functions exists.

The solution of the considered physical variables involved in equation (6) can be decomposed in terms of normal modes as follows [22, 25, 26]

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$$(\phi,\psi)=\{\phi^*(x_3),\psi^*(x_3)\}\exp(\omega t+i\eta x_1) \tag{9}$$

where ω is the (complex) time constant, $i=\sqrt{-1}$ and η is the wave number in the x_1-direction and $\phi^*(x_3)$ and $\psi^*(x_3)$ are the amplitude of the functions.

Insertion of (9) in (6) gives

$$\begin{aligned}&\left(\frac{d^2}{dx_3^2}-\gamma_1^2\right)\phi^*+\frac{i\eta R_H}{D_1^*-P}\frac{d\psi^*}{dx_3}=0\\&\left(\frac{d^2}{dx_3^2}-\gamma_2^2\right)\psi^*-\frac{\eta^2R_H}{D_2^*-P}\psi^*=0\end{aligned} \tag{10}$$

where $R_H=\frac{c_A^2}{c_0^2}$, $\varepsilon_H=\frac{c^2}{c_0^2}$, $c_0=\sqrt{\frac{\mu_0}{\rho}}$, $P=\frac{p_0}{\rho c_0^2}=$ initial stress parameter, $c=\frac{\omega}{\eta}=$ phase velocity and

$$\gamma_1^2=\eta^2\left(1+\frac{\varepsilon_H}{D_1^*-P}\right),\gamma_2^2=\eta^2\left(1+\frac{\varepsilon_H}{D_2^*-P}\right),D_1^*=\frac{D_T^*}{c_0^2},D_2^*=\frac{D_S^*}{c_0^2} \tag{11}$$

in which

$$D_T^*=\frac{D_\lambda^*+2D_\mu^*}{\rho D_\eta^*},D_S^*=\frac{D_\mu^*}{\rho D_\eta^*},D_\lambda^*=\sum_{k=0}^{n}\lambda_k\omega^k,D_\mu^*=\sum_{k=0}^{n}\mu_k\omega^k,D_\eta^*=\sum_{k=0}^{n}\eta_k\omega^k$$

The general solutions of equations (10) may be taken as

$$\begin{aligned}&\phi^*=A\sinh(\zeta_1x_3)+B\cosh(\zeta_1x_3)+C\sinh(\zeta_2x_3)+D\cosh(\zeta_2x_3)\\&\psi^*=C_1\sinh(\zeta_2x_3)+D_1\cosh(\zeta_2x_3)\end{aligned} \tag{12}$$

Plugging (12) in (10) one obtains the following

$$C=\alpha_1D_1\ ,\quad D=\alpha_1C_1 \tag{13}$$

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where

$$\alpha_1 = \frac{-i\eta R_H}{D_1^* - P}\frac{\zeta_2}{\zeta_2^2 - \gamma_1^2} \tag{14}$$

and

$$\zeta_1^2 = \gamma_1^2, \;\; \zeta_2^2 = \eta^2\left(1 + \frac{V_H}{D_2^* - P}\right) \tag{15}$$

in which $V_H = \varepsilon_H + R_H$.

Employing the boundary conditions (7) we obtain

$$\begin{aligned}
&A\xi_1 p_1 - Bx_1 p_1 + C_1\xi_2 p_2 - D_1 x_2 p_2 = 0\\
&Ax_1 q_1 - B\xi_1 q_1 + C_1 x_2 q_2 - D_1\xi_2 q_2 = 0\\
&An_1 p_1 - Bl_1 p_1 + C_1 n_2 p_2 - D_1 l_2 p_2 = 0\\
&Al_1 q_1 - Bn_1 q_1 + C_1 l_2 q_2 - D_1 n_2 q_2 = 0
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
&\xi_1 = 0, \;\; \xi_2 = \left(\zeta_2^2 + \eta^2\right) - 2i\eta\alpha_1\zeta_2, \; x_1 = 2i\eta\zeta_1, \; x_2 = n_2 = l_1 = 0,\\
&n_1 = D_T^*\left(\zeta_1^2 - \eta^2\right) + 2D_S^*\eta^2, \; l_2 = -\left[D_T^*\left(\zeta_2^2 - \eta^2\right)\alpha_1 + 2D_S^*\eta\left(\eta\alpha_1 + i\zeta_2\right)\right]
\end{aligned} \tag{17}$$

and

$$p_j = \sinh\left(\zeta_j H\right), \quad q_j = \cosh\left(\zeta_j H\right), \qquad j = 1,2 \tag{18}$$

Elimination of indispensable constants A, B, C_1 and D_1 from (16) gives

$$\Delta = 0 \tag{19}$$

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where

$$\Delta = \begin{vmatrix} \xi_1 & x_1 & \xi_2 \dfrac{\tanh \zeta_2 H}{\tanh \zeta_1 H} & x_2 \dfrac{\tanh \zeta_2 H}{\tanh \zeta_1 H} \\ n_1 \dfrac{\tanh \zeta_1 H}{\tanh \zeta_2 H} & l_1 \dfrac{\tanh \zeta_1 H}{\tanh \zeta_2 H} & n_2 & l_2 \\ x_1 & \xi_1 & x_2 & \xi_2 \\ l_1 & n_1 & l_2 & n_2 \end{vmatrix}$$

On simplification equation (19) leads to the transcendental equation

$$\frac{\tanh \zeta_1 H}{\tanh \zeta_2 H} + \frac{\tanh \zeta_2 H}{\tanh \zeta_1 H} = \frac{x_1^2 l_2^2 + n_1^2 \xi_2^2}{x_1 n_1 l_2 \xi_2} \tag{20}$$

Equation (20) represents the wave velocity equation for waves propagated in a higher order magneto viscoelastic plate (including both stress rate and strain rate) under the initial state of hydrostatic stress. This equation contains $c\ (=\omega/\eta)$ and η as only unknown quantities and hence c can be expressed as a function of η indicating the dispersive nature of wave considered. The above frequency equation $\Delta = 0$ can be expressed as $\Delta_1 . \Delta_2 = 0$

where

$$\Delta_1 = \begin{vmatrix} x_1 \tanh \zeta_1 H & n_1 \\ \xi_2 \tanh \zeta_2 H & l_2 \end{vmatrix}$$

and

$$\Delta_2 = \begin{vmatrix} n_1 \tanh \zeta_1 H & x_1 \\ l_2 \tanh \zeta_2 H & \xi_2 \end{vmatrix}$$

Hence (19) implies either

$$\begin{vmatrix} x_1 \tanh \zeta_1 H & n_1 \\ \xi_2 \tanh \zeta_2 H & l_2 \end{vmatrix} = 0 \tag{21}$$

or

$$\begin{vmatrix} n_1 \tanh \zeta_1 H & x_1 \\ l_2 \tanh \zeta_2 H & \xi_2 \end{vmatrix} = 0 \tag{22}$$

Special cases:

(i) For symmetric displacement, the solutions of equations (10) may be taken as

$$\begin{aligned} \phi_1 &= \{B\cosh(\zeta_1 x_3) + D\cosh(\zeta_2 x_3)\}\exp[(\omega t + i\eta x_1)] \\ \psi_1 &= \{C_1 \sinh(\zeta_2 x_3)\}\exp[(\omega t + i\eta x_1)] \end{aligned} \tag{23}$$

It is verified from (2) that the displacement components u_1 and u_3 are symmetric with respect to the plane $x_3 = 0$.

Proceeding similarly as in the general case one obtains the wave velocity equation for symmetric vibration in the following form

$$\Delta_1 = \begin{vmatrix} x_1 \tanh \zeta_1 H & n_1 \\ \xi_2 \tanh \zeta_2 H & l_2 \end{vmatrix} = 0 \tag{24}$$

(ii) For antisymmetric displacement, we now consider another set of interesting solutions of (10) given by

$$\begin{aligned} \phi_2 &= \{A\sinh(\zeta_1 x_3) + C\sinh(\zeta_2 x_3)\}\exp[(\omega t + i\eta x_1)] \\ \psi_2 &= \{D_1 \cosh(\zeta_2 x_3)\}\exp[(\omega t + i\eta x_1)] \end{aligned} \tag{25}$$

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In this case also it is verified that the displacement components u_1 and u_3 are antisymmetric with respect to the plane $x_3 = 0$.

Proceeding similarly one obtains the following frequency equation

$$\Delta_2 = \begin{vmatrix} n_1 \tanh \zeta_1 H & x_1 \\ l_2 \tanh \zeta_2 H & \xi_2 \end{vmatrix} = 0 \tag{26}$$

In view of the above analysis, it is interesting to note that the wave velocity equation for waves propagated in a higher order viscoelastic plate under the influence of initial stress and magnetic field can be decomposed to two special cases one of which corresponds to symmetric displacements and the other corresponds to antisymmetric displacements.

We now discuss each of the above cases separately as follows:

Case A $(\Delta_1 = 0)$:

In this case we have

$$\frac{\tanh \zeta_1 H}{\tanh \zeta_2 H} = \frac{\xi_2 n_1}{x_1 l_2} \tag{27}$$

(i) If the length of the wave is large in comparison with the thickness 2H of the plate the hyperbolic tangents can be replaced by their arguments and the equation (27) becomes

$$\frac{\zeta_1}{\zeta_2} = \frac{\xi_2 n_1}{x_1 l_2} \tag{28}$$

The equation (28) determines the wave velocity of plane waves in a magneto viscoelastic initially stressed plate and may be considered as more generalized form of the result obtained by Rayleigh [27] and Lamb [28] for an elastic layer.

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(ii) If the length of the wave is very small in comparison with the thickness 2H of the plate we may assume that the ratios of the hyperbolic tangents in equation (27) approaches to unity and hence we have

$$x_1 l_2 - \xi_2 n_1 = 0 \tag{29}$$

Equation (29) determines the velocity of Rayleigh surface waves in a higher order viscoelastic initially stressed plate in presence of magnetic field.

In the absence of initial stress the equation (29) reduces to

$$2\sqrt{1+\frac{\varepsilon_H}{D_1^*}}\sqrt{1+\frac{V_H}{D_2^*}}\left[2\left\{1-\frac{R_H}{D_1^* D_R}\right\}-\frac{D_L R_H V_H}{D_1^* D_2^* D_R}\right]-\left[2+\frac{D_L \varepsilon_H}{D_1^*}\right]\left[2+\frac{V_H}{D_2^*}-\frac{2R_H\left(1+\frac{V_H}{D_2^*}\right)}{D_1^* D_R}\right]=0 \tag{30}$$

where $D_L = \dfrac{D_T^*}{D_S^*}$, $D_R = \dfrac{V_H}{D_2^*} - \dfrac{\varepsilon_H}{D_1^*}$.

It is noted that the result deduced from equation (30) for first order viscoelasticity (n=1) is in agreement with the corresponding result obtained by Das et al [9].

When the viscous effect and stress rate and strain rate are neglected the equation (30) transforms to

$$x_1 l_2 - \xi_2 n_1 = 0 \tag{31}$$

in which

$$x_1 = 2i\eta\zeta_1,\ l_2 = -\left[D_T^*\left(\zeta_2^2-\eta^2\right)\alpha_1 + 2D_S^*\eta\left(\eta\alpha_1 + i\zeta_2\right)\right]$$
$$n_1 = \left[D_T^*\left(\zeta_1^2-\eta^2\right)+2D_S^*\eta^2\right],\ \xi_2 = \left(\zeta_2^2+\eta^2\right)-2i\eta\alpha_1\zeta_2$$

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with

$$\zeta_1^2 = \eta^2\left(1+\frac{\varepsilon_H}{D_1^* - P}\right), \qquad \zeta_2^2 = \eta^2\left(1+\frac{V_H}{D_2^* - P}\right),$$

$$D_1^* = \frac{\lambda_0 + 2\mu_0}{\rho c_0^2}, D_2^* = \frac{\mu_0}{\rho c_0^2}, \alpha_1 = \frac{-i\eta R_H}{D_1^* - P}\frac{\zeta_2}{\zeta_2^2 - \gamma_1^2}$$

This equation represents magneto elastic Rayleigh wave velocity equation under the initial state of hydrostatic stress and is in agreement with the result obtained by Acharya and Sengupta [8].

In the absence of initial stress and magnetic field equation (29) reduces to

$$\left[2+\frac{D_L\varepsilon_H}{D_1^*}\right]\left[2+\frac{\varepsilon_H}{D_2^*}\right]-4\sqrt{1+\frac{\varepsilon_H}{D_1^*}}\sqrt{1+\frac{\varepsilon_H}{D_2^*}}=0 \tag{32}$$

in which D_1^* and D_2^* are given by (11).

Case B $(\Delta_2 = 0)$:

Equation (26) transforms to

$$\frac{\tanh\zeta_1 H}{\tanh\zeta_2 H} = \frac{x_1 l_2}{\xi_2 n_1} \tag{33}$$

(i) If the length of the wave is very large in comparison with the thickness of the plate, the hyperbolic tangents can be replaced by the first two terms of their expansion into series and hence the equation (33) becomes

$$\frac{\zeta_1}{\zeta_2} = \frac{x_1 l_2}{\xi_2 n_1} \tag{34}$$

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Equation (34) may be regarded as the generalized form of classical result obtained by Rayleigh and Lamb for elastic plate

(ii) If the length of the wave is very small in comparison with the thickness 2H of the plate the ratio of the hyperbolic tangents in equation (33) approaches to unity and hence reduces to (29) which determine the velocity of Rayleigh surface wave in a higher order viscoelastic initially stressed plate in presence of magnetic field.

Numerical Results.

For numerical calculations following assumptions are made:

(a) We consider the viscosity of the medium upto third order only i.e. $n = 1, 2$ and 3.

(b) Since ω is complex in general we write $\omega = \omega_0 + i\omega_1$. Therefore $e^{\omega t} = e^{\omega_0 t}(\cos\omega_1 t + i\sin\omega_1 t)$ and hence, for small values of time, we can take $\omega = \omega_0$ (real).

(c) The medium is made of Poisson's solid so that we assume $\lambda_0 = \mu_0$.

(d) Following Eringen [29] and Acharya and Mandal [30] the concept of Poisson's material is extended to include $\lambda_1 = \mu_1, \lambda_2 = \mu_2, \lambda_3 = \mu_3$.

In view of the above assumptions the frequency equation (24) transforms to

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$$\frac{\tan\eta H\sqrt{1+\frac{\varepsilon_H}{L}}}{\tan\eta H\sqrt{1+\frac{V_H}{T}}} = \frac{\left[2+\frac{3\varepsilon_H}{L}\right]\left[2+\frac{V_H}{T}-\frac{2R_H\left(1+\frac{V_H}{T}\right)}{L\left(\frac{V_H}{T}-\frac{\varepsilon_H}{L}\right)}\right]}{2\sqrt{1+\frac{\varepsilon_H}{L}}\sqrt{1+\frac{V_H}{T}}\left[2\left\{1-\frac{R_H}{L\left(\frac{V_H}{T}-\frac{\varepsilon_H}{L}\right)}\right\}-\frac{3R_H V_H}{LT\left(\frac{V_H}{T}-\frac{\varepsilon_H}{L}\right)}\right]} \quad (35)$$

where $L = D_1^* - P$, $T = D_2^* - P$.

The numerical constants of the problem are taken as $\omega_0 = 1$, $\eta H = 1$, $\frac{c_1^2}{c_0^2} = \frac{1}{50}$,

$\frac{c_2^2}{c_0^2} = \frac{1}{100}$, $\frac{c_3^2}{c_0^2} = \frac{1}{200}$, $\eta_0 = \eta_1 = \eta_2 = \eta_3 = 1$.

Using the numerical techniques outlined above the normalized wave velocity (c^2/c_0^2) is found out for different values of c_A^2/c_0^2 (square of the normalized Alfven wave velocity) and for different viscoelastic order $(n = 1,2,3)$. Computations are carried out with the help of commercially available software MathCAD-12. The frequency equation (35) gives infinite numbers of values of wave velocity c^2/c_0^2 for a particular value of c_A^2/c_0^2. Each of the figs 1-3 gives the variation of c^2/c_0^2 against c_A^2/c_0^2, in the range $0.4 < c_A^2/c_0^2 < 1.8$, for different viscoelastic order $(n = 1,2,3)$ and initial stress parameter($P = -0.2$, $P = 0$, $P = 0.2$), always selecting minimum values of c^2/c_0^2. The ranges of c_A^2/c_0^2 for minimum values of c^2/c_0^2 are different for different viscoelastic order irrespective of the presence or absence of the initial stress. The curves of these figures are drawn for comparison. Contour plots (figs 4-6), indicated by continuous lines with discreet zeros $(-0-0-)$ are depicted to highlight a few values of c^2/c_0^2 for a particular value of c_A^2/c_0^2 for which the equation (35) is

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satisfied. Though the figures are self explanatory yet we point out some of the peculiarities of the figs 1-3. For any viscoelastic order c^2/c_0^2 diminishes continuously as c_A^2/c_0^2 increases. It is observed from fig 1 and fig 2 that c^2/c_0^2 diminishes with the increase of viscoelastic order. However exceptions are found out in fig 3 which corresponds to the compressive initial stress, $P = 0.2$. Here no such conclusion could be made. In this case curves for $n = 1$ and $n = 3$ intersect which indicates that there exists a particular value of c_A^2/c_0^2 for which same value of wave velocity is obtained for $n = 1$ and $n = 3$.

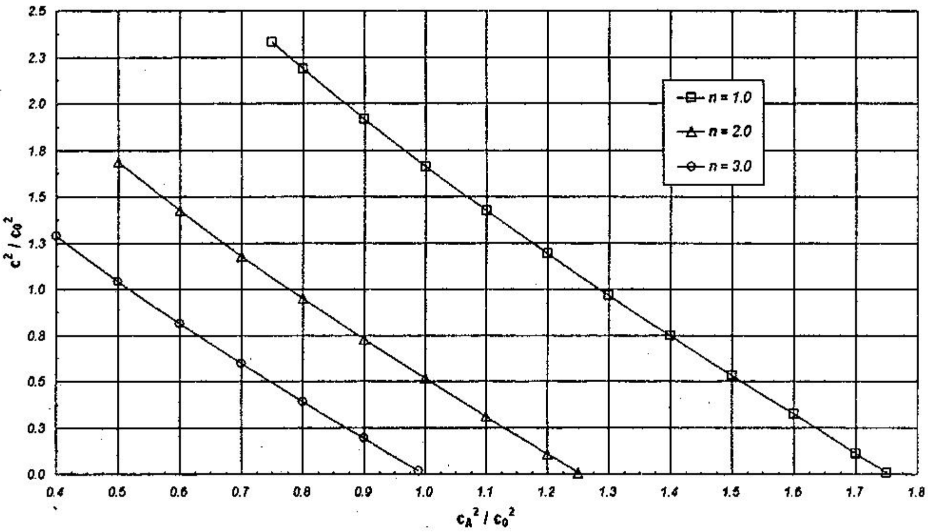

Fig1. Variation of c^2/c_0^2 versus c_A^2/c_0^2 for different viscoelastic order (n=1,2,3) when initial stress parameter $P = -0.2$

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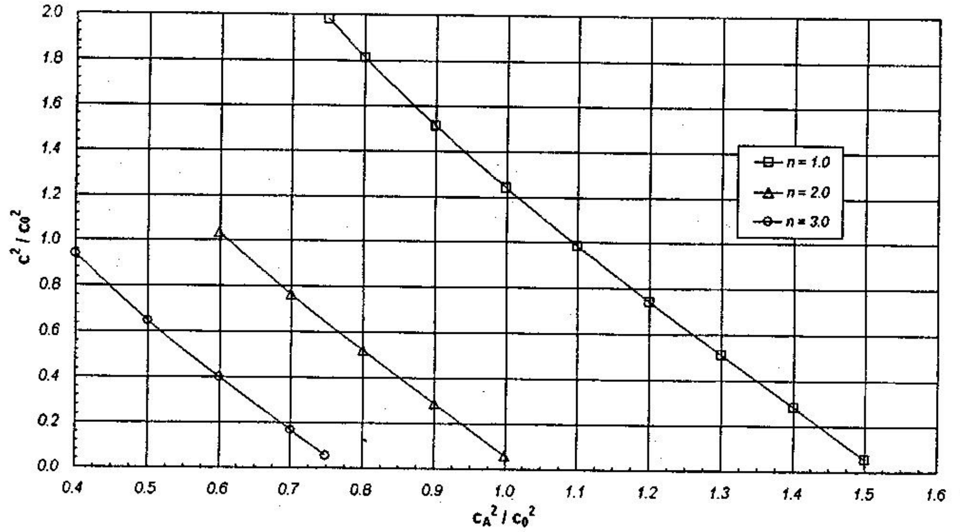

Fig.2 Variation of c^2/c_0^2 versus c_A^2/c_0^2 for different viscoelastic order (n=1,2,3) when initial stress parameter P=0

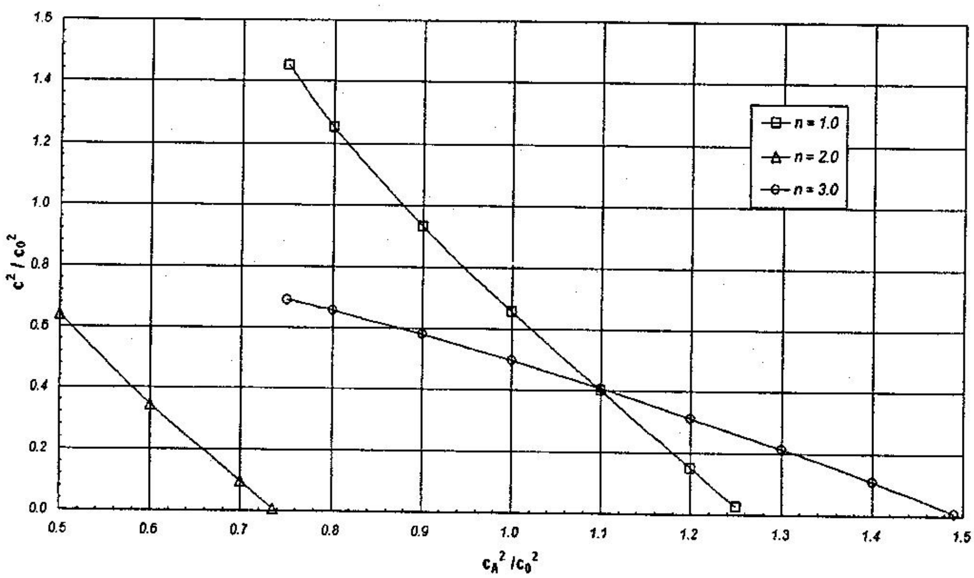

Fig3. Variation of c^2/c_0^2 versus c_A^2/c_0^2 for different viscoelastic order (n=1,2,3) when initial stress parameter $P = 0.2$

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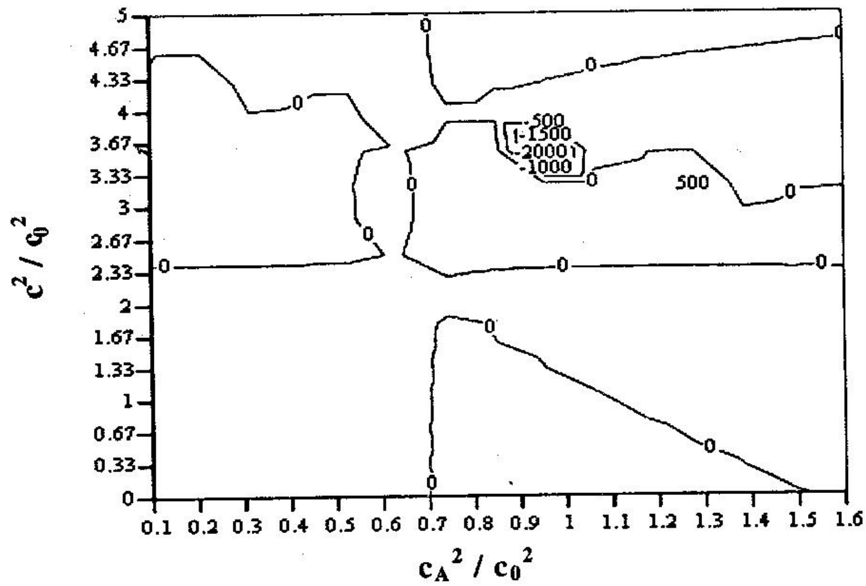

Fig. 4 Contour plot of the equation for $P = 0$, ηH=1 and n=1, where the lines indicate zero (0) value are possible solution

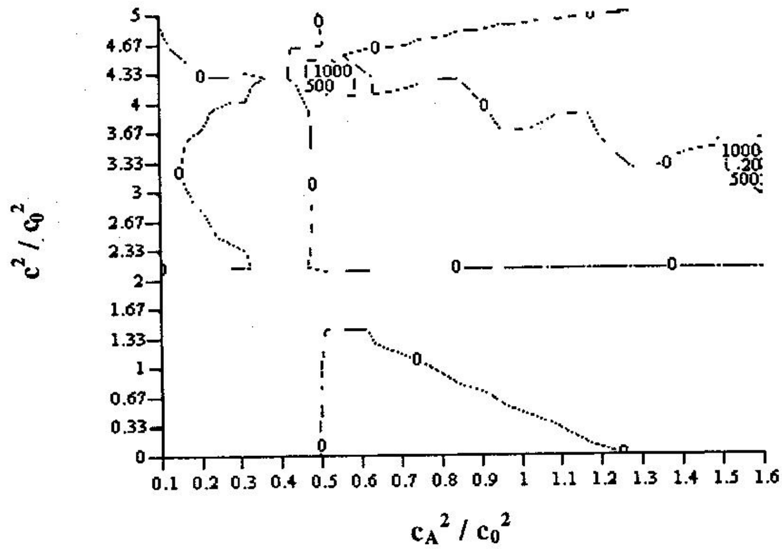

Fig. 5 Contour plot of the equation for $P = -0.2$, ηH=1 and n=2, where the lines indicate zero (0) value are possible solution

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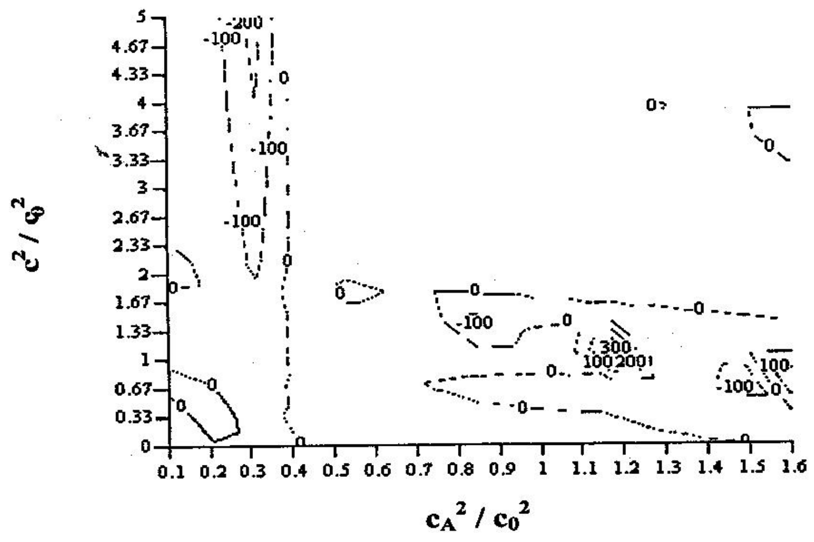

Fig. 6 Contour plot of the equation for $P = 0.2$, $\eta H=1$ and $n=3$, where the lines indicate zero (0) value are possible solution

Conclusions.

From the analysis presented in this paper following concluding remarks are :

(i) Due to the complicated nature of the governing equations for generalized magneto viscoelastic problem with initial stress a few attempts have been made to solve the problem in this field. The present attempt utilized an approximate method and contour plot that is valid only for a specified range of some parameters.

(ii) Normal mode analysis introduced in this paper also shows the dispersive nature of the wave as that of classical case.

(iii) Magnetic field, initial stress and the viscoelastic character of the medium modulate the velocity of the waves propagated in a plate to a considerable extent. Further modulation of wave velocity occurs due to their combined effect.

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